CHAPTER 26

INTERMEDIATE FREQUENCY AMPLIFIERS

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SECTION 1 : CHOICE OF FREQUENCY

(i) Reasons for selection of different frequencies (ii) Commonly accepted intermediate frequencies.

(i) Reasons for selection of different frequencies

The choice of a particular intermediate frequency in a receiver is governed by a number of factors.

(a) If the frequency is very low the circuits will be generally too selective, resulting in side-band cutting.   Very high selectivity will also make a receiver more difficult to tune, and imposes severe requirements on oscillator, and other circuit, stability.

(b) The lower the intermediate frequency the more difficult it becomes to eliminate image interference.   This difficulty becomes greater as the carrier frequency is increased.

(c) The intermediate frequency should not fall within the tuning range of the receiver, as this would lead to instability and severe heterodyne interference. Also lower harmonics (principally second and third) of the i-f should not fall within the tuning range if this can be avoided.   This requirement is not always easy to meet because of the other factors discussed in this section.

(d) Too high a value of intermediate frequency should be avoided as it generally leads to a serious reduction in selectivity and, usually, gain.

(e) The intermediate frequency should not be such that it approaches the range of frequencies over which the receiver is required to tune.

(f) If the intermediate frequency is made too high, tracking difficulties between the signal and oscillator circuits may be experienced.

(g) In some cases, where the intermediate frequency is higher than the highest received signal frequency (as in single span receivers), it is possible to tune the oscillator only and use low-pass filters for the signal circuits.   However, it is generally preferable to use tunable signal circuits.

(ii) Commonly accepted intermediate frequencies

As a result of the experience gained over a number of years in addition to the considerations stated previously the values selected for the intermediate frequencies of most commercial receivers have become fairly well standardized. For the majority of broadcast receivers tuning the bands 540-1600 Kc/s and 6-18 Mc/s, an i-f of about 455 Kc/s is usual. A frequency of 110 Kc/s has been extensively used in Europe where the long wave band of 150-350 Kc/s is in operation. Receivers for use only on the short wave bands commonly employ an i-f of 1600 Kc/s or higher. Frequency modulation receivers covering the 40-50 Mc/s band generally use a 4.3 Mc/s i-f, and for the 88-108 Mc/s band they use 10.7 Mc/s. This latter value has been adopted as standard in U.S.A., and some other countries, for v-h-f receivers.

SECTION 2 : NUMBER OF STAGES

The number of stages required in the i-f amplifier is generally a compromise between the factors of selectivity, gain, and cost. For the usual broadcast receiver having a 455 Kc/s i-f, one stage employing two transformers is generally considered as being adequate (when using an i-f valve with a g_m of about 2000 micromhos). The transformers in common use are two parallel tuned circuits coupled by mutual inductance and stray capacitance. The windings are arranged so that any capacitive coupling aids the mutual inductance. Care should be taken to keep capacitive coupling small as it alters the symmetry of the overall response curve of frequency versus attenuation.

Common values of capacitance tuning the primary and secondary windings in commercial transformers are from about 50 $\mu\mu$F to 120 $\mu\mu$F to which must be added valve and circuit stray capacitances. Values as high as 800 $\mu\mu$F are used in special circumstances, however. This point will receive further discussion in Section 7 of this chapter. Unloaded values for the primary and secondary Q's are from about 70 to 130. The transformer windings are in most cases coupled from 0.8 of critical to critical. The amount of coupling when the transformer is wired into the receiver is the important factor as stray coupling (including regeneration and alteration of tuning slug positions) will often alter any value of k or conditions for critical k measured external to the receiver.

It should be noted particularly that the second i-f transformer usually feeds into a diode detector and this will appreciably affect the selectivity and gain of the preceding stage because of the loading across the transformer secondary. This loading is commonly taken as half the d.c. diode load resistance, which in most cases does not exceed 0.5 megohm because of the detector circuit requirements. If improved selectivity is required then the secondary of the i-f transformer may be tapped and the diode circuit fed from this tapping point to reduce loading on the tuned circuit ; but this generally involves a loss in stage gain.

For small battery receivers of the portable type it may be advantageous to employ three i-f transformers (i.e. two stages) rather than a r-f stage and one i-f stage. Although there is some reduction in signal-to-noise ratio with this arrangement, additional gain is often the main requirement. Further, it is possible to operate the two i-f valves in a very economical condition, e.g. by using reduced screen voltage, as the full gain available can seldom be utilized without difficulties arising from instability. The Q's of the i-f's need not be more than 60 or 70 with this arrangement as the selectivity is more than adequate for ordinary reception.

With receivers using higher intermediate frequencies at least two stages (three transformers) are often required to give improved selectivity. Another important consideration, however, is that the dynamic impedance of the load presented by the transformer to the valve is lower, and stage gain is decreased. Valves used in the i-f stages of receivers having a high value of i-f generally have a mutual conductance

(g_m) of the order of 4000 micromhos to allow additional gain to be obtained to offset the loss due to the lower circuit dynamic impedance. These conditions apply particularly in F-M receivers using an i-f of 10.7 Mc/s. Short wave receivers using 1600 Kc/s i-f transformers commonly employ two stages (3 transformers) although one stage is often used and generally, but not always, the valves are similar to those used at lower frequencies and have a g_m of about 2000 micromhos or less. The passband and gain requirements will largely determine the values of the constants for the transformers.

A 10.7 Mc/s transformer in a F-M receiver has to pass a band of frequencies about 240 Kc/s wide and at the same time must not introduce such an appreciable amount of amplitude modulation (due to the selectivity of the transformers) that the limiter (or whatever device is used that is insensitive to amplitude variations) cannot give substantially constant output. To secure these results, some designers use combinations of overcoupled and critically-coupled transformers, while others prefer to use only critically-coupled transformers because of the simplification in the alignment procedure. For the type of F-M receiver using a ratio detector, two i-f stages are generally considered as being adequate (although this may lead to difficulties because of insufficient adjacent channel selectivity) with the second i-f valve feeding into a discriminator transformer. For most F-M receivers using limiters, three 10.7 Mc/s i-f transformers are used in cases where one limiter stage is included ; or for two limiter stages an additional very wide band i-f transformer may be incorporated. The purpose of the additional i-f transformer is to provide interstage coupling, but at the same time the design must be such as to introduce no appreciable amplitude modulation of the signal after it has passed through the first limiter stage. If a locked oscillator type of F-M detector (such as the Bradley circuit) is employed, then it is common practice to use three normal i-f transformers plus a fourth transformer giving a voltage step down of about 6 to 1 to provide a low impedance voltage source for driving the detector. An additional i-f stage is included in receivers using limiters for the purpose of giving the greater gain required to obtain satisfactory amplitude limiting with very weak signals ; the additional i-f stage also provides improved selectivity outside the required passband.

The number, and type, of stages used in any i-f amplifier is decided, of course, by the various requirements of the particular receiver and the selectivity and gain are under the control of the designer. Discussion as to the number of i-f stages used in typical A-M and F-M broadcast receivers is meant only to serve as a guide to common practice. However, it should not be overlooked that although gain and pass band requirements can sometimes be met with fewer stages than suggested above, adjacent channel selectivity will generally call for additional transformers. It is important that both pass band and adjacent channel selectivity be considered during the design, and if this is done there is usually little difficulty in selecting the number of i-f stages required.

Special i-f requirements can be fulfilled by making preliminary calculations for any number of stages which a designer may consider necessary. In Section 4 it is proposed to carry out the complete design of several transformers showing the various factors to which attention must be paid to meet a given set of conditions.

SECTION 3 : COMMONLY USED CIRCUITS

(i) *Mutual inductance coupling* (ii) *Shunt capacitance coupling* (iii) *Composite i-f transformers.*

In this section it is proposed to confine attention, particularly, to the most commonly used i-f transformer arrangement which uses mutual inductance coupling. Shunt capacitance coupling is not very widely used, but is included here as being of some interest because of its occasional application in F-M receivers. Details of various coupled circuit arrangements are given in Chapter 9 ; the references should be consulted for a more comprehensive survey of circuit arrangements, and analysis of their

properties. Of course, almost any type of coupling can be used in i-f circuits but transformers other than those using two windings coupled by mutual inductance are the exception rather than the rule in ordinary broadcast and communications receivers.

(i) Mutual inductance coupling

Fig. 26.1 shows the most widely used circuit arrangement for an i-f coupling transformer ; C_1, L_1 and C_2, L_2 are the primary and secondary capacitances and inductances respectively. The two windings are coupled together by mutual inductance. The total capacitances tuning the circuits are due to valve input and output, wiring and coil capacitances as well as C_1 and C_2.

The transformer is set to the required i-f by using powdered iron " slugs " which are moved inside the primary and secondary windings to vary the inductance values. Variable capacitance trimmers are often used as an alternative, and in some cases this may be advantageous since the inductance values can be pre-set fairly accurately and the capacitance, which is not known accurately because of additional strays, can be set to give the required resonant frequency ; this arrangement also allows close control on the coefficient of coupling. However, capacitance trimmers are not always completely reliable and it is often of greater practical convenience to use variable iron cores in transformer windings. Further, the fixed capacitors C_1 and C_2 can be of high quality (e.g. silvered mica) to improve the circuit stability.
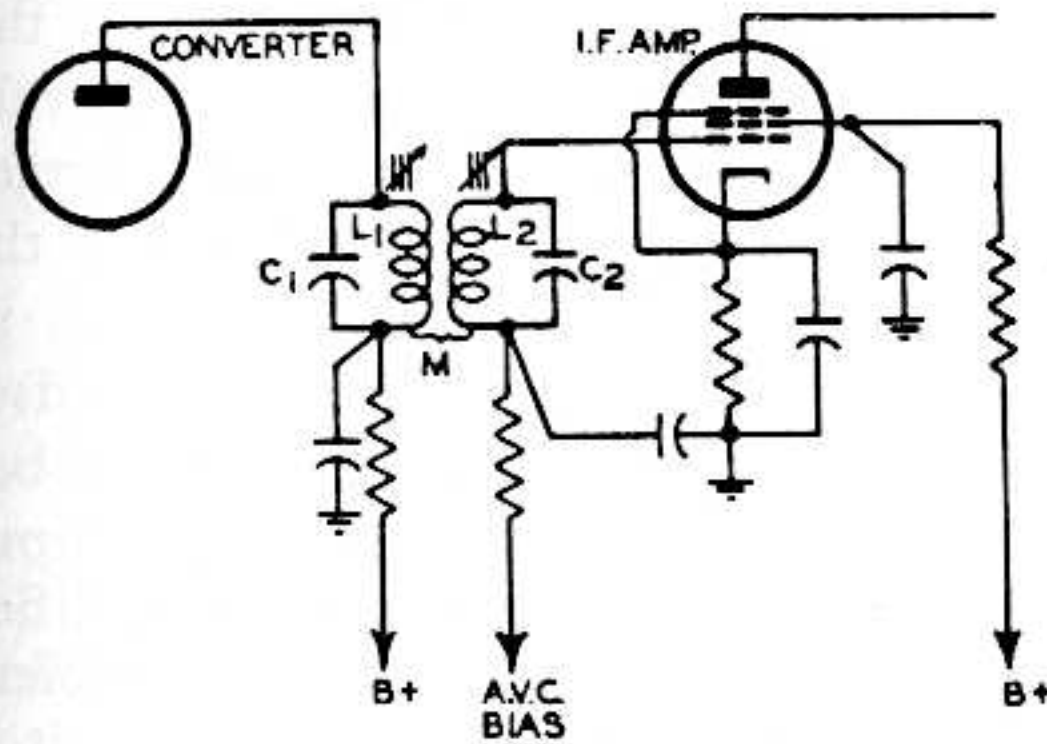

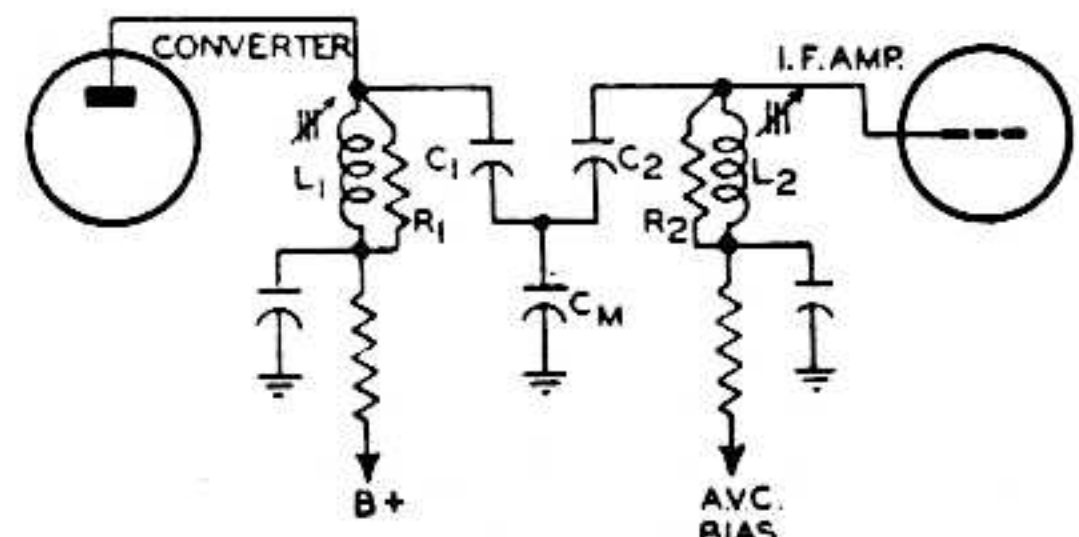FIG. 26·1   TYPICAL TRANSFORMER COUPLED I.F. STAGE

FIG. 26·2   SHUNT CAPACITANCE COUPLED I.F. CIRCUIT

The type of circuit being considered is discussed also in Chapter 9 Sect. 6(iii), (iv), (v). Graphical methods of determining selectivity are given in Chapter 9 Sect. 10.

Section 4 below gives detailed design methods and examples for several transformers of this type.

(ii) Shunt capacitance coupling

This type of coupling arrangement is illustrated in Fig. 26.2. Circuits of this type can be conveniently arranged to give a fairly wide pass band and are sometimes used in F-M receivers, in particular, to couple two cascaded limiter stages. Some discussion of this type of circuit is given also in Chapter 9 Sect. 8(ii).

The design of circuits of this type is carried out in a similar manner to those using mutual inductance coupling. Usually the values selected for C_1 and C_2 are about 50 $\mu\mu$F and the value for C_M is determined by the required coefficient of coupling However, C_M is generally fairly large, being of the order of 1000 to 2000 $\mu\mu$F in typical cases. L_1 and L_2 (which usually have equal values) are determined from :

$$L_1 = L_2 = 25\,330/f^2 C$$

where L_1 is in microhenrys

C is in $\mu\mu$F and is equal to $\dfrac{C_1 C_M}{C_1 + C_M} \left(\text{or } \dfrac{C_2 C_M}{C_2 + C_M}\right)$ + stray capacitance across the primary circuit

and f is in Mc/s, and is taken as the intermediate frequency.

The resistors R_1 and R_2 are for the purpose of lowering the Q of the tuned circuits to the values required for the bandwidth desired.

The coefficient of coupling is given by

$$k = \sqrt{\frac{C_1 C_2}{(C_1 + C_M)(C_2 + C_M)}}$$

$$\approx \frac{\sqrt{C_1 C_2}}{C_M} \qquad \text{when } C_M \gg (C_1, C_2).$$

(iii) Composite i-f transformers

Receivers used for F-M and A-M reception, on the 88-108 Mc/s and the 540-1600 Kc/s bands respectively, generally have the i-f amplifiers arranged so that the intermediate frequency is 10.7 Mc/s for the F-M band and 455 Kc/s for the A-M range. For reasons of economy many manufacturers use the same amplifier valves for both i-f's, and so it is necessary to find a solution to this problem which does not require elaborate circuit arrangements.
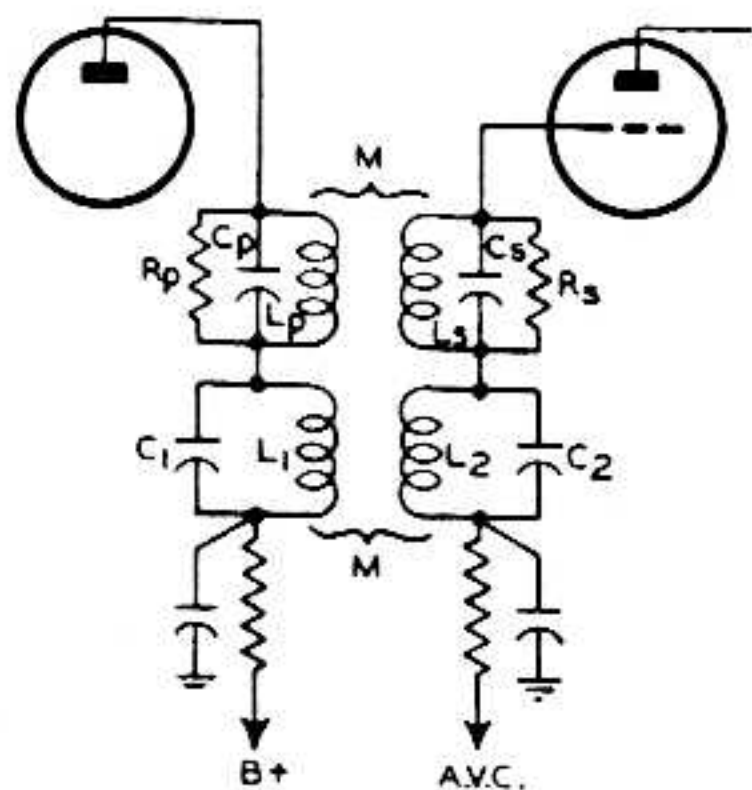

FIG. 26-3   ARRANGEMENT FOR F-M AND A-M I-F STAGE

One solution is the circuit shown in Fig. 26.3. The 10.7 Mc/s transformer is represented by C_p, L_p etc. and the 455 Kc/s transformers by L_1, C_1 etc. When the carrier output from the converter stage is 10.7 Mc/s, the primary and secondary circuits of the 455 Kc/s i-f act as capacitances which allow the lower ends of the 10.7 Mc/s i-f to be effectively earthed. At 455 Kc/s the 10.7 Mc/s i-f transformer tuned circuits act as inductances in series with the transformers in use. The effective inductance, in series with the 455 Kc/s transformer, tends to offset any loss in gain due to the voltage drop across it because it reduces the loading on the valve input circuit. Any possible loss in gain is not important, however, as the g_m of the amplifier valves is about 4000 micromhos, and there are usually two i-f stages. The problem is generally one of getting rid of excess gain on the lower frequency channel; it is for this reason that very low L/C ratios are used in the 455 Kc/s i-f transformers, in receivers for this purpose, and the primary and secondary capacitances are often of the order of 750 $\mu\mu$F. This system of gain reduction is sometimes preferred to tapping down on 455 Kc/s i-f transformers using the more conventional component values as discussed in Sect. 2.

In receivers using a common converter, for F-M and A-M, it is usual to short out whichever transformer is not in use. It is not considered necessary, in most cases, to use switching with all i-f stages. Receivers adopting the method of using separate converters for improved oscillator stability on the v-h-f band, and a common i-f channel, do not require any switching in the i-f amplifier; the converter not in use having the high tension removed. Switching of detector circuits is also unnecessary; the last 455 Kc/s A-M transformer can be connected directly to a diode in the first a-f voltage amplifier valve, and a separate double diode, connected to the discriminator transformer, is used for detection in the F-M channel after F-M to A-M conversion has been carried out. Some valves, such as type 6T8, incorporate three diodes, a triode, and two separate cathodes, and so will allow all the detection and first a-f voltage amplification functions to be performed with one valve without switching being required. An alternative arrangement, common in receivers using ratio detectors, is to use the grid-cathode circuit of the last i-f valve (the driver stage) as a diode for A-M detection. When the receiver has a.v.c. applied to the driver stage, it is possible to select suitable values for the a.v.c. load resistor and the i-f by-pass capacitor to provide a satisfactory R-C combination for A-M detection.

The two transformers may have all the windings in one can, or two cans may be used for each i-f, one for the 10.7 Mc/s i-f, and the other for the 455 Kc/s i-f. This latter arrangement has the advantage that it reduces interaction between the windings,

and also permits both transformers to be "slug" tuned without using elaborate mechanical arrangements. In some cases where the transformers are both mounted in one can, combinations of "slug" and capacitance adjustment are provided; an example of this is illustrated in Ref 12.

Many arrangements have been suggested for combined i-f transformers, and some of these are described in Ref. 22. However, the circuit shown, or modifications of it, has received fairly wide acceptance as providing quite a good solution to the problem.

SECTION 4 : DESIGN METHODS

(*i*) *General*
(*ii*) *Critically-coupled transformers*
      (*A*) *Design equations and table*   (*B*) *Example*
      (*C*) *Design extension*   (*D*) *Conclusions*
      (*E*) *k measurement*
(*iii*) *Over-coupled transformers*
      (*A*) *Design equations and table*   (*B*) *Example*
      (*C*) *k measurement (when k is high)*
(*iv*) *Under-coupled transformers and single tuned circuits*
      (*A*) *Single tuned circuit equations*   (*B*) *Example*
      (*C*) *Under-coupled transformer equations*
      (*D*) *Example*
(*v*) *F-M i-f transformers*
      (*A*) *Design data*   (*B*) *Example*
(*vi*) *I-F transformer construction*
(*vii*) *Appendix : Calculation of coupling coefficients.*

(i) General

The design procedure for i-f transformers can be greatly simplified by the use of charts and tables. If certain assumptions are made, which approximate to practical conditions, the design procedure can be reduced to a few routine operations. Here we will consider only the two winding transformer using mutual inductance coupling; the added capacitance coupling, which is always present, does not seriously affect the results particularly as its presence is taken into account when setting the coefficient of coupling (k).

The methods given below can be applied to inductive or capacitive coupling provided that the coefficient of coupling (k) is suitably interpreted [see appendix (vii) below for methods of calculating coupling coefficients]. The practical two winding transformer, as previously mentioned, has mixed coupling but this does not cause any difficulty when the two forms of coupling assist each other [suitable connections are given in (vi) below]. However, if the two forms of coupling are in opposition a rejection frequency is possible which can occur within the pass band of the transformer. This effect is well known to receiver designers who have accidentally reversed the connections of one of the i-f transformer windings, and found that the i-f stage gives practically no gain. This fact can also be made the basis of a useful method for measuring k, the capacitive coupling being increased (using a calibrated capacitor) until it equals the inductive coupling and zero voltage transfer then occurs at the working frequency. For further data on mixed coupling see Ref. 4.

The initial assumptions will be that the primary and secondary inductances L_1 and L_2 are given by $L = \sqrt{L_1 L_2}$. Also, it will be assumed that the values of Q do not alter appreciably over the range in which the selectivity curves are taken. We will **not** always take the primary and secondary Q's as being equal, and the advantages

to be gained will become clear as we proceed. The magnification factor, or Q, will be defined as

$$Q = \sqrt{Q_1 Q_2}$$

and provided the ratio of Q_1/Q_2 or Q_2/Q_1 is not greater than 2, the error in the usual design equations is negligible for most practical purposes.

As long as $\sqrt{Q_1 Q_2} \approx (Q_1 + Q_2)/2$ the error in any of the usual approximate design equations will be small. If Q_1 and Q_2 differ by large amounts then the exact design equations are necessary and can be obtained from Refs. 2, 3, 6 and 8, or the design can be modified by using the universal selectivity curves to obtain the required results. It is of interest to note that the simplified equations given by Kelly Johnson (Ref. 1), Ross (Ref. 2) and Maynard (Ref. 6 and Figs. 9.17 and 9.18 of this book) are identical when it is assumed that $Q = Q_1 = Q_2$ and the various notations are made the same.

It may be thought that writing $Q = \sqrt{Q_1 Q_2}$ and $L = \sqrt{L_1 L_2}$ will be inconvenient since the i-f transformer, as constructed, will have its primary and secondary inductances and Q's equal. However, in the majority of cases $L = L_1 = L_2$ is applied, and the method is extended to fulfil the condition that the unloaded primary and secondary Q's should be equal while allowing the required $Q = \sqrt{Q_1 Q_2}$ to be obtained in the receiver, without further adjustment.

Critical-coupling, or a close approach to it, is most often employed in i-f transformers but there is little difficulty in designing transformers for almost any degree of coupling. All cases will be treated.

Universal selectivity and phase shift curves are given in Chapter 9, Sect. 10 (Figs. 9.17 and 9.18). Additional charts and tables are given, to be used as described in the appropriate sections.

The design procedure generally consists of finding values of Q, k and stage gain for given bandwidths at some value of i-f; or of finding the required bandwidth for values of Q and k previously determined. For clarity the cases of critical-, over-, and under-coupled transformers will be dealt with separately. Single tuned circuits are also included as they are sometimes required in i-f amplifiers. Additional data for the design of F-M transformers will be given in Sect. 4(v).

Stagger tuning (Refs. 8, 13, 17 and 38) of i-f transformers (e.g. tuning primary and secondary to different frequencies) to give substantially the same bandwidths as over-coupled transformers, does not have a very wide application in F-M and A-M receivers (except in cases where variable selectivity is to be used) and will not be discussed in detail.

Stagger tuning of single and double tuned circuits is widely used in television receivers, but this is a different application from the case where it is applied to the comparatively narrow bandwidths of ordinary sound receivers. In this case the transformer primary and secondary are tuned to the same frequency, but this is not necessarily the intermediate frequency.

·Since i-f transformers for television receivers involve special problems they are not treated here (see Refs. 27, 28, 94, 95, 96 and 97) Also, triple tuned transformers are not discussed, but the article of Ref. 26 gives an excellent treatment.

Finally, the design methods do not make allowance for regenerative effects, nor should they be applied for finding the shape of resonance curves at frequencies far removed from resonance. The selectivity curve shapes are assumed to be perfectly symmetrical although in practice it will be found that this is seldom true.

(ii) Critically-coupled transformers

(A) Design equations and table

In general the procedure given is similar to that due to Kelly Johnson (Ref. 1) and Ross (Ref. 2). However, these procedures assume that $Q = Q_1 = Q_2$; we shall take $Q = \sqrt{Q_1 Q_2}$. For most cases it is not advisable to use the equations given below for Q_1/Q_2 or Q_2/Q_1 greater than about 2, unless some additional adjustment is made from the universal selectivity and phase shift curves.

For N critically-coupled transformers,

$$\rho = \left(1 + \frac{X^4}{4}\right)^{N/2} \qquad (1)$$

$$X = \sqrt{2}\,(\rho^{2/N} - 1)^{\frac{1}{4}} \qquad (2)$$

$$Q = \frac{X f_0}{2\Delta f} \qquad (3)$$

and $\quad \theta = \tan^{-1}\dfrac{2X}{2 - X^2} \qquad (4)$

where ρ = attenuation at Δf c/s off resonance

N = number of identical transformers used ($N = 1$ for one transformer)

f_0 = resonant frequency (the i-f in our case)

$Q = \sqrt{Q_1 Q_2} = 1/k_c$ (in which Q_1 and Q_2 are actual primary and secondary Q's ; k_c is critical-coupling coefficient)

$2\Delta f$ = total bandwidth for a given attenuation (ρ)

and $\quad \theta$ = phase shift between secondary current at resonance and secondary current at Δf c/s off resonance.

The required design information is given in equations (1) to (4). Usually either ρ is stated for a given bandwidth and a known i-f, or X can be found to allow ρ to be determined. Once these two factors have been found, the determination of k and/or Q is a simple matter.

Table 1 lists various values of ρ and X. Suppose ρ is known, then X is read from the table and used in equation (3) to find Q (since $2\Delta f/f_0$ is already known). The coefficient of coupling is then $k_c = 1/Q$.

If complete resonance and/or phase shift curves are required, then the universal curves of Figs. 9.17 and 9.18 (Chapter 9, Sect. 10) are used. These curves apply to one transformer only. For N identical transformers the attenuation in decibels is multiplied by N ; for transformers which are not identical, the individual attenuations (in db) are added. Resonance and phase shift curves can also be determined directly, by using table 1 and eqns. (1) to (4).

In the application of the universal curves take $D = X = Q2\Delta f/f_0$ (this Q being $\sqrt{Q_1 Q_2}$ as determined) and $b = k/k_c = Qk$ (for critical coupling $b = 1$, in this case) ; which are the same as for $Q_1 = Q_2$. If the values of Q_1 and Q_2 differ by more than about 2 to 1, then the more exact expressions for D and b^2 are applied to check how closely the required conditions are approached, it being carefully noted that in all expressions on the curves involving a that the Q shown is Q_2.

TABLE 1   CRITICALLY-COUPLED TRANSFORMERS
For use with equations (1), (2) and (3)

N = Number of Transformers

Attenuation (ρ)		$N = 1$	$N = 2$	$N = 3$
Times Down	db Down	X	X	X
$\sqrt{2}$	3	1.41	1.14	1.01
2	6	1.86	1.41	1.25
4	12	2.76	1.86	1.57
7	17	3.73	2.21	1.81
10	20	4.46	2.46	1.95
20	26	6.32	2.96	2.26
40	32	8.96	3.54	2.56
70	37	11.9	4.08	2.82
100	40	14.1	4.46	3.02
1000	60	---	7.96	4.46
10 000	80	---	14.1	6.66

The maximum stage gain is given by

$$\text{Gain} = g_m Q \omega_0 L / 2 \tag{5}$$

where g_m = mutual conductance of i-f valve (if conversion gain is required, conversion conductance (g_c) is substituted for g_m)

ω_0 = $2\pi \times$ resonant frequency (f_0) i.e. $f_0 = i\text{-}f$

$L = \sqrt{L_1 L_2}$; L_1 and L_2 are primary and secondary inductances

and $Q = \sqrt{Q_1 Q_2}$; Q_1 and Q_2 are primary and secondary magnification factors.

A condition, not specifically stated in the equations, is that $L_1 C_1 = L_2 C_2$ in all cases.

The maximum gain is usually converted to decibels, so that the gain at any point on the resonance curve can be found by subtraction of the attenuation, also expressed in decibels.

(B) Example and additional design extension

A 455 Kc/s i-f transformer, using critical-coupling, is required to give a total bandwidth of 20 Kc/s for an attenuation of 20 db (10 times).

(a) $f_0/2\Delta f = 455/20 = 22.75$.

(b) From table 1 we have $X = 4.46$ (since $N = 1$).

(c) From eqn. (3), $Q = 4.46 \times 22.75 = 101$.

(d) $k_c = 1/Q = 0.0099$.

(e) Select a suitable value for C_1 ($= C_2$); a capacitance of 100 $\mu\mu$F is satisfactory (made up of fixed + stray capacitances).

Then $L = \dfrac{25.33}{f^2 C} = \dfrac{25.33}{0.455^2 \times 100} = 1.22$ mH

(f in Mc/s; C in $\mu\mu$F)

and take $L = L_1 = L_2$ since this is convenient in this case.

(f) The i-f valve (e.g. type 6SK7) has $g_m = 2$ mA/volt ($= 2000$ μmhos). From equation (5),

Max. stage gain $= \dfrac{2 \times 10^{-3} \times 101 \times 2\pi \times 455 \times 10^3 \times 1.22 \times 10^{-3}}{2}$

$= 352$ times (or 51 db).

(g) Some designs might stop here and the magnification factor would be taken as $Q = Q_1 = Q_2 = 101$. It would be realized that valve loading would have an effect although possibly nothing more would be done (or else some attempt would be made to allow for the plate and grid resistances by finding new values of Q_1 and Q_2).

Let us proceed further and ask whether the transformer as it stands fulfils the design conditions in a radio receiver. The answer is that obviously it does not, since it would be connected in most cases between two i-f valves, a converter and i-f valve or between an i-f valve and a diode detector. Suppose the connection between two i-f amplifier valves (type 6SK7 would be representative) is considered since this appears a fairly innocuous case. The plate resistance (r_p) of a type 6SK7 under the usual conditions of operation is 0.8 megohm. The short circuit input resistance, also under one set of operating conditions, is 6.8 megohms (this is calculated from the data given in Chapter 23, Sect. 5); other effects, which would alter this value, will be ignored for simplicity, although they may not be negligible. It is first required to determine what values of Q_1 and Q_2 are required to give $Q = Q_1 = Q_2 = 101$.

This is found from

$$Q_u = \frac{QR}{R - Q\omega_0 L} \tag{6}$$

where Q_u = unloaded Q

Q = loaded Q

R = parallel resistance across winding

ω_0 = $2\pi f_0$; (resonant frequency $= f_0$)

and L = inductance.

For the primary
$$Q = 101 \; ; \; R = r_p = 0.8 \text{ M}\Omega \; ;$$
$$\omega_0 = 2\pi \times 455 \times 10^3 \; ; \; L = L_1 = 1.22 \text{ mH}$$
and $Q\omega_0 L = 101 \times 2\pi \times 455 \times 10^3 \times 1.22 \times 10^{-3} = 0.352 \text{ M}\Omega$.

Then $Q_u = \dfrac{101 \times 0.8}{0.8 - 0.352} = 180.$

For the secondary
$$Q_u = \frac{101 \times 6.8}{6.8 - 0.352} = 106.8.$$

(C) Design extension

The value $Q_u = 180$ could not be obtained very easily, if at all, with a normal type of i-f transformer. In addition, the disadvantage of unequal primary and secondary Q's should be apparent. For values of Q only about 10% higher than that given, or where the transformer is coupled to a diode detector, the situation becomes so much worse that it is clear that a revised approach is necessary. What is actually needed is

(1) A transformer with equal values of primary and secondary Q's when unloaded. These will be denoted by $Q_u = Q_{u1} = Q_{u2}$.

(2) The values of Q_{u1} and Q_{u2} to be such that when the transformer is connected into the i-f amplifier, and loaded by the valve output and input resistances, the desired value of $Q = \sqrt{Q_1 Q_2}$ will be obtained.

(3) The required coefficient of coupling (k) (critical for this particular example) to be unchanged. It will be described later how k can be pre-set to any desired value for any two circuits coupled together.

(4) Excessive values of Q_{u1} and Q_{u2} are to be avoided (see the previous method of determining Q_{u1}) as far as possible, because of the practical difficulties involved.

(5) The response curve of frequency versus attenuation to be that specified (or very close to it).

All of these conditions can be fulfilled very closely, provided the approximations made in deriving the design equations hold. A simple analysis of the circuits involved, and including the required conditions, gives

$$Q_u = \frac{\alpha + \sqrt{\alpha^2 + 2Q^2 R_1 R_2 \beta}}{\beta} \tag{7}$$

where $Q_u = Q_{u1} = Q_{u2}$ (unloaded primary and secondary Q)
$\quad Q = \sqrt{Q_1 Q_2}$ (in which Q_1 and Q_2 are loaded primary and secondary Q's)
$\quad R_1 =$ parallel resistance (r_p in our case) shunted across trans. primary
$\quad R_2 =$ parallel resistance (grid input in our case) shunted across trans. secondary
$\quad \alpha = Q(Q\omega_0 L)(R_1 + R_2)$; in all cases it will be taken that $L = L_1 = L_2$
and $\quad \beta = 2[R_1 R_2 - (Q\omega_0 L)^2]$.

For our example :
$$Q = 101 \; ; \; Q^2 = 1.02 \times 10^4 \; ; \; R_1 = 0.8 \text{ M}\Omega \; ; \; R_2 = 6.8 \text{ M}\Omega$$
$$Q\omega_0 L = 0.352 \text{ M}\Omega \text{ (found previously)}$$
$$\alpha = 101 \times 0.352 \times 7.6 = 270 \; ; \; \alpha^2 = 7.29 \times 10^4$$
$$\beta = 2[5.44 - 0.124] = 10.63 \; (\text{M}\Omega)^2$$
$$Q_u = \frac{270 + \sqrt{7.29 \times 10^4 + 2 \times 1.02 \times 10^4 \times 5.44 \times 10.63}}{10.63} = 131$$

so that $Q_{u1} = Q_{u2} = 131$.

To check that the transformer, when placed in the receiver, gives the desired value of $Q = \sqrt{Q_1 Q_2}$ use

$$Q = \frac{Q_u R}{Q_u \omega_0 L + R} \tag{8}$$

from which
$$Q_1 = \frac{131 \times 0.8}{0.458 + 0.8} = 83.4$$

$$Q_2 = \frac{131 \times 6.8}{0.458 + 6.8} = 123$$

and so

$$Q = \sqrt{Q_1 Q_2} = \sqrt{83.4 \times 123} = 101, \text{ which is the desired value (as}$$
determined previously).

(D) Conclusions

All that is required to design the specified transformer is to go through the simple steps (a) to (e) and, knowing R_1 and R_2, apply eqn. (7). Overall response and phase shift are determined from the universal curves, as explained previously.

It should be obvious that eqn. (7) will not hold under all practical conditions, but it is not limited by the ratio of Q_1/Q_2 or Q_2/Q_1, and failing cases can be checked by the condition for $\beta = 0$. It has been assumed for simplicity that $L = L_1 = L_2$ but this is not essential, and the design equation could be extended to the case of $L = \sqrt{L_1 L_2}$. In some failing case, if it is essential to fulfil the specified conditions, Q_1 and Q_2 (and if necessary L_1 and L_2) can be selected to give the desired values of Q and L; this will be illustrated in the section on the design of variable bandwidth crystal filters.

(E) k Measurement

The coefficient of coupling, k, for two circuits resonant at the same frequency, can be set on a Q meter (provided Q_b lies within the useful working range) using the relationship

$$Q_b = \frac{Q_{u1}}{1 + Q_{u1} Q_{u2} k^2} \tag{9}$$

If $\quad Q_{u1} = Q_{u2} = Q_u$

then $\quad Q_b = \dfrac{Q_u}{1 + (Q_u k)^2}$ \hfill (9A)

where Q_{u1} = primary Q (sec. o/c or detuned by large amount)

$\qquad Q_{u2}$ = secondary Q (pri. o/c or detuned by large amount)

$\qquad Q_b$ = Q to be obtained when primary and secondary are coupled and the secondary tuned to make the primary Q a minimum.

(Usual precautions as to can and earthy side of secondary winding being grounded to be observed—it is preferable to use the same order of connections for measurement as those to be used in the receiver.)

When the transformer has different primary and secondary Q's, it is often advantageous to use the higher Q winding as the primary when setting the coefficient of coupling; this applies particularly when the coupling is very loose.

The actual capacitance values tuning the primary and secondary for Q meter measurements should include the allowance made for stray capacitance otherwise incorrect slug positions (i.e. incorrect inductance values) will give rise to an error which can be avoided.

In our example we desire a value for $k = 0.0099$ (for critical-coupling when $Q = \sqrt{Q_1 Q_2} = 101$); $Q_u = 131$.

Then from (9A)

$$Q_b = \frac{131}{1 + (1.297)^2} = 48.9.$$

All that is required is to adjust the spacing between the two resonant circuits until the Q meter reads 48.9. The desired co-efficient of coupling has then been obtained. Alternatively, by transposing terms in the equation, k is given for any values of Q_b, Q_{u1}, and Q_{u2},

so that $\quad k = \sqrt{\dfrac{Q_{u1} - Q_b}{Q_{u1} Q_{u2} Q_b}}$ \hfill (9B)

The method applies directly to under-, over-, or critically-coupled transformers and is useful within the limits set by the usable range of the Q meter. For over-coupled transformers additional methods are sometimes required, and the procedure will be indicated in Sect. 5.

It is sometimes required to measure k in terms of critical or transitional coupling. In this case the circuits are loaded to give the values of Q_1 and Q_2 required when the transformer is connected into the receiver, and the following expressions can be applied :

$$\frac{k}{k_c} = \sqrt{\frac{Q_1 - Q_b}{Q_b}} \tag{9C}$$

and $\dfrac{k}{k_t} = \sqrt{\dfrac{2Q_1Q_2(Q_1 - Q_b)}{Q_b(Q_1{}^2 + Q_2{}^2)}}$ \hfill (9D)

It might be noted that when $Q_1 = Q_2$ the expressions (9C) and (9D) are identical, as would be expected.

(iii) Over-coupled transformers

(A) Design equations and table

Here the method to be followed is based on that due to Everitt (Ref. 3).

Fig. 26.5 illustrates the terms used regarding bandwidth.

It should be noted that when the primary and secondary Q's differ appreciably, two peaks of secondary output voltage do not appear immediately critical-coupling is exceeded. The actual value of k, which corresponds to the condition for two peaks of secondary voltage, is called the transitional-coupling factor [see Chapter 9, Sect. 6(v)], and Ref. 8.

In what follows we shall use $Q = \sqrt{Q_1Q_2}$ and $L = \sqrt{L_1L_2}$, as was done for critically-coupled transformers, but this is not an approximation in the derivation of the design equations (10), (11) and (12) provided that $L_1C_1 = L_2C_2$. It will also be assumed, for simplicity, that the peaks of the response curve are of equal height and symmetrically placed in regard to f_0.

$$Q_k = A + \sqrt{A^2 - 1} \tag{10}$$

$$A = \frac{(Qk)^2 + 1}{2Qk} \tag{11}$$

$$\frac{2\Delta f_p}{f_0} = k\sqrt{1 - \frac{1}{(Qk)^2}} \tag{12}$$

$$\theta = \tan^{-1}\frac{2X}{1 - X^2 + (Qk)^2} \tag{13}$$

where $Q = \sqrt{Q_1Q_2} = \dfrac{1}{k_c}$ (in which Q_1 and Q_2 are primary and secondary Q's respectively ; k_c is coefficient of critical-coupling)

k = any coefficient of coupling equal to or greater than critical

A = gain variation from peak to trough (i.e. difference in transmission level)

$2\Delta f_p$ = bandwidth between peaks ; $\sqrt{2}(2\Delta f_p)$ is the total bandwidth for two other points on the resonance curve with the same amplitude as at f_0

θ = phase shift between the secondary current at resonance and the secondary current at Δf c/s off resonance

$X = (2\Delta f/f_0)Q$

and f_0 = resonant frequency of transformer (i-f).

The universal resonance and phase shift curves of Figs. 9.17 and 9.18 (Chapter 9 Sect. 10) are directly applicable, using the exact expressions if desired and taking Q as Q_2 for all terms involving a. It is more convenient, and sufficiently accurate, to use the conditions for $Q_1 = Q_2$ when Q_1/Q_2 or $Q_2/Q_1 \not> 2$; in this case $b = Qk$ (or k/k_c), $D = (2\Delta f/f_0)Q$ and since these expressions do not involve a, the value $Q = \sqrt{Q_1Q_2}$ as determined in the design problem is used. A check will reveal that it is difficult to read any difference from the curves whichever method is used.

It should be observed that five points on the resonance curve are given directly from the design equations.

To find the maximum stage gain which occurs at the peaks, the equation (5) as given for critically-coupled transformers, is applied directly. Generally it is the average gain in the pass band ($\sqrt{2} \times 2\Delta f$) which is required and this is given by multiplying eqn. (5) by

$$\frac{(Qk + 1)^2}{2[(Qk)^2 + 1]} \quad \quad (14)$$

If the gain at f_0 (i.e. at the trough of the curve) is required, equation (5) is multiplied by

$$\frac{2Qk}{(Qk)^2 + 1} \quad \quad (15)$$

which is the same as multiplying equation (5) by $1/A$, since equation (15) is equivalent to $1/A$.

Equations (14) and (15) can be evaluated directly from Fig. 26.4 when Qk is known ; the dotted line being for eqn. (14) and the solid line for eqn. (15). Equation 5 is multiplied by the gain reduction factor so found. The gain reduction indicated by eqn. (15) can also be read directly from the $1/A$ column in table 2.
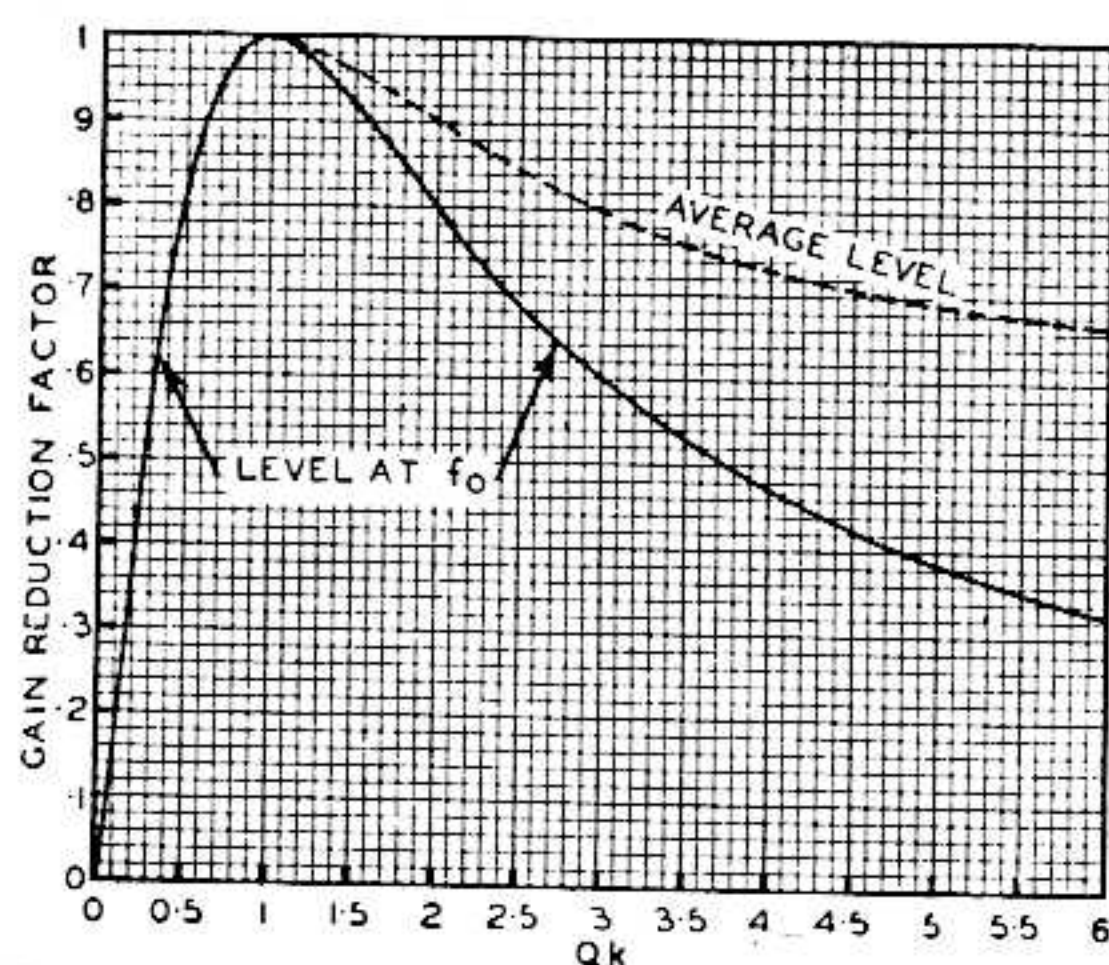

FIG.26·4 GAIN REDUCTION FACTORS FOR COUPLING
OTHER THAN CRITICAL

TABLE 2. OVER-COUPLED TRANSFORMERS
For use with equations (10), (11), (12), (14), (15)

A (= peak to trough gain variation)		Qk	$\sqrt{1 - \dfrac{1}{(Qk)^2}}$	$(Qk)^2$	$\dfrac{1}{A}$
db	Times Down				
0	1.00	1.00	0.000	1.00	1.00
0.25	1.03	1.27	0.616	1.61	0.971
0.50	1.06	1.41	0.707	2.00	0.943
1	1.12	1.73	0.817	3.00	0.893
1.9	1.25	2.00	0.866	4.00	0.800
2	1.26	2.02	0.869	4.08	0.794
3	1.41	2.41	0.910	5.81	0.709
3.1	1.43	2.45	0.913	6.00	0.670
4	1.59	2.81	0.935	7.90	0.629
4.4	1.67	3.00	0.943	9.00	0.599
5	1.78	3.25	0.952	10.56	0.562
6	2.00	3.72	0.963	13.84	0.500
6.6	2.13	4.00	0.968	16.00	0.469
7	2.24	4.24	0.972	17.98	0.446

(B) Example

A 455 Kc/s i-f transformer is required to pass a band of frequencies 16 Kc/s wide (i.e. $\pm$ 8 Kc/s). The variation in gain across the pass band is not to exceed 0.5 db.

(a) From Fig. 26.5 it is reasonable to take the **total** bandwidth as 16 Kc/s and so the peak separation is $16/\sqrt{2} = 11.3$ Kc/s.

(b) $2\Delta f/f_0 = 11.3/455 = 0.0248$.

(c) From Table 2, $\sqrt{1 - 1/(Qk)^2} = 0.707$.

(d) From eqn. (12), $k = 0.0248/0.707 = 0.035$.

(e) From Table 2, $Qk = 1.41$
$$Q = 1.41/0.035 = 40.2.$$

(f) Assuming a value for $C_1 (= C_2)$ of 80 $\mu\mu$F (including strays)
$$L = 25.33/0.455^2 \times 80 = 1.53 \text{ mH}$$
and take $L = L_1 = L_2 = 1.53$ mH.

(g) To determine the average stage gain in the pass band. From Fig. 26.4 (or eqn. 14), with $Qk = 1.41$, reading from the dotted curve, gain reduction factor equals 0.97. Assuming we use a type 6J8-G converter valve having a conversion conductance of 290 μmhos (0.29 mA/volt), then from eqn. (5) and the gain reduction factor,
Average stage gain $= 0.97 \times \pi \times 0.29 \times 40.2 \times 0.455 \times 1.53$
$$= 24.7 \text{ times (or 27.9 db)}.$$

(h) Assume that the transformer is connected between a type 6J8-G converter and a type 6SK7 voltage amplifier and that both valves are working under a particular set of operating conditions.

For type 6J8-G the conversion plate resistance $r_p = 4\text{M}\Omega = R_1$ and for the type 6SK7 the short circuit input resistance $= 6.8$ M$\Omega = R_2$ (as determined from Chapter 23, Sect. 5).

From Equation (7)
$$\alpha = 40.2 \times 0.17 \times 10.8 = 73.8 \text{ ; } \alpha^2 = 0.544 \times 10^4$$
$$\beta = 2[27.2 - 0.17^2] = 54.4 \text{ approximately}$$
$$Q_u = \frac{73.8 + \sqrt{0.544 \times 10^4 + 4.76 \times 10^6}}{54.4} = 41.4,$$

so that $Q_u = Q_{u1} = Q_{u2} = 41.4$ which is the unloaded value for primary and secondary Q's before the transformer is connected between the two valves ; the additional refinement in design is hardly necessary here, and it would be sufficient to make $Q = Q_1 = Q_2 = 40$ (approx.).
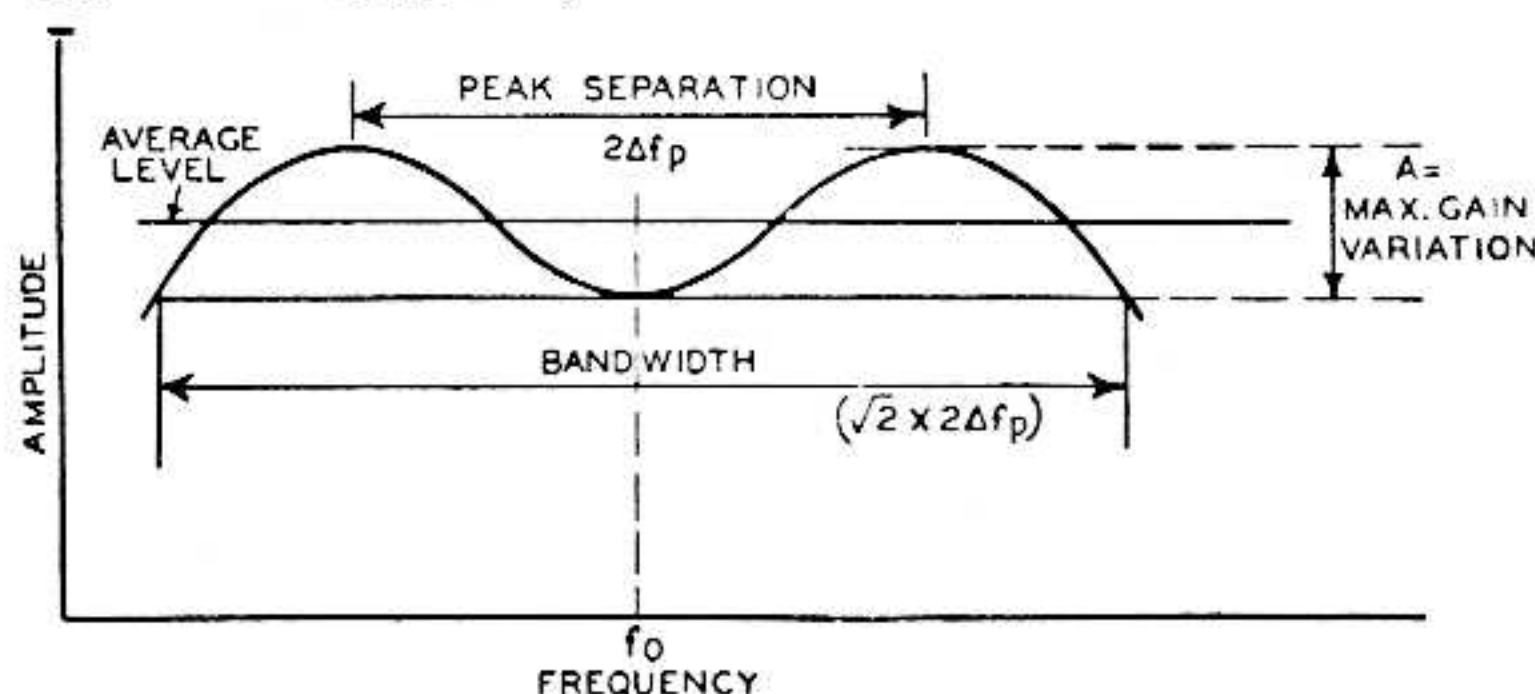

FIG. 26·5   ILLUSTRATION OF TERMS USED FOR OVER-COUPLED TRANSFORMERS.

It can easily be checked, using the procedure set out for critically-coupled transformers, that the geometrical-mean of the loaded values Q_1 and Q_2 is 40.2 as required. Complete resonance and phase shift curves are plotted as previously explained. The values for k can be set in exactly the same way as explained in the section on critically-coupled transformers, if this is convenient. If k is high it is preferable to apply one of the following methods.

(C) k measurement (when k is high)

When k is high, one of the following methods may be used

The first of these uses the relationship
$$\Delta C = C_1 k^2/(1 - k^2)$$

or $\qquad k = \sqrt{\dfrac{\Delta C}{\Delta C + C_1}}$

where C_1 = capacitance required to tune the primary to resonance with the secondary open circuited.

$\qquad k$ = coefficient of coupling required (say k greater than 0.1 or so)

and $\quad \Delta C$ = increment in capacitance required to tune the primary to resonance when the secondary is short circuited.

As an illustration suppose $k = 0.2$ and $C_1 = 200$ $\mu\mu$F (the exact working frequency may not always be convenient) then $\Delta C = 8.34$ $\mu\mu$F. Using a " Q " meter, the spacing between primary and secondary is adjusted until this increment in capacitance is obtained ; this gives the required value of k.

With some transformers, neither of the two methods given previously will be convenient, and a third method is required. In this case a " Q " meter is again used, and the two transformer windings are connected firstly " parallel aiding " and then " parallel opposing." Any convenient resonance frequency may be selected, and the two capacitance values (C_1 and C_2) required to resonate the coils with the two different connections are noted. The coefficient of coupling is then given by

$$k = \left(\frac{1 + a^2}{2a}\right)\left(\frac{C_1 - C_2}{C_1 + C_2}\right)$$

where $a^2 = L_2/L_1$. (This result was determined by J. B. Rudd).

For a pre-determined value of k the relationship

$$\Delta C = \frac{4akC_2}{1 + a(a - 2k)}$$

is applied ; where ΔC = change in capacitance = $C_1 - C_2$.

As an example, suppose $L_1 = L_2$ (i.e. $a^2 = 1$) the capacitance $C_2 = 100$ $\mu\mu$F and $k = 0.1$ is required. Then

$$\Delta C = \frac{4 \times 1 \times 0.1 \times 100}{1 + 1(1 - 2 \times 0.1)} = 22.2 \ \mu\mu\text{F}.$$

This method should not be used for small values of k (say below about 0.02) as it does not take into account capacitive coupling. For k equal to 0.02 or less the method of Sect. (ii)E should always be used. For k greater than about 0.2 the short-circuit/open-circuit method, given previously, is usually the most convenient. All measurements must be made with the transformer in its can, and the can should be earthed.

In cases where a " Q " meter is not available, the operation of the transformer can be checked using a single stage amplifier and measuring the selectivity curve with a signal generator and a valve voltmeter. Alternatively, measurements can be made in the receiver, and a typical case is illustrated in Chapter 27 Sect. 2(iv) in connection with measurements on ratio detectors (see also Chapter 14 of Ref. 17).

(iv) Under-coupled transformers and single tuned circuits

It is sometimes necessary to use combinations of single tuned circuits or under-coupled transformers in conjunction with over-coupled circuits to give a substantially level response over the pass band. Another application for the under-coupled transformer often arises when an improvement in selectivity is needed, without an excessive loss in stage gain. It has been shown by Adams (Ref. 9) that the optimum conditions for selectivity and gain for a given Q, are obtained when the coefficient of coupling is approximately 0.82 of critical. However, it is readily shown for a transformer having k equal to 0.5 k_c that the loss in gain is only about 2 db (approx. 0.8 of maximum gain).

The design equations (Ref. 1) that follow are applicable to single tuned circuits, which will be considered first :

(A) Single Tuned Circuits

$$\rho = (1 + X^2)^{N/2} \tag{16}$$

$$X = (\rho^{2/N} - 1)^{\frac{1}{2}} \tag{17}$$

$$Q = \frac{Xf_0}{2\Delta f} \tag{18}$$

and $\quad \theta = \tan^{-1} \dfrac{2\Delta f}{f_0} Q = \tan^{-1} X$ (19)

where ρ = attenuation at Δf c/s off resonance

$\quad\quad N$ = number of identical tuned circuits (for a single tuned circuit $N = 1$)

$\quad\quad f_0$ = resonant frequency

and $\quad \theta$ = phase shift between current at resonance and the current at Δf c/s off resonance.

The design of the single tuned circuit presents no difficulty, and it is only necessary to use Table 3 in conjunction with eqns. (16), (17) and (18). Selectivity and phase shift can be found from the universal curves of Figs. 9.17 and 9.18 (Chapter 9 Sect. 10), or selectivity can be evaluated directly from eqns. (16), (17) and (18) used in conjunction with Table 3.

TABLE 3. SINGLE TUNED CIRCUITS
For use with equations (16), (17) and (18)
N = number of identical tuned circuits

Attenuation (ρ)		$N = 1$	$N = 2$	$N = 3$
Times Down	db Down	X	X	X
$\sqrt{2}$	3	1.00	0.644	0.509
2	6	1.73	1.00	0.767
4	12	3.87	1.73	1.23
7	17	6.93	2.45	1.63
10	20	9.95	3.00	1.91
20	26		4.36	2.52
40	32		6.25	3.27
70	37		8.31	4.00
100	40		9.95	4.53
1000	60			9.95

(B) Example

Two single tuned circuits are required to give an attenuation of 17 db for a total bandwidth of 10 Kc/s. The i-f is 455 Kc/s.

(a) From Table 3, $X = 2.45$ (since $N = 2$).

(b) $Q = 2.45 \times 455/10 = 111.5$ from eqn. (18).

(c) Assuming $C = 200 \ \mu\mu\text{F}$ (including all strays)
$\quad\quad L = 25.33/0.455^2 \times 200 = 0.611 \text{ mH}$.

(d) The unloaded Q required depends on the combined effects of valve input and output resistance. Take the loading for the two transformers as being the same, for simplicity.

    Then suppose $r_p = 0.8 \ \text{M}\Omega$ and grid input resistance $= 6.8 \ \text{M}\Omega$ the effective shunt resistance is $0.715 \ \text{M}\Omega$.

    From eqn. (6)
$\quad\quad Q_u = 111.5 \times 0.715/(0.715 - 0.194) = 153$.

(e) The gain of each stage (twice that for a critically-coupled transformer) is $g_m Q \omega_0 L$, so that taking $g_m = 2 \text{ mA/volt}$ (2000 μmhos) in each case, stage gain $= 2\pi \times 2 \times 111.5 \times 0.455 \times 0.611 = 390$ times (or 51.8 db).

(f) Suppose we have the loaded Q given as 111.5 (as in our previous problem using $N = 2$), and we require the bandwidth for 6 db attenuation ; from table 3 obtain $X = 1.0$ and from eqn. (18) the total bandwidth ($2\Delta f$) is 4.08 Kc/s. In a similar manner the attenuation can be found when the bandwidth is stated. The resonance curves could also be used to find X ($= D$ in Chapter 9, in this case) and ρ.

(C) Under-coupled transformers

If the transformers use very loose coupling, the methods for single tuned circuits could be applied ($N = 2$ for each transformer) in conjunction with eqn. (7). This

approach does not lead to very accurate results since the values of coupling are seldom less than 0.1 of critical and more often are of the order of 0.5 to 0.8 of critical.

General design equations applicable to transformers having any degree of coupling are given below, but it will be seen that they are not quite as tractable as in previous cases unless an additional factor such as Q, k, or Qk (i.e. a given proportion of critical-coupling) is specified. However, this will offer little difficulty.

$$X = [\{(1 - \alpha^2)^2 + (\rho^{2/N} - 1)(1 + \alpha^2)^2\}^{\frac{1}{2}} - (1 - \alpha^2)]^{\frac{1}{2}} \tag{20}$$

$$\rho = \left[\frac{(1 + \alpha^2 - X^2)^2 + 4X^2}{(1 + \alpha^2)^2}\right]^{N/2} \tag{21}$$

$$X = (2\Delta f/f_0)Q \tag{22}$$

$$Qk = \alpha = \left[\frac{X\{X^2 + (X^2 + 4)(\rho^{2/N} - 1)\}^{\frac{1}{2}} - (X^2 + \rho^{2/N} - 1)}{\rho^{2/N} - 1}\right]^{\frac{1}{2}} \tag{23}$$

$$\theta = \tan^{-1} 2X/[1 - X^2 + \alpha^2] \tag{13}$$

where ρ = attenuation at Δf c/s off resonance

$\alpha = Qk = k/k_c$

$Q = \sqrt{Q_1 Q_2} = 1/k_c$ (in which Q_1 and Q_2 are the primary and secondary Q's and k_c is critical coupling coefficient)

k = any coefficient of coupling

N = number of identical transformers

f_0 = resonant frequency

and θ = phase shift between secondary current at and off resonance.

The restriction of Q_1/Q_2 or $Q_2/Q_1 \not> 2$ is applied, as previously explained.

It may be observed that these equations are the most general ones, e.g. if $Qk = 1$ the equations reduce to those for critical-coupling.

Stage gain is given by evaluating eqn. (5) and multiplying by eqn. (15) (or reading the gain reduction factor from Fig. 26.4).

The coefficient of coupling can be set as described for critically-coupled transformers.

(D) Example

A 455 Kc/s i-f transformer is required to give a total bandwidth of 20 Kc/s for an attenuation of 4 times. The transformer is to be connected between a voltage amplifier, having a plate resistance of 0.8 MΩ (e.g. type 6SK7), and a diode detector having a load resistance of 0.5 MΩ.

In this case other loading effects due to the a.v.c. system etc. will be neglected. For a typical case where the a.v.c. diode plate is connected by a fixed capacitor to the i-f transformer primary, there will be appreciable damping of the primary circuit due to the diode circuit (approx. $R_L/3$ when diode is conducting). This damping will not be constant for all signal input voltages, particularly if delayed a.v.c. is used.

(a) The difficulty first arises in evaluating X. If we select a suitable value for Q the problem becomes quite straightforward.

To select a value for Q it is necessary to realize that an unloaded value of 150 would be about the absolute maximum with normal types of construction, and even this figure is well on the high side unless an " iron pot " or a fairly large can and former are used. Assume $Q = 150$ for this problem (so far as the procedure is concerned it is unimportant if a lower value is selected).

The next point is that circuit loading will set a limit to the value of $Q\omega_0 L$ ($= Q/\omega_0 C$). Now L will be set, normally, by the minimum permissible value of C. Suppose $C = 85$ $\mu\mu$F including strays, then $L = 1.44$ mH ; and we will take $L = L_1 = L_2$ and $C = C_1 = C_2$ for practical convenience. Better performance would be possible by making $L_1 > L_2$ but the improvement is only small, and hardly worthwhile unless the secondary load is very small.

(b) For our problem

$\omega_0 L = 1/\omega_0 C = 4120$ Ω ; also $R_1 = 0.8$ MΩ and

$R_2 = 0.5/2 = 0.25$ MΩ (half the d.c. diode load resistance).

Then applying eqn. (8)

$$Q_2 = \frac{150 \times 0.25}{0.25 + (150 \times 4.12 \times 10^{-3})} = 43.3$$

and $Q_1 = \dfrac{150 \times 0.8}{(0.8 + 0.618)} = 84.6$

so that

$$Q = \sqrt{Q_1 Q_2} = \sqrt{84.6 \times 43.3} = 60.5.$$

(c) From eqn. (22)

$$X = \frac{20 \times 60.5}{455} = 2.66.$$

(d) From eqn. (23)

$$Qk = \left[\frac{2.66\{2.66^2 + (2.66^2 + 4)(4^2 + 1)\}^{\frac{1}{2}} - (2.66^2 + 4^2 - 1)}{4^2 - 1}\right]^{\frac{1}{2}} = 0.93$$

and $k = 0.93/60.5 = 0.0154$.

(e) From eqns. (5) and (15) (the solid curve of Fig. 26.4)

$$\text{Stage gain} = \frac{(2 \times 60.5 \times 4.12)}{2} \, 0.996$$

$$= 249 \text{ times or } 47.9 \text{ db.}$$

(f) The completed transformer has primary and secondary Q's of 150 (before connection into the receiver), a coefficient of coupling equal to 0.0154 (which is 0.93 of critical-coupling when the transformer windings are loaded) primary and secondary inductances of 1.44 mH and tuning capacitances of 85 $\mu\mu$F (including strays). The stray capacitances across the primary would be valve output (7 $\mu\mu$F for type 6SK7) plus distributed capacitance of winding, plus capacitances due to wiring and presence of shield can ; across the secondary there would be diode input capacitance (about 4 $\mu\mu$F for a typical case), plus distributed capacitance of secondary winding plus wiring and shield can capacitances. The total capacitances can be measured in the receiver or estimated using previous experience as a guide ; typical values would be 10-20 $\mu\mu$F depending on the type of i-f transformer, valves etc. If the second valve is not a diode, the input capacitance should also include that due to space charge, Miller effect etc. as discussed in Chapter 2 Sect. 8 ; Chapter 23 Sect. 5 and Sect. 7 of this chapter. However, in most practical cases the total capacitance across the secondary is estimated by adding a suitable value to the valve input capacitance. Changes in input resistance which would affect the loading across the i-f transformer secondary, are also discussed in these same sections. Input capacitance changes with a.v.c. are considered in Sect. 7 of this chapter.

(g) A complete resonance curve can be obtained from Fig. 9.17 (Chapter 9 Sect. 10), by taking

$$D = Q2\Delta f/f_0 \, (= X) \text{ and } b = Qk = k/k_c,$$

or directly from the design equations (20), (21) and (22).

(v) F-M i-f transformers
(A) Design data
The design methods given so far are applicable to both F-M and A-M transformers, but there are additional data available which will be of assistance (see also Sect. 9(ii) of this chapter for an alternative design procedure based on permissible non-linear distortion).

Bandwidth requirements are of importance, and it is fairly generally accepted that the i-f amplifier should be capable of passing all significant sideband frequencies of the frequency modulated wave ; where significant sidebands are taken as those having amplitudes which are greater than about 1% of the unmodulated carrier amplitude. The bandwidths for this condition can be found from Table 4 (see also Ref. 19) for commonly occurring values of modulation index.

TABLE 4. BANDWIDTHS FOR USE WITH F-M TRANSFORMERS

$$\text{Modulation Index} = \frac{\Delta F}{f} = \frac{\text{carrier frequency deviation}}{\text{audio modulating frequency}}$$

Values for $\dfrac{\Delta F}{f}$ may be interpolated with sufficient accuracy

$\Delta F/f$ =	0.01-0.4	0.5	1.0	2.0	3.0	4.0	5.0	6.0
Bandwidth =	$2f$	$4f$	$6f$	$8f$	$12f$	$14f$	$16f$	$\cdot 18f$

$\Delta F/f$ =	7.0	8.0	9.0	10.0	12.0	15.0	18.0	21.0
Bandwidth =	$22f$	$24f$	$26f$	$28f$	$32f$	$38f$	$46f$	$52f$

As an example, for the F-M broadcast band $\Delta F = \pm 75$ Kc/s and the highest audio frequency is 15 Kc/s, then $\Delta F/f = 75/15 = 5$.

Then the required bandwidth is $16 \times 15 = 240$ Kc/s.

The highest audio frequency is chosen because this imposes the most severe requirements on bandwidth ; e.g. suppose we had taken $f = 7.5$ Kc/s, then the modulation index would be 10, and the bandwidth $= 28f = 28 \times 7.5 = 210$ Kc/s.

The bandwidths actually employed in a receiver should also make allowance for possible drift in the oscillator frequency. A reasonably good oscillator should not drift by more than about ± 20 Kc/s when operating around 110 Mc/s ; so that an additional 40 Kc/s should be added to the bandwidth. Of course, this is only a rough approximation, since the oscillator frequency variation is random and would introduce additional frequency modulation ; the determination of the true bandwidth would be quite a difficult problem, unless several simplifying assumptions are made.

From what has been said, it appears that the receiver total bandwidth should be about 280 Kc/s to fulfil the most severe requirements. However, most practical receivers limit the total bandwidth to about 200 Kc/s, which is not unreasonable since the average frequency deviation is about ± 50 Kc/s.

For such large bands of frequencies to be passed through tuned circuits which do not give transmission (for the bandwidth desired) without attenuation it is necessary to have some criterion which will allow the permissible amount of attenuation to be estimated. To eliminate non-linear distortion the circuits should provide a uniform amplitude and a linear phase characteristic over the operating range. Curvature of the phase characteristic of the tuned circuits will cause non-linear a-f distortion, while curvature of the amplitude characteristic may cause additional distortion if the amplitude happens to drop below the operating voltage range of the amplitude limiting device incorporated in the receiver. A suitable criterion can be determined from the phase angle/frequency characteristics of the tuned circuits (the phase angle being that between the secondary current at resonance to that at Δf c/s off resonance). Inspection of universal phase shift curves will show that the greatest range of linearity of phase shift versus frequency change, is given by critically-coupled transformers (Ross, Ref. 2) ; but slight overcoupling does not lead to excessive non-linearity. Overcoupling has some advantages, in particular slightly greater adjacent channel selectivity can be obtained ; but there is the disadvantage of more difficult circuit alignment. As an extension of this work, Ross (Ref. 2) has also shown that for a critically-coupled transformer a suitable criterion of permissible non-linearity is that

$$X \not> 2 \tag{24}$$

where we will take

$X = (2\Delta f/f_0)Q$ (this is the same X as previously)

$2\Delta f$ = total bandwidth

f_0 = central carrier frequency

and $Q = \sqrt{Q_1 Q_2}$ (where Q_1 and Q_2 are the primary and secondary magnification factors).

The amount of introduced amplitude modulation can be estimated from

$$m = \frac{\rho - 1}{\rho + 1} \qquad (25)$$

where m = amplitude modulation factor

and ρ = attenuation at bandwidth of twice deviation frequency (i.e. $2\Delta f = 2\Delta F$).

Fig. 26.6 shows directly values of m for various values of X. Values of m are of importance since they allow an estimate to be made of the amplitude limiting requirements demanded from whatever device is incorporated in the receiver to "iron out" amplitude variations.

Many F-M receiver designs allow anything from 20% to 50% of introduced amplitude modulation, but these figures should always be considered in connection with the amount of non-linear distortion introduced by the tuned circuits [see Sect. 9(8)]. Good designs often allow considerably less than 20% of introduced amplitude modulation.

FIG. 26·6 PERCENTAGE OF INTRODUCED A-M IN AN F-M CURRENT FOR N CRITICALLY COUPLED TRANSFORMERS

It is also worth noting, before leaving this section, that the carrier frequency should be regarded as a reference point only, since, unlike amplitude modulation, its amplitude varies and becomes zero under some conditions of modulation. This is the basis of a method due to Crosby (Ref. 25) used for measuring frequency deviation.

(B) Example

A F-M i-f amplifier is required using three critically-coupled transformers. The i-f is 10.7 Mc/s and the frequency deviation $\pm$ 75 Kc/s ; the highest a-f modulating frequency is 15 Kc/s (critical-coupling has been selected in this case but a combination of critical and over-coupled transformers might lead to a better solution).

The converter valve to be used has a conversion r_p of 1.5 MΩ and a conversion conductance of 475 μmhos ; and the two i-f valves each have plate resistances of 2 MΩ and g_m = 5000 μmhos.

(a) For simplicity it will be taken that any additional selectivity, due to the other tuned circuits in the receiver, is negligible. Also, as will be illustrated, the dynamic impedances of the i-f transformers will be so low as to render additional damping due to plate and grid input resistances negligible ; this is not true, however, if the final transformer is connected to a limiter stage because of grid current damping—some consideration will be given to this later.

(b) From previous considerations regarding bandwidth, in connection with Table 4, we will adopt 220 Kc/s as a compromise.

If we design on the limit of $X = 2$, then from eqn. (3)

$$Q = \frac{2 \times 10.7 \times 10^6}{220 \times 10^3} = 97.4$$

and $k_c = 1/Q = 0.0103$.

(c) Taking $2\Delta f = 2\Delta F = 150$ Kc/s, for maximum frequency deviation, since $Q = 97.4$ and $N = 3$, then, from eqn. (3), $X = 1.365$ and, from equation (1), $\rho = 2.55$, whence from eqn. (25) or Fig. 26.6,

$$m = \frac{2.55 - 1}{2.55 + 1} = 0.436 \text{ or } 43.6\% \text{ amplitude modulation.}$$

(d) This is a severe additional requirement for limiters etc., and could also lead to high distortion, although the condition for linearity of phase shift with frequency is fulfilled. The distortion can be found from eqn. (54) for each of the transformers.

(e) Select suitable values for C_1 and C_2. To obtain the highest possible dynamic impedance these are usually made rather small. Take

$$C_1 = C_2 = 60 \ \mu\mu F \text{ (including strays)}$$

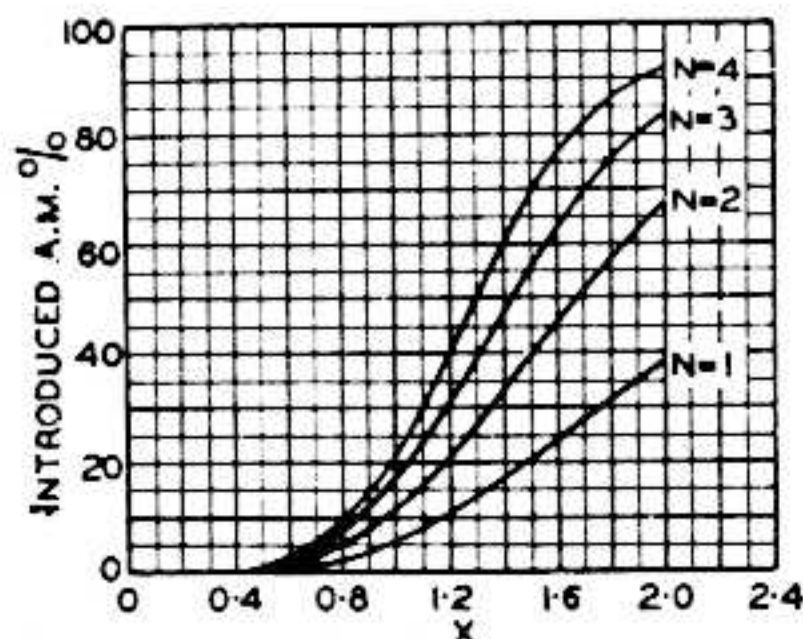$$L(= L_1 = L_2) = \frac{25\,330}{10.7^2 \times 60} = 3.68 \ \mu H.$$

There would be an advantage in making $C_1 < C_2$ and $L_1 > L_2$ but this is awkward for winding i-f's on a machine.

(f) The dynamic impedance of each winding (considered uncoupled from one another) is
$$R_0 = Q\omega_0 L = 97.4 \times 2\pi \times 10.7 \times 3.68 = 24\,000\ \Omega$$
which is very much less than the valve plate or input resistances in typical stages.

(g) To find the overall gain of the i-f stages :—For the first stage, connected to the converter, we are concerned with conversion gain.

From eqn. (5) and step (f),

Conversion gain = $475 \times 10^{-6} \times 24\,000/2 = 5.7$ times (15.1 db). For the second and third stages in each case, gain = $5.7 \times 5/0.475 = 60$ times (35.6 db). Hence the overall gain is $15.1 + (2 \times 35.6) = 86.3$ db.

(h) If the third transformer connects to a limiter its design should be modified for best results ; however, this is not done in many receivers. Grid current will alter the effective input capacitance of the limiter valve and cause very appreciable detuning of the transformer secondary. To overcome this detuning it may be necessary to make the pass band of this transformer somewhat greater than for the other two, or else to use a very large capacitance (of the order of 600 $\mu\mu$F) to tune the secondary (tapping down is also effective). It has also been suggested that Q_1 should equal Q_2 in this case (Ref. 21). The important point, apart from possible distortion, is that the susceptibility to certain types of impulse noise is increased if the tuned circuits are not accurately aligned to the centre frequency of the discriminator. Further, when the i-f circuit is detuned, additional amplitude modulation will be introduced, and this increases the difficulty of obtaining effective limiting. Grid current damping, of course, would tend to offset the effect to some extent (see also Refs. 21, 24). Since this transformer would probably be non-standard in any case, it would be advantageous to make L_1 as large as possible, to assist in keeping the stage gain high. This follows as a result of the stage gain being directly proportional to $\sqrt{L_1 L_2 Q_1 Q_2} = LQ$.

(i) It is important to check whether the i-f transformers will provide sufficient adjacent channel selectivity. This will depend on the frequency allocations of the various transmitters. In the U.S.A. local F-M transmissions are usually spaced 400 Kc/s apart, and the past experience of some designers has indicated that adjacent selectivity is adequate if the receiver bandwidth is not more than about 800 Kc/s for a relative attenuation of 60 db.

In the example above there are three critically-coupled transformers each having a nominal Q of 97 :

From Table 1, $X = 4.46$ for 60 db attenuation.

From eqn. (3)
$$2\Delta f = \frac{Xf_0}{Q} = \frac{4.46 \times 10.7 \times 10^6}{97} = 494 \text{ Kc/s.}$$

However, commercial F-M receivers have given reasonably satisfactory performance with two i-f transformers with Q's of about 75 leading to a bandwidth of 1140 Kc/s for 60 db attenuation. It might also be noted that larger commercial receivers generally use three transformers with Q's of about 70 to 75 and not 97 as found in the worked example, which is not to be taken as indicating good design practice.

Whether the selectivity performance is entirely adequate will depend very largely on the relative field strengths of the desired and undesired transmissions at the point of reception, and it is to be expected that more exact figures will only be decided on after a number of years' experience with practical operating conditions. The problem is similar to that of the A-M broadcast receivers (using 455 Kc/s i-f transformers) where it is usually taken that adjacent channel selectivity will be adequate if the overall band-

width does not exceed about 40 Kc/s for 60 db attenuation relative to the response at the centre frequency, although many commercial receivers have bandwidths of less than 30 Kc/s for the same attenuation.

(vi) I-F transformer construction

The methods of (i) to (v) in this section will allow the necessary design data for an i-f amplifier to be collected together. The final step is to determine the winding details and physical arrangement of the transformers. It is not proposed to discuss the merits of various types of windings, but merely to give a few details which have proved helpful in practice.

For transformers working at the higher frequencies, the windings are quite often solenoids and the determination of the number of turns required is a simple matter. Satisfactory results can be obtained by applying Hayman's modification of Wheeler's formula as given in Chapter 10. Methods are also set out for determining the number of turns per inch, and suitable wire diameters for obtaining optimum values of Q. Usually wire gauges between 18 and 28 s.w.g. are suitable as they are not so heavy as to be awkward to bend and they do not tear the usual type of coil former when the construction requires the leads to be passed through the inside of the former down to the base connections. Some error in the calculated number of turns will be apparent unless allowance is made for the inductance of leads. The number of turns required is finally determined experimentally in any case so that the calculated number of turns provides a good starting point.

Measurements must always be made with the coils in the cans because the effects of the can, brass mounting bosses, slugs etc. on inductance and Q are quite large.

Powdered iron " slugs " are commonly used for setting the inductance values and, for 10.7 Mc/s in particular, the iron must be very finely divided if the coil Q is not to be seriously changed as the cores are moved through the windings. Sufficient inductance variation also requires that the " slugs " have a certain minimum size.

Silvered mica fixed capacitors are to be preferred for good frequency stability with temperature changes, particularly at high frequencies such as 10.7 Mc/s, and temperature compensation using negative temperature coefficient capacitors is essential if the best results are to be obtained. Cheaper mica types are very often used at the lower frequencies around 455 Kc/s. If capacitance trimmers are to be used, care must be taken in their choice as pressure types are often mechanically unstable and sometimes have very low Q values. Suitable wax or other treatment should be applied to fixed capacitors to offset the effects of changes in humidity.

Coils should be baked* to remove moisture and then given suitable wax or varnish treatment to prevent humidity changes altering their properties. The electrical characteristics of the coils will be altered by wax etc. and it is essential to check the final values for Q, k etc. after the treatment is complete.

Typical former diameters for i-f transformers range from about $\frac{3}{8}$ in. to $\frac{3}{4}$ in., and the grade of material used for the former will affect the stability of the transformer directly when variations in temperature and humidity occur.

The design of transformers with pie windings (a larger number of pies generally reduces distributed capacitance and increases Q) is not as simple as for solenoids since most equations require a knowledge of coil dimensions which are not always available. There are also optimum sizes of winding to give the highest possible Q—see Chapter 10. A method which has proved satisfactory in practice is to make measurements on various types of coils which are likely to be used fairly often and apply the relationship

$$N = B\sqrt{L} \tag{26}$$

where N = turns per pie

L = inductance in μH

and B = experimentally determined constant.

As an example : A winding is to be made to have an inductance of 1.44 mH. Previous experiments have shown that, in the frequency range of 300-900 Kc/s and for

*See also Chapter 11 Sect. 7.

an inductance of about 0.5-2 mH, a two pie winding on a 9/16 in. former (each pie 5/32 in. wide, with 3/32 in. spacing between the pies and using 5/44 A.W.G. Litz wire) has the factor $B = 4.33$. Then from eqn. (26)

$$N = 4.33\sqrt{1440} = 165 \text{ turns per pie ;}$$

the two pies each of 165 turns, being connected series aiding. The same method can be applied in cases where it is convenient, to any type of winding.

Since the presence of the iron " slug " will affect inductance (and r-f resistance) it is necessary to determine its effect and also to calculate the variation in inductance which can be made. The turns required are found for the condition with the " slug " in the winding and in the position giving the mean inductance value. This means that the value of L used in eqn. (26) will be less than the calculated value by the increase due to the iron.

The value of the inductance (L) using a powdered iron core (e.g. magnetite) can be found from

$$L = L_0\left[1 + a\left(\frac{r_1}{r_2}\right)^2\left(\frac{l_1}{l_2}\right)(\mu_{eff} - 1)\right] \tag{27}$$

where L_0 = inductance of air cored coil

r_1 = radius of iron core

r_2 = mean radius of coil

l_1 = length of core

l_2 = length of coil

μ_{eff} = effective permeability of iron core (Refs. 88 and 89 list values for μ and μ_{eff} for various types of iron powders ; typical values for μ_{eff} are from 1.5 to 3, depending on the type of iron)

$a = 0.8$ when $l_1 < l_2$

and $a = 1$ when $l_2 < l_1$.

The iron cores in common use are about $\frac{3}{8}$ in. to $\frac{3}{4}$ in. in diameter and range in length from about $\frac{1}{4}$ in. to 1 in. An inductance change of about $\pm 10\%$ when the " slug " is moved through the winding, is generally sufficient for most requirements. The dimensions required for solving eqn. (27) are available if the experimental procedure previously suggested has been carried out on air cored coils ; or the whole procedure can be carried out experimentally.

Some manufacturers make up 455 Kc/s i-f transformers completely enclosed in powdered iron pots. There is often little difficulty with this construction in obtaining Q's in the order of 150. Stray capacitances are often large, however, and sometimes lead to very unsymmetrical resonance curves.

At the lower frequencies (up to about 1 or 2 Mc/s) Litz wire is advantageous for obtaining high Q values and 3, 5, 7 and 9 strands of about 44 A.W.G. (or near S.W.G. or A.W.G. gauges) wire are common ; Q values greater than about 120 will require some care in the transformer construction, and the size of can selected will materially affect the value obtainable (see Ref. 91 for illustration).

The required values of L_1, L_2, k, etc. for developmental purposes are conveniently found using a Q meter. Methods for setting k, using a Q meter, have already been outlined in this section. For an experimental transformer it is of assistance to place the windings on strips of gummed paper (sticky side outwards) wrapped around the former. In this way the windings can be moved quite readily along the former.

It should be noted that some variation in k can be expected when the transformer is connected in the receiver because of alteration of " slug " position, added top capacitance coupling (this occurs for example because of capacitance between the a.v.c. and detector diodes) etc. Regeneration is also troublesome as it alters the effective Q values, and hence the conditions for critical k. To avoid overcoupling it is sometimes desirable to make the value of k somewhat less than is actually required (often about 0.8 to 0.95 of the critical value depending on the receiver construction and i-f). If all added coupling is accounted for and the Q values are those specified then no difficulty arises. Some slight increase in coupling, in the receiver, is not necessarily serious, because the loaded primary and secondary Q's are not always equal. In this

case a double hump in the secondary voltage does not appear until transitional coupling has been exceeded, and the k required for this to occur is always higher than k critical ; transitional and critical k are the same, of course, when primary and secondary Q's are equal.

For the capacitance and mutual inductance coupling to be aiding, the primary and secondary windings are arranged so that if the plate connects to the start of the primary, then the grid (or diode plate) of the next stage connects to the finish of the secondary winding ; both coils being wound in the same direction. This method of connection also assists in keeping the undesired capacitance coupling to a minimum. The order of base connections is also important in reducing capacitance coupling and the grid and plate connections should be as far from one another as possible.

The cans to be used with i-f transformers should be as large as is practicable. They are generally made from aluminium, although copper was extensively used at one time. Cans should preferably be round and seamless. Perfect screening is not obtained, in general, and care is necessary in the layout of the various stages to ensure that the transformers are not in close proximity to one another. When mounting the transformer into a can, if there is a choice as to the position of the leads (although this is largely determined by the valve type available) it is always preferable to bring the connections for each winding out to opposite ends, as this reduces stray capacitance coupling. The effects of the coil shield on inductance and r-f resistance can be calculated (see Chapter 10 and Ref. 4, p. 134) and the results serve as a useful guide, but direct measurement on the complete transformer is the usual procedure. Mechanical considerations generally ensure that the can is thick enough to provide adequate shielding (for considerations of minimum thickness see Ref. 4, p. 135).

Methods for determining gear ratios, winding pitch and so on, for use with coil winding machines are discussed in the literature (Refs. 29, 30, 31, 32) ; see also Chapter 11, Sect. 3(iv) in particular.

Finally, the measure of the success of any i-f transformer design will be how closely the predicted performance approaches the actual results obtained when the transformer is connected into the receiver.

(vii) Appendix : Calculation of Coupling Coefficients

For the calculation of the coefficient of coupling (see also Chapter 9) it is helpful to use the following rules of procedure :

(a) If the coupling circuit is drawn as a T section, the coefficient of coupling is given by the ratio of the reactance of the shunt arm divided by the square root of the product of the sum of the reactances of the same kind in each arm including the shunt arm,

i.e. $$k = \frac{X_m}{\sqrt{(X_1 + X_m)(X_2 + X_m)}}.$$

(b) If the coupling circuit is drawn as a Π section, the coefficient of coupling is given by the ratio of the susceptance of the series arm divided by the square root of the product of the sum of the susceptances of the same kind in each arm including the series arm,

i.e. $$k = \frac{B_m}{\sqrt{(B_1 + B_m)(B_2 + B_m)}}$$

A few examples will now be given to illustrate the procedure. From rule (a applied to Fig. 9.15,

$$k = \frac{X_m}{\sqrt{(X_1 + X_m)(X_2 + X_m)}} = \frac{L_m}{\sqrt{(L_1 + L_m)(L_2 + L_m)}}.$$

For Fig. 9.12 using rule (b)

$$k = \frac{B_m}{\sqrt{(B_1 + B_m)(B_2 + B_m)}} = \frac{C_m}{\sqrt{(C_1 + C_m)(C_2 + C_m)}}.$$

For Fig. 9.14 using rule (a)

$$k = \frac{1/C_m}{\sqrt{\left(\dfrac{1}{C_1} + \dfrac{1}{C_m}\right)\left(\dfrac{1}{C_2} + \dfrac{1}{C_m}\right)}} = \sqrt{\frac{C_1 C_2}{(C_1 + C_m)(C_2 + C_m)}}.$$

For Fig. 9.13 using rule (b)

$$k = \frac{1/L_m}{\sqrt{\left(\dfrac{1}{L_1} + \dfrac{1}{L_m}\right)\left(\dfrac{1}{L_2} + \dfrac{1}{L_m}\right)}} = \sqrt{\frac{L_1 L_2}{(L_1 + L_m)(L_2 + L_m)}}.$$

It is sometimes helpful to make use of the obvious relationship in the form of k for the two types of circuit, by drawing dual networks and writing down the value of k for the duals as though capacitances were inductances and vice versa, e.g. the inductances of Fig. 9.13 are the duals of the capacitances of Fig. 9.14, and so the value of k is written down for, say, Fig. 9.14 and if k is required for Fig. 9.13 we merely substitute the symbols $L_1 L_2 L_m$ for $C_1 C_2 C_m$.

As a more difficult example the coefficient of coupling for Fig. 9 16(A) will be calculated. Firstly redraw the two coupled circuits as two T sections and then combine the two directly connected series arms. The complete coupling circuit now comprises a series arm $L_1 - M_1$; a Π section made up from M_1, $L_1' - M_1 + L_2' - M_2$, M_2; a series arm $L_2 - M_2$. Transforming the Π section to a T section and adding the two series arms of the previous circuit to the new series arms, we have a single T section in which the first series arm is

$$\frac{M_1(L_1' - M_1 + L_2' - M_2)}{L_1' + L_2'} + L_1 - M_1 = \frac{L_1(L_1' + L_2') - M_1(M_1 + M_2)}{L_1' + L_2'}$$

the second series arm is

$$\frac{M_2(L_1' - M_1 + L_2' - M_2)}{L_1' + L_2'} + L_2 - M_2 = \frac{L_2(L_1' + L_2') - M_2(M_1 + M_2)}{L_1' + L_2'}$$

and the shunt arm is $\dfrac{M_1 M_2}{L_1' + L_2'}$.

Applying rule (a), and writing $L_m = L_1' + L_2'$,

$$k = \frac{M_1 M_2}{\sqrt{[L_1 L_m - M_1(M_1 + M_2) + M_1 M_2][L_2 L_m - M_2(M_1 + M_2) + M_1 M_2]}}$$

$$= \frac{M_1 M_2}{\sqrt{(L_1 L_m - M_1^2)(L_2 L_m - M_2^2)}}$$

$$= \frac{k_1 k_2}{\sqrt{(1 - k_1^2)(1 - k_2^2)}}$$

where $k_1 = \dfrac{M_1}{\sqrt{L_1 L_m}}$ and $k_2 = \dfrac{M_2}{\sqrt{L_2 L_m}}$.

As a further example a bridged T coupling section will be considered, made up from inductances bridged by a capacitance. This corresponds to the familiar case of a mutual inductance coupled transformer with added top capacitance coupling (e.g Fig. 9.16D) and also refers to any practical i-f transformer because of the presence of stray capacitance affecting the coupling between the primary and secondary windings.

The procedure in this case is to use rule (a) and determine the coupling coefficient due to mutual inductance (the inductances are considered as forming a T network); call this k_m. Next use rule (b) to determine the coupling coefficient due to capacitance coupling (the capacitances form a Π network); call this k_c. Then if the connections are such that the two forms of coupling aid each other, the resultant coupling coefficient is

$$k_T = \frac{k_m + k_c}{1 + k_m k_c}$$

and if $k_m k_c \ll 1$ then

$k_T = k_m + k_c$ (approximately).

For the case of the two forms of coupling being in opposition

$$k_T = \frac{k_m - k_c}{1 - k_m k_c}$$

and if $k_m k_c \ll 1$ then

$$k_T = k_m - k_c \text{ (approximately)}.$$

The above expressions for k_T can be derived quite simply from data given in Ref. 101, or directly using a similar method to that set out below. The results are most helpful in determining rapidly the effect of additional top capacitance coupling e.g. an i-f transformer is connected between the last i-f amplifier valve and a diode detector, and the a.v.c. diode is connected to the primary of the transformer in the usual manner. Then clearly there is top capacitance coupling added across the i-f transformer due to the direct capacitance between the two diodes, and a knowledge of the .value of this capacitance will allow the added coupling to be taken into account when designing the transformer. If the capacitance value is not known its effect can be estimated when a response curve is taken.

This method for finding k_T can be applied in exactly the same way to a bridged T coupling network in which the two main series arms are inductances with mutual inductance coupling between them, the shunt arm is a capacitance and the T is bridged by a capacitance.

An alternative procedure, and a most important one, for determining k will now be discussed. This method can be applied quite generally to determining coupling coefficients for Π and T networks which have $L_1 C_1 = L_2 C_2$. The method is due to Howe and the reader should consult Refs. 99 and 100 for a more detailed explanation. This procedure uses the relationship

$$k = \frac{\omega_1^2 - \omega_2^2}{\omega_1^2 + \omega_2^2}$$

where ω_1 is the higher angular frequency of free oscillation in the circuit and ω_2 is the lower angular frequency of free oscillation.
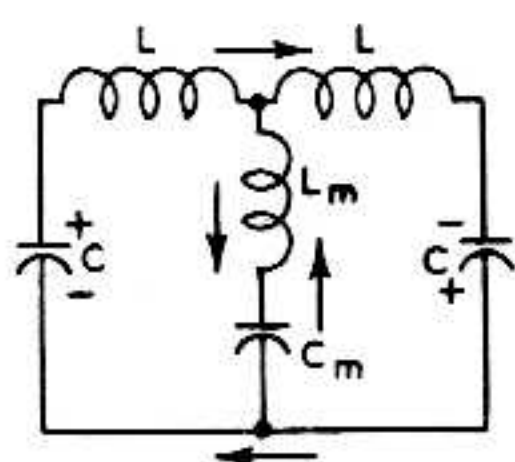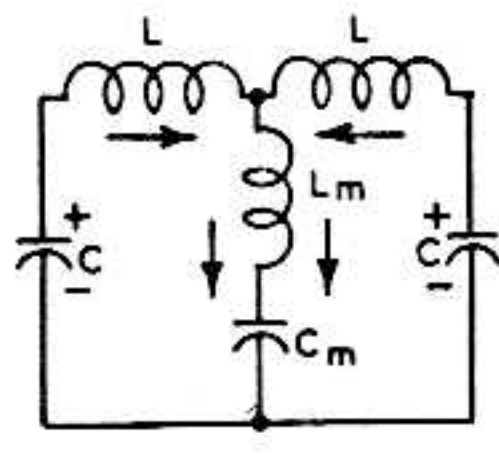

FIG. 26.6 A   FIG. 26.6B

ω_1 is calculated from Fig. 26.6A, which shows a symmetrical T network with mixed coupling, by considering the capacitors C to be charged as shown. During the discharge of the capacitors it is clear that $L_m C_m$ have no effect on the resonant frequency in this case, and

$$\omega_1^2 = 1/LC.$$

ω_2 is calculated from Fig. 26.6B in which the capacitors C are charged as shown. In this case, when the capacitors are discharging, it is convenient to consider the shunt arm as being made up of two equal parts each carrying the primary and secondary current alone; the reactances of L_m and C_m being doubled to allow for the single current. Then

$$\omega_2^2 = \frac{1}{(L + 2L_m)\left(\dfrac{C C_m/2}{C + C_m/2}\right)}$$

so that $k = \dfrac{L_m C_m - LC}{L_m C_m + LC + LC_m}$.

A somewhat similar procedure can be applied to a Π network, or an equivalent T network can be found and the above procedure applied.

For cases where the sections are unsymmetrical some modification in the procedure is necessary. Taking the case of a T section using inductance coupling (with $L_1 C_1 = L_2 C_2$) the shunt arm is replaced by two inductances (x_1 and x_2) in parallel such that their combined inductance equals that of the shunt arm (L_m) and such that the values selected will give equal resonant frequencies for the two circuits

$$\text{i.e. } \frac{L_1 + x_1}{L_2 + x_2} = \frac{C_2}{C_1}.$$

The case of capacitance coupling is treated in a similar way. The procedure is then exactly as before for the symmetrical coupling network. There would be little difficulty in extending this procedure to the case of the unsymmetrical T section using mixed coupling, but it is worth noting that numerical solutions are much easier to handle than a general algebraic solution even in the case of simple coupling.

For the two mesh network just considered, using mixed coupling and in which $L_1 C_1 = L_2 C_2$ it is usually simpler to use the relationship

$$k_T = \frac{k_m \pm k_c}{1 \pm k_m k_c}$$

as was done for the bridged T network. Taking the example just considered,

$$k_m = \frac{L_m}{L + L_m}$$

$$k_c = \frac{C}{C + C_m}$$

and
$$k_T = \frac{\dfrac{L_m}{L + L_m} - \dfrac{C}{C + C_m}}{1 - \dfrac{L_m C}{(L + L_m)(C + C_m)}}$$

$$= \frac{L_m C_m - LC}{L_m C_m + LC + LC_m}$$

exactly as before.

It should be carefully noted that in all cases of mixed coupling the sign of L_m is most important. For the example just given L_m was taken as being positive ; this is the case of the two forms of coupling being in opposition, and zero voltage transfer occurs when the shunt arm is series resonant. With L_m negative the two forms of coupling aid each other and there is no series resonant frequency. The aiding condition, which is the one usually required, is possible only when the coupling between the two inductances is due to mutual induction, as a physical coupling inductance would act so as to oppose the capacitive coupling.

The methods given so far, for determining the coupling coefficient, are useful for the types of circuit considered. However, it should be observed that they can be applied directly only to the following cases :

(1) Circuits having coupling elements all of the one kind i.e. all capacitances or all inductances (simple coupling).

(2) Circuits having mixed coupling in which the primary and secondary circuits are tuned to the same frequency. However, it is only necessary that $L_1 C_1 = L_2 C_2$ and the primary and secondary circuit elements need not be identical.

Determination of k for the cases just stated usually presents little difficulty and is simplified by choosing one or other of the procedures outlined ; , these cases cover most practical requirements. A real difficulty does arise when the primary and secondary are tuned to different frequencies and the coupling is mixed.

It may now be of interest to consider the procedure given below which makes use of the result obtained by Howe (Ref. 98) and follows from the definition that the coupling between two circuits is the relation between the possible rate of transfer of energy and the stored energy of the circuits ; by the possible rate of energy transfer is meant the rate of energy transfer in the absence of all resistance other than that utilized for coupling.

The definition leads to the relationship

$$k = \frac{1}{\sqrt{\omega_1 \omega_2}} \sqrt{\frac{E_{12}}{W_1} \cdot \frac{E_{21}}{W_2}}$$

where we have taken for our purposes that

$\omega_1 = 2\pi \times$ resonant frequency of primary circuit with sec. o/c

$\omega_2 = 2\pi \times$ resonant frequency of secondary circuit with pri. o/c

E_{12} = voltage transferred from primary to secondary

    = $Z_{m_{12}}$ for a primary alternating current of 1 amp. and angular frequency ω_1

E_{21} = voltage transferred from secondary to primary

    = $Z_{m_{21}}$ for a secondary alternating current of 1 amp. and angular frequency ω_2

W_1 = maximum energy stored in primary circuit for an alternating current of 1 amp.

W_2 = maximum energy stored in secondary circuit for an alternating current of 1 amp.

This method can be applied to any two oscillatory circuits coupled together in any manner, when all the elements are linear and bilateral.

As a simple example consider the case covered previously of an unsymmetrical T section of inductances. Then

$$Z_{m_{12}} = \omega_1 L_m ; \quad Z_{m_{21}} = \omega_2 L_m ; \quad W_1 = L_1 + L_m ; \quad W_2 = L_2 + L_m$$

and $$k = \frac{1}{\sqrt{\omega_1 \omega_2}} \sqrt{\frac{\omega_1 L_m \omega_2 L_m}{(L_1 + L_m)(L_2 + L_m)}}$$

$$= \frac{L_m}{\sqrt{(L_1 + L_m)(L_2 + L_m)}} \text{ exactly as before.}$$

It can be seen that the primary and secondary frequencies need not be considered in cases of this type involving simple coupling only.

Now consider an unsymmetrical T section with mixed coupling, and different primary and secondary resonant frequencies :

$$Z_{m_{12}} = \omega_1 L_m - \frac{1}{\omega_1 C_m} = \omega_1 \left(L_m - \frac{1}{\omega_1^2 C_m} \right)$$

$$Z_{m_{21}} = \omega_2 L_m - \frac{1}{\omega_2 C_m} = \omega_2 \left(L_m - \frac{1}{\omega_2^2 C_m} \right)$$

$$W_1 = L_1 + L_m$$
$$W_2 = L_2 + L_m$$

$$k = \frac{1}{\sqrt{\omega_1 \omega_2}} \sqrt{\frac{\omega_1 \omega_2 \left(L_m - \frac{1}{\omega_1^2 C_m} \right) \left(L_m - \frac{1}{\omega_2^2 C_m} \right)}{(L_1 + L_m)(L_2 + L_m)}}$$

$$= \sqrt{\frac{\left(L_m - \frac{1}{\omega_1^2 C_m} \right) \left(L_m - \frac{1}{\omega_2^2 C_m} \right)}{(L_1 + L_m)(L_2 + L_m)}}$$

and since

$$\omega_1^2 = \frac{C_1 + C_m}{(L_1 + L_m)C_1 C_m}$$

$$\omega_2^2 = \frac{C_2 + C_m}{(L_2 + L_m)C_2 C_m}$$

the expression for k can be rewritten as

$$k = \sqrt{\frac{(L_m C_m - L_1 C_1)(L_m C_m - L_2 C_2)}{(L_1 + L_m)(L_2 + L_m)(C_1 + C_m)(C_2 + C_m)}}.$$

It should be carefully noted that the ω_1 and ω_2 used here are not the same as found previously for the natural resonant frequencies of the coupled circuits.

If now we take $L_1 = L_2 = L$ and $C_1 = C_2 = C$,

$$k = \frac{L_m C_m - LC}{(L + L_m)(C + C_m)}.$$

Since this case corresponds to that used in connection with Figs. 26.6A and 26.6B it would be expected that the results would correspond. However, it is seen that there is an additional term $L_m C$ in the denominator of the second solution. This is explained by the method used for determining ω_1 and ω_2 in the latter case, and so the method as given here cannot be considered exact. The approximation involved in this particular example is that the result is equivalent to

$$k_T = k_m \pm k_c \text{ and not } k_T = \frac{k_m \pm k_c}{1 \pm k_m k_c}.$$

This is seen quite readily because

$$k_m = \frac{L_m}{L + L_m} ; \ k_c = \frac{C}{C + C_m}$$

and so $k_T = \dfrac{L_m(C + C_m) - C(L + L_m)}{(L + L_m)(C + C_m)} = \dfrac{L_m C_m - LC}{(L + L_m)(C + C_m)}$

which is the result just obtained.

A rather similar difficulty occurs when determining the central reference frequency for the transformer of Fig. 26.2. The angular frequency (ω) is determined exactly as above by considering the secondary on open circuit, and for this case

$$\omega = \sqrt{\frac{\omega_1^2 + \omega_2^2}{2}}$$

where ω_1 and ω_2 are now determined in the same way as for Figs. 26.6A and 26.6B.

SECTION 5 : VARIABLE SELECTIVITY

(i) *General considerations*   (ii) *Automatic variable selectivity*.

(i) General considerations

Any method giving variable-coupling may be used to provide variable selectivity.

Variable-coupling by means of pure mutual inductance is the only system in which mistuning of the transformer does not occur without introducing compensation of other component values. As the coupling is increased above the critical value, the trough of the resonance curve remains at the intermediate frequency (Ref. 8).
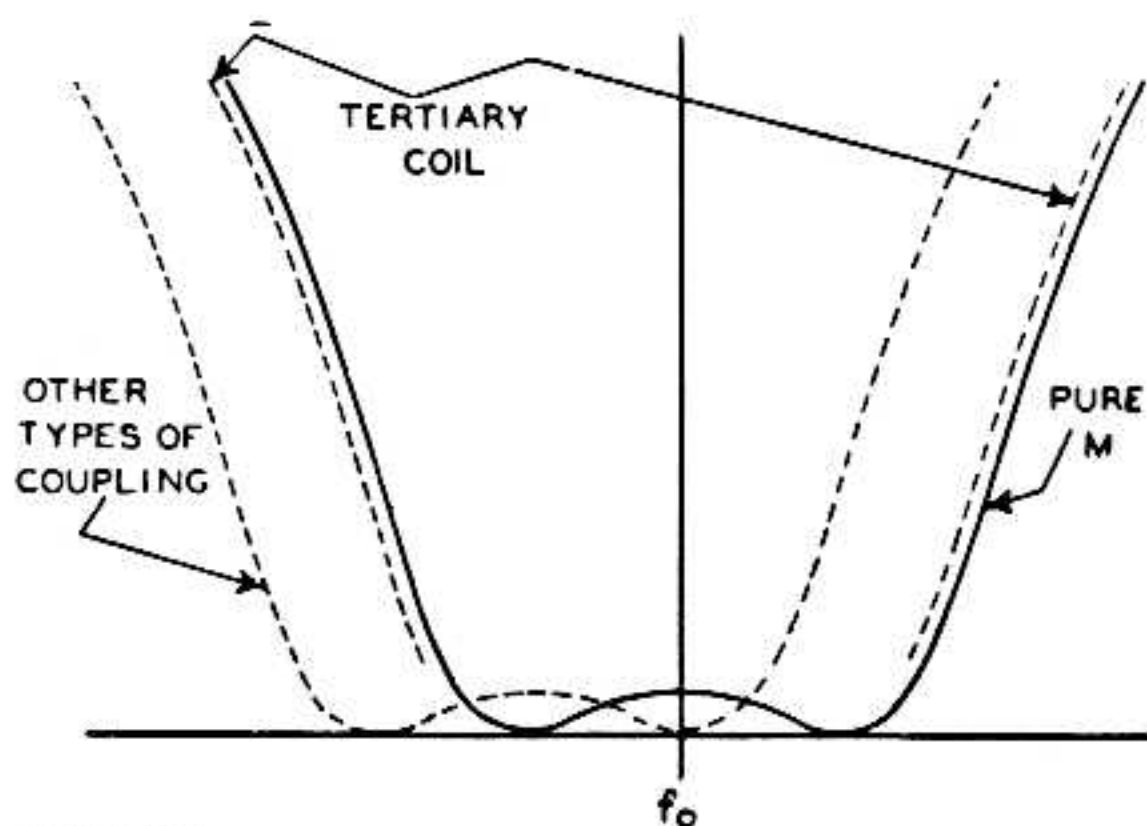

FIG. 26·7

With any other type of coupling which has no mutual inductance component, one of the two humps remains at the intermediate frequency. The mistuning is then half the frequency bandwidth between humps.

The method of switching tertiary coils approximates far more closely to the curve for pure mutual inductance (M in Fig. 26.7) than it does to the other curve. The

tertiary coil may have no more than 5% of the turns on the main tuning coil and less than 0.5% ratio of inductance. The symmetry of the overall selectivity curves is usually good.

Variable capacitance coupling may be used and the coupling capacitor may be either a small capacitor linking the top end of the primary to the top end of the secondary or it may be a common capacitor in series with both primary and secondary circuits. This latter arrangement is commonly known as " bottom coupling."

For " top coupling " very small capacitances are required and the effect of stray capacitances is inclined to be serious, particularly in obtaining low minimum coupling. It may be used, however, with a differential capacitor arrangement whereby continuously variable selectivity is obtained (Refs. 41, 42). The differential capacitance in this case adds to or subtracts from the capacitance in the primary and secondary circuits to give the requisite tuning. The disadvantage of this arrangement is that sufficiently low minimum coupling is very difficult to obtain and the capacitor is a non-standard type.

" Bottom coupling " has results similar to " top coupling," but is easier to handle for switching and also has advantages for low coefficients of coupling. A two or three step tapping switch may be used to give corresponding degrees of bandwidth provided that simultaneously other switch contacts insert the necessary capacitances in the primary and secondary circuits, for each switch position, to give correct tuning.

TWO POSITION SELECTIVITY   FIG. 26. 7A
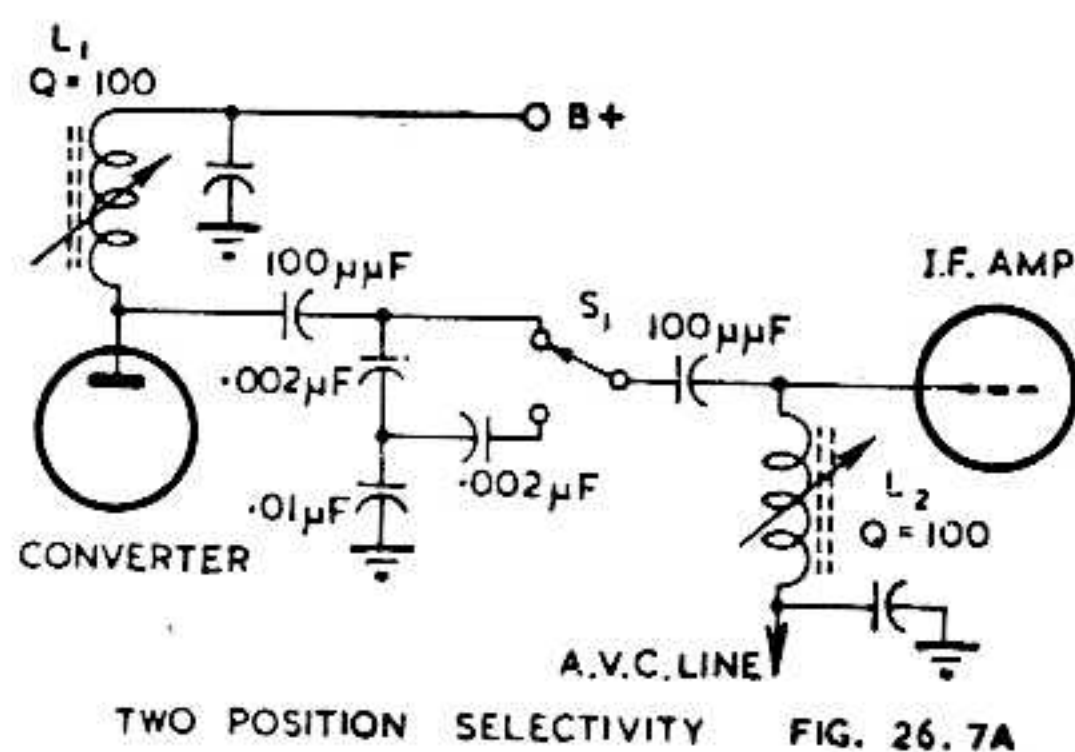
One such bottom coupling method which requires the minimum of switching for two degrees of coupling is shown in Fig. 26.7A (Ref. 42a). The two coils L_1 and L_2 should each be in a separate shield can. With switch S_1 in the position shown, the selectivity is broad ; in the lower position the selectivity is normal. Capacitances as shown are only illustrative. The same principle can be used for three values of coupling, using seven capacitors.

In most cases, the second i-f transformer is slightly under-coupled and its peak is used to fill in the " trough " between the peaks of the first transformer in its broad position.

A wide variety of circuit arrangements for obtaining variable selectivity has been described in the literature. Design methods and practical constructional details for many of these can be found in the references listed at the end of this chapter. Further discussion is also given in Chapter 11, Sect. 3(iii).

(ii) Automatic variable selectivity

Automatic variable selectivity allows the bandwidth of the i-f amplifier to be varied with signal strength, the pass band being a maximum for the strongest signal.

Many types of circuits have been developed using variable circuit damping, varying coupling reactance and circuit detuning. A number of the circuits are not very practical arrangements since they involve the addition of a number of valves whose sole function is to provide variable bandwidth.

One interesting circuit arrangement (Ref. 46), which does not require any additional valves, is shown in Fig. 26.8.

To understand the operation of the circuit it is first necessary to consider what happens in an ordinary mutual inductance coupled transformer. It is well known,

when two circuits are coupled together, that there is a back e.m.f. induced in the primary winding from the secondary which will have a marked effect on the magnitude and phase characteristics of the primary current as a function of frequency. The primary current response curve exhibits a double peak at values of k even less than critical, because the back e.m.f. represents a voltage drop that decreases rapidly on either side of the resonant frequency. Double peaking of the primary current is masked in its effect on the secondary current by the selectivity of the secondary circuit. When the coupling exceeds critical (actually transitional) coupling the secondary current exhibits two peaks, but to a lesser extent than the primary current.
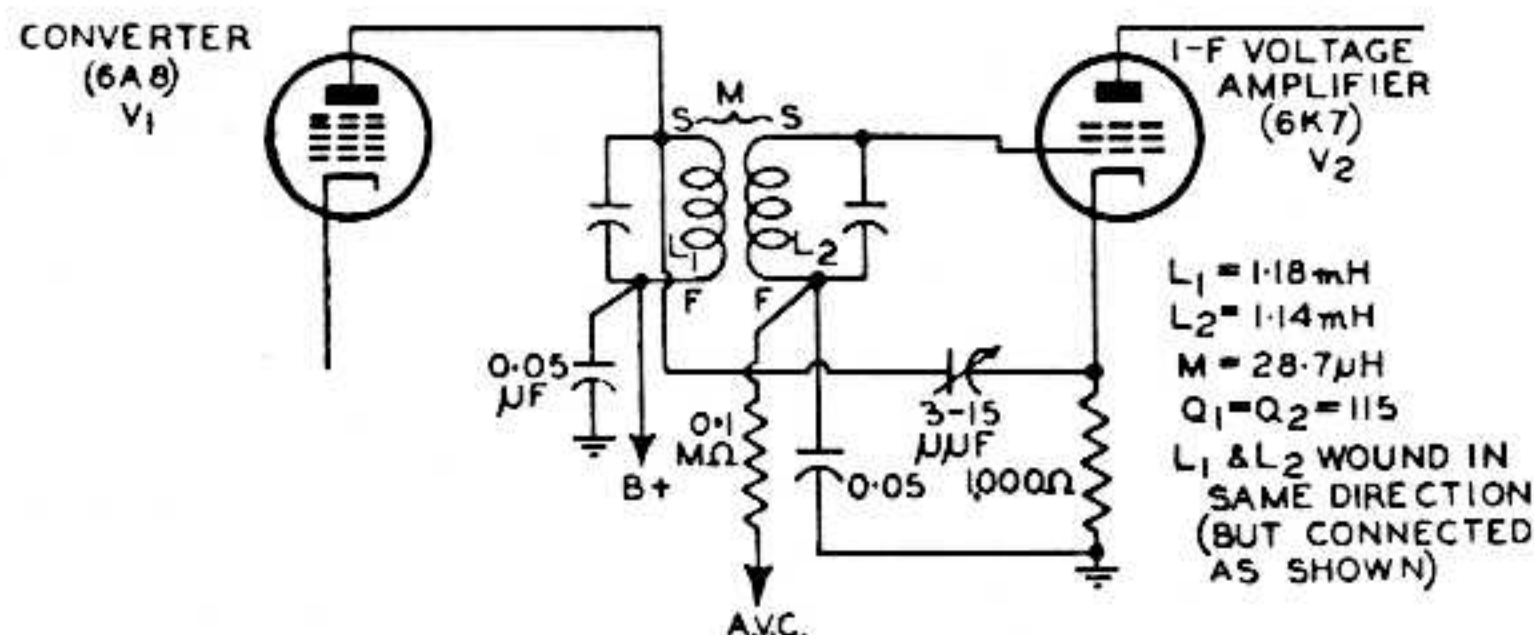

FIG. 26·8   AUTOMATIC SELECTIVITY ARRANGEMENT

If the magnitude of the counter e.m.f. is increased by increasing the mutual inductance, and at the same time a voltage of almost equal magnitude and opposite phase is added in series with the primary circuit, a single peak of output current can be obtained in the transformer secondary. The additional series voltage, so introduced, will increase the stage gain and selectivity from the condition which existed prior to its introduction ; in our case the prior condition is that the stage gain is less and the secondary current has two peaks, i.e. the transformer is initially over-coupled.

In the circuit of Fig. 26.8 the valve V_2 performs the dual functions of amplification and production of the series voltage to be added to the primary circuit. A resistance in the cathode of V_2 produces an a.c. voltage drop, which is proportional to the g_m of the valve and the secondary current of the transformer ; this voltage drop across the cathode resistor is the one applied back to the transformer primary. When a strong signal is being received a.v.c. will increase the bias on the grid of V_2 and so its g_m is reduced. When the g_m (and consequently the voltage drop across the cathode resistor) is sufficiently small, peak separation is obtained ; the width between the peaks depending on the value of mutual inductance. Weak signals reverse the process and narrow down the overall response curve.

A quantitive analysis (setting out design data and instability conditions) is given in the Ref. (46). For the circuit of Fig. 26.8 the peak separation is given as 8 Kc/s for an i-f of 450 Kc/s. Other designs are shown for peak separations of 15 Kc/s and also for obtaining two position automatic selectivity ; the latter arrangement necessitates the use of a relay.

Other types of automatic selectivity are discussed in the references listed.

SECTION 6 : VARIABLE BANDWIDTH CRYSTAL FILTERS

(i) Behaviour of equivalent circuit   (ii) Variable bandwidth crystal filters   (iii) Design of variable bandwidth i-f crystal filter circuits (A) Simplifying assumptions (B) Gain (C) Gain variation with bandwidth change (D) Selectivity (E) Crystal constants (F) Position of filter in circuit (G) Other types of crystal filters   (iv) Design example.

(i) Behaviour of equivalent circuit

Fig. 26.9 shows the generally accepted equivalent electrical circuit for a quartz crystal of the type used in the i-f stage of a communications receiver.

In this network suppose we consider that R is zero, then between terminals 1 and 2 there is a reactance

$$X = \frac{\omega L - 1/\omega C}{\omega C_0 \left[\omega L - \frac{1}{\omega}\left(\frac{1}{C} + \frac{1}{C_0}\right)\right]}. \tag{28}$$

When $\omega L = 1/\omega C$ the reactance is zero and the circuit is series resonant.

When $\omega L = \frac{1}{\omega}\left(\frac{1}{C} + \frac{1}{C_0}\right)$ the reactance is infinitely large, and the circuit is parallel resonant (anti-resonant).

The difference in frequency between f_p and f_r is given approximately by

$$\Delta f = \frac{C}{2C_0} f_r.$$

From the values of the 455 Kc/s crystal constants which are given in (iii)E below, it will be seen that Δf is about 250 c/s in a typical case.

The presence of R will slightly modify the conditions for which parallel resonance occurs. For our purposes the conditions given are near enough.

It should be clear from the circuit that the parallel resonance frequency (f_p) will be higher than the series resonance frequency (f_r). For frequencies above f_r the resultant reactance due to L and C in series is inductive, and this is connected in parallel with a capacitive reactance due to C_0. A typical curve of output voltage versus frequency change, for this type of circuit, is shown in Fig. 26.10.

If the value of C_0 could be altered as required, then the position of f_p would be variable. Suppose by some means we are able to connect a negative capacitance C_N (or a parallel inductance will give somewhat similar results) across C_0 ; then the value of the capacitance shunted across the series circuit will be

$$C' = C_0 - C_N. \tag{29}$$
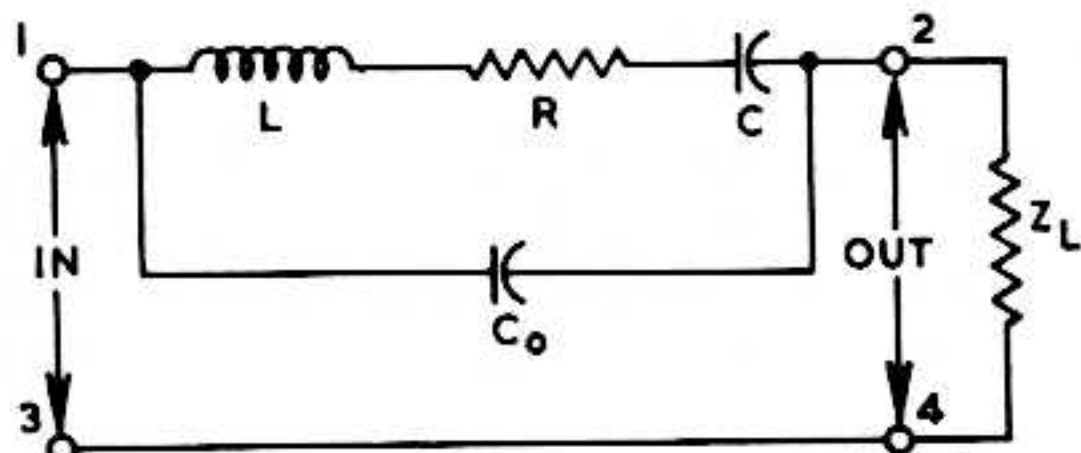
ELECTRIC CIRCUIT REPRESENTATION
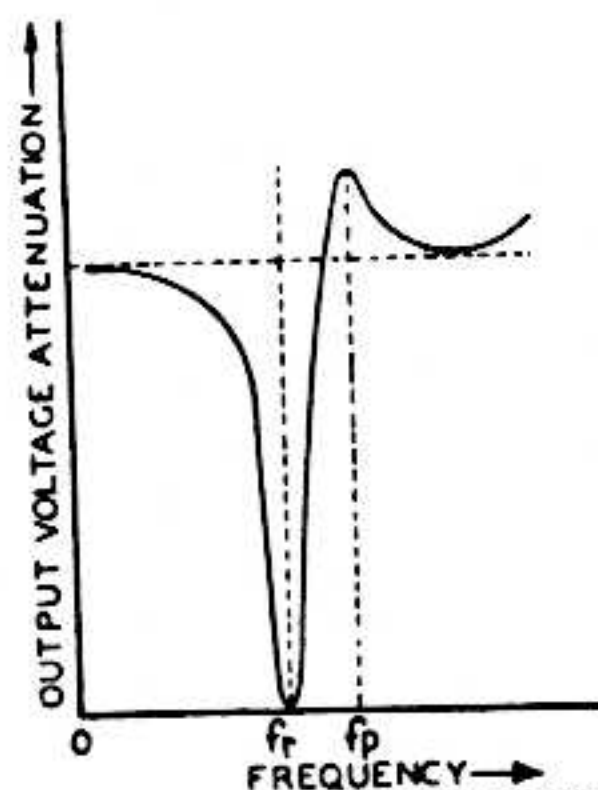FIG. 26.9   OF QUARTZ CRYSTAL

FIG. 26·10 RESPONSE CURVE FOR CIRCUIT OF FIG. 26·9

From this, it follows that C' can be reduced from its initial value of C_0 (when C_N is zero) until it becomes zero, and C_N has then exactly neutralized C_0. The response curve would now be that for the series circuit (made up from L, R and C) alone, which behaves as a pure series resistance at f_r. If the magnitude of C_N is further increased, then C' becomes negative and the series circuit is shunted by a negative capacitance ; which is equivalent to shunting an inductance across the series circuit (which behaves like a capacitive reactance for frequencies below f_r). This means that the parallel resonance frequency (f_p) will now be lower than the series resonance frequency (f_r). Any frequency above or below the series resonance frequency can now be chosen as the rejection, or parallel resonance, frequency and the current through the circuit, at this point, will be reduced ; the values for f_p and the current depending on the circuit constants.

To achieve the variation in C' the circuit of Fig 26.9 (i.e. the crystal) is incorporated in the bridge circuit of Fig. 26.11, in which C_N is made variable to achieve the results discussed. In this circuit Z_L is the load impedance ; Z_s is the impedance of the voltage source ; Z_1 and Z_2 are any two impedances used to make up the resultant

bridge circuit. It will be realized that the presence of Z_s and Z_L would have some modifying effect on our previous discussion, but the general principles remain unchanged ; these impedances will be taken into account when the design of the crystal filter stage is carried out.

FIG. 26-11 CIRCUIT ARRANGEMENT FOR VARYING $C'(=C_0-C_N)$

The arrangement of any practical circuit using a single crystal can be reduced to the general form of Fig. 26.11.

Two typical circuit arrangements are shown in Fig. 26.12. Fig. 26.12(A) shows that Z_1 and Z_2 are obtained by using two capacitances C_1 and C_2 for the bridge arms. Fig. 12(B) uses a tap on the i-f transformer secondary (L_1 and L_2) to obtain the bridge arms Z_1 and Z_2.

In any circuit arrangement such as those shown, the best overall selectivity is obtained when C' is zero (i.e. $C_N = C_0$). Under this condition, if the circuit is designed with some care, the response curve of output voltage with frequency change will be reasonably symmetrical. If C_N is varied to place f_p above or below f_r, although we may improve the selectivity (and so the rejection) against an unwanted signal at f_p, the resonance curve is no longer symmetrical and the selectivity is decreased on the opposite side of the curve.
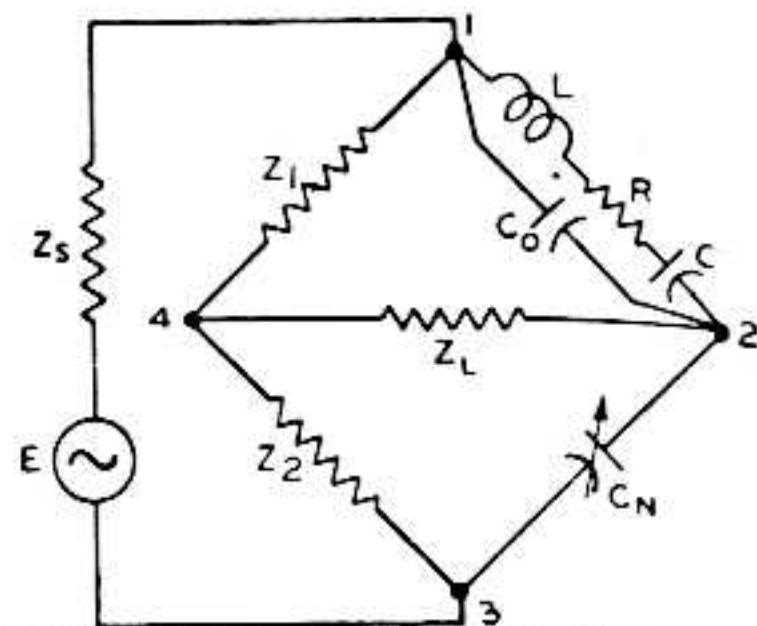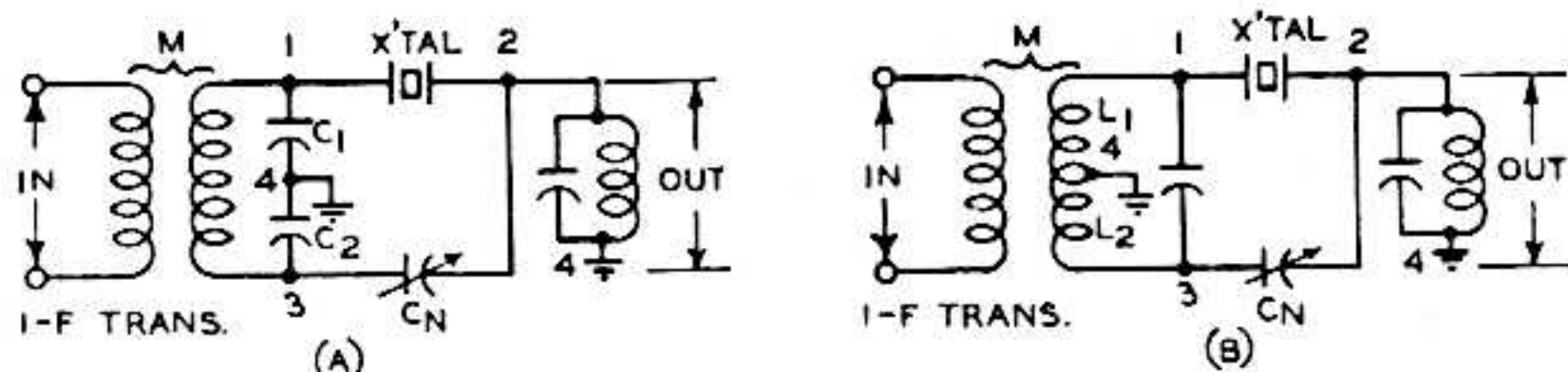

FIG. 26-12   PRACTICAL CIRCUIT ARRANGEMENTS FOR QUARTZ CRYSTAL FILTERS

(ii) Variable bandwidth crystal filters

It is often necessary, in practical receivers, to have means available for varying the bandwidth of the crystal filter circuit. This can be achieved in a number of ways, such as detuning the input and (or) output circuits or varying their resonant impedances. Also, as well as having variable bandwidth it is desirable that there should not be appreciable change in stage gain as the bandwidth is altered. Further, the overall response curve of voltage output versus frequency should remain as symmetrical as possible with changes in bandwidth when the bridge circuit is arranged so that C_0 is neutralized by C_N.

To fulfil all of the conditions above, a simple solution is to use a tuned circuit as the load for the filter circuit, and to alter its dynamic impedance by switching-in either series or parallel resistors. Simple detuning of the input circuit, or the output circuit, is not very satisfactory and the best results can only be achieved by detuning both the input and output circuits in opposite directions by an amount depending on their relative Q's. (For a description of circuits using the latter method see Refs. 48 and 63). To obtain constancy of gain it is necessary that the voltage source impedance be low, and this again suggests the switched output circuit when the bandwidth is to be varied. The choice of series or shunt resistors to alter bandwidths is largely bound up in stray capacitances across switch contacts and the resistors themselves ; careful consideration here suggests that resistors in series with the inductor, or the capacitor, of the tuned load will probably give the least detuning effects. Unless the values of the tuning capacitances for the input and load circuits are fairly large, alteration of C_N will have some appreciable effect on the resonant frequencies of these circuits, giving rise to asymmetry of the response curve and loss in sensitivity. This effect can sometimes be overcome, partly, by the use of a differential type of neutralizing capacitor. However, circuits using this arrangement should be carefully examined as usually there are still some detuning effects. For most practical cases it is sufficient to choose suitable values of tuning capacitance, particularly when

operation is confined to an i-f of 455 Kc/s, and to tap down on the circuit when this is possible.

Two other points are worth mentioning. The first is that the crystal stage gives high selectivity around resonance, but the " skirt " selectivity may be quite poor; for this reason the other stages in the receiver must provide the additional " skirt " selectivity required. In addition, good " skirt " selectivity is a requirement of the i-f amplifier to minimize any possible undesirable effects which may arise because of crystal subsidiary resonances. The other point is that having C_N as a variable control is not necessarily a great advantage, and simpler operation is obtained when C_N can be pre-set to neutralize C_0. This cannot be done with all types of variable bandwidth circuits, since the conditions for neutralization may be altered as the bandwidth is changed.

(iii) Design of variable bandwidth i-f crystal filter circuits
(A) Simplifying assumptions

From Fig. 26.11, when a capacitance balance is obtained as far as C_0 is concerned,
$$C_N \approx C_0(Z_1/Z_2). \tag{30}$$
Since C_0 is generally about 17 $\mu\mu$F it is convenient to make $Z_1 = Z_2$. Also, C_N must have very low losses if the attenuation at the rejection frequencies is to be high. The general design will call for all capacitors to be of the low loss type. When the condition of eqn. (30) is fulfilled, the equivalent circuit reduces to that of Fig. 26.13(A).
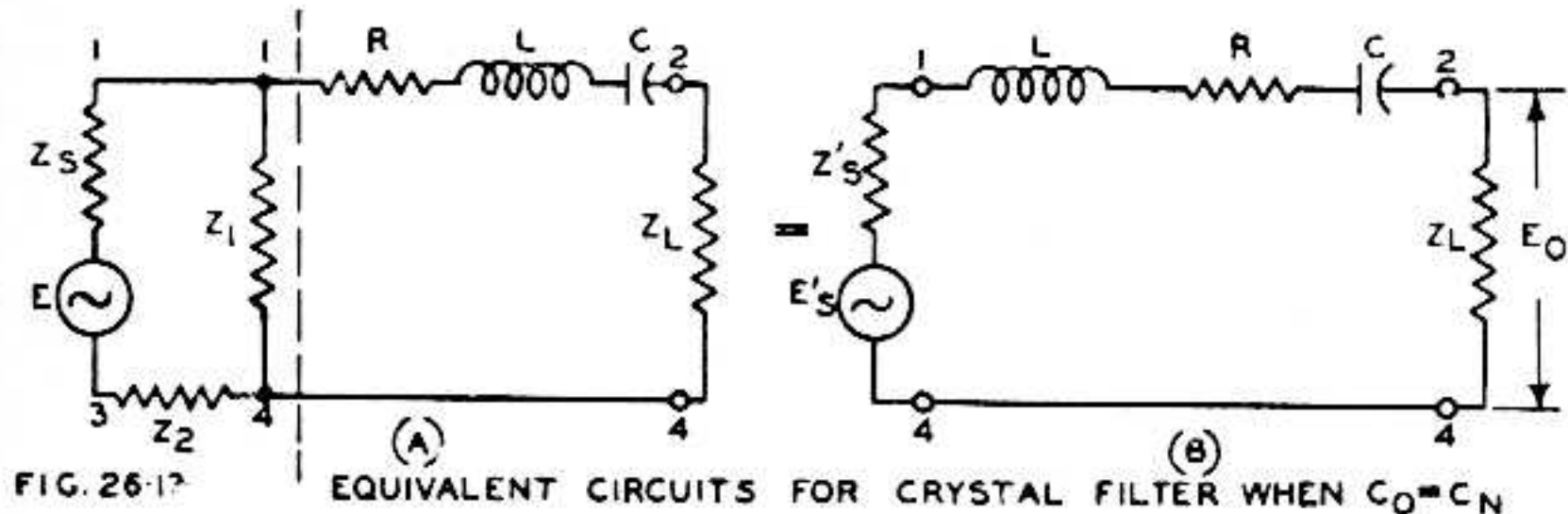

FIG. 26.13   EQUIVALENT CIRCUITS FOR CRYSTAL FILTER WHEN $C_0 = C_N$

For purposes of analysis it is convenient to rearrange Fig. 26.13(A) as shown in Fig. 26.13(B), and, if we take $Z_1 = Z_2$, the value of Z'_s will be a quarter of the total dynamic impedance of the secondary circuit of the i-f input transformer. E'_s will be half the voltage developed across the two series capacitors tuning the i-f transformer secondary.

(B) Gain

First we will derive an expression for the overall gain of the i-f stage.
From Fig. 26.13(B),
$$\frac{E_0}{E'_s} = \frac{Z_L}{Z_L + Z'_s + Z_x} \tag{31}$$
where E_0 = output voltage across load circuit applied to grid of following i-f amplifier valve

Z_L = load impedance

Z'_s = voltage source impedance

Z_x = crystal impedance (C_0 neutralized)

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= R \text{ at resonance (i.e. i-f)}$$

FIG. 26.14   CIRCUIT FOR DETERMINING E_s'

and E'_s = available voltage developed at tap on input i-f transformer secondary.

To compute the complete stage gain we must now consider Fig. 26.14. Here, since we require maximum gain, a critically-coupled transformer is connected to a voltage amplifier valve (V_1) whose grid to cathode input voltage is E_g. Then since E'_s is half the total voltage appearing across C_1 and C_2

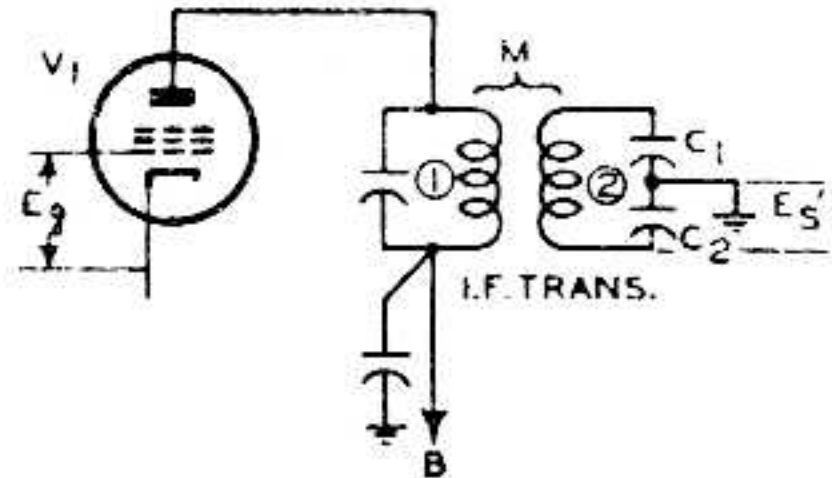$$E'_s = \frac{1}{2}\left(\frac{g_m E_g Q \omega_0 L}{2}\right) \tag{32}$$

where g_m = mutual conductance of V_1

$$Q = \sqrt{Q_1 Q_2} = 1/k_c$$
$$\omega_0 = 2\pi \times i\text{-}f$$

and $L = \sqrt{L_1 L_2}$ (always provided that $L_1 C_1 = L_2 C_2$).

Combining eqns. (31) and (32), the gain from the grid of V_1 to the grid of the next amplifier valve (V_2) is

$$\text{Stage gain} = \frac{E_0}{E'_s} \times \frac{E'_s}{E_g} = \frac{E_0}{E_g} = \left(\frac{Z_L}{Z_L + Z'_s + Z_x} \right) \left(\frac{g_m Q \omega_0 L}{4} \right). \tag{33}$$

Eqn. (33) gives a great deal of information about the circuit. For constancy of gain, it follows that Z_L should be very much greater than $(Z'_s + Z_x)$. Since Z_x (or R at resonance) cannot be altered by the receiver designer, it is necessary to make Z'_s as small as possible. However, as Z'_s is made smaller the overall gain will be affected since it is a centre tap on the output of the input i-f transformer. The maximum value for Z_L will be limited by the maximum bandwidth requirements, and the permissible values of circuit constants. The other factors affecting gain are g_m, Q and L. For a given valve under a fixed set of operating conditions, g_m is practically outside the designer's control ; the value of $L(=\sqrt{L_1 L_2})$ is made as large as possible consistent with the requirements of minimum permissible tuning capacitance ; $Q = \sqrt{Q_1 Q_2}$ is adjusted so that Q_1 is made as high as possible, and since we desire Z'_s to be low a suitable value is selected for Q_2. The method of determining Q has a large effect on the stage gain which can be obtained, as will be seen later in the illustrative example.

(C) Gain variation with bandwidth change

For constancy of maximum gain at f_0 (i.e. the i-f) it has been suggested that Z'_s should be small. There is a limit, however, to the minimum gain variation that can be obtained for a given change from maximum to minimum bandwidth. From the preceding equations it may be deduced that the maximum stage gain (A_{max}) is obtained when the bandwidth is greatest, and the minimum gain (A_{min}) when the bandwidth is least. This limit in gain variation (since R for the crystal is fixed) is the condition for which the voltage source impedance (Z'_s) becomes zero, which occurs when

$$\alpha_0 = \frac{A_{min}}{A_{max}} = \frac{R_{T_2}(R_{T_1} - R)}{R_{T_1}(R_{T_2} - R)} \tag{34}$$

where R_{T_1} = total series resistance (at f_0) when the bandwidth is least

                $(= R + Z'_s + Z_L$; in which Z_L has its smallest value)

and R_{T_2} = total series resistance (at f_0) when the bandwidth is greatest

                $(= R + Z'_s + Z_L$; in which Z_L now takes up its largest value).

To determine the voltage source impedance (Z'_s) for the usual condition where Z'_s is not zero, we use

$$Z'_s = \frac{R_{T_2}(R_{T_1} - R) - \alpha R_{T_1}(R_{T_2} - R)}{R_{T_2} - \alpha R_{T_1}} \tag{35}$$

which, for convenience, is rewritten in terms of eqn. (34) as

$$Z'_s = \frac{\alpha_0 - \alpha}{\dfrac{\alpha_0}{R_{T_1} - R} - \dfrac{\alpha}{R_{T_2} - R}} \tag{35A}$$

where R_{T_1} and R_{T_2} have the values given immediately above,

     R = equivalent series resistance of crystal

and $\alpha = \dfrac{A_{min}}{A_{max}}$ = gain variation desired, and must always be less than one.

If a value is selected for Z'_s, then

$$\alpha = \frac{R_{T_2}(R_{T_1} - R) - Z'_s R_{T_2}}{R_{T_1}(R_{T_2} - R) - R_{T_1} Z'_s}. \tag{36}$$

(D) Selectivity

Selectivity around resonance can be calculated by the methods to be outlined, but for most purposes it is sufficient to determine the bandwidth for a given attenuation at the half power points (3 db attenuation or $1/\sqrt{2}$ of the maximum voltage output) on each of the required selectivity curves.

Considerable simplification in the design procedure is possible, if the bandwidths for 1 db attenuation (or less) are known. In this case it would be sufficiently accurate to take Z_L and Z_s equal to their dynamic resistances at resonance, at least for most practical conditions, without introducing appreciable error. The advantage obtained being that, in what follows, $R_{T_1} = R'_{T_1}$ and $R_{T_2} = R'_{T_2}$ for all conditions. The value of Q_x would be given by $Q_x = f_0/4\Delta f$ for 1 db attenuation, and this expression would be used in place of eqn. (38). However, the procedure given is more general and there should be little difficulty in applying the simplified procedure, if necessary. Further the method given illustrates (in reverse) how the bandwidths near resonance can be calculated for various amounts of attenuation.

To determine the bandwidth at the half power points proceed as follows : first it may be taken that the equivalent inductive reactance of the crystal is very much greater than the inductive reactance of the load and source impedances. Also, provided the variations in gain are not allowed to become excessve, at and near resonance the value of Z'_s is very closely R'_s. For very narrow bandwidths, near resonance, it is also sufficiently close to take $Z_L = R_L$; for large bandwidths the resistive component (R_L) of Z_L at the actual working frequency will have to be found. From these conditions, we have

$$Q_x = \frac{\omega_0 L}{R_L + R'_s + R} = \frac{\omega_0 L}{R'_T} = \frac{1}{\omega_0 C R'_T} \tag{37}$$

where Q_x = the equivalent Q of the circuit of Fig. 26.13(B)

      L = equivalent inductance of the crystal

      C = equivalent series capacitance of the crystal

      R = equivalent series resistance of the crystal

      R_L = resistive component of Z_L, the load impedance, at the frequency being considered ($= Z_L$ at resonance)

      R'_s = resistive component of Z'_s the voltage source impedance

      $R'_T = R_L + R'_s + R$ = total series resistance at frequencies away from f_0 (for very narrow bandwidths $R_L \approx Z_L$ and $R'_T \approx R_{T_1}$)

and $\omega_0 = 2\pi \times f_0$ (where f_0 is the i-f).

From the principles of series resonant circuits we know that the total bandwidth $(2\Delta f)$ for the half power points (3 db atten.) is given by

$$2\Delta f = f_0/Q_x. \tag{38}$$

It follows from eqns. (38) and (37) that for large bandwidths Q_x should be small and so R'_T should be large. For narrow bandwidths the reverse is true, and the limiting case for the narrowest bandwidth would be when $R'_T = R$ (i.e for the crystal alone) ; a condition impossible to achieve in practice because of circuit requirements.

Since $Z_L \gg (Z'_s + R)$ for constancy of maximum gain at f_0, under conditions of varying bandwidth, it also follows that Q_x will mainly be determined by the magnitude of the resistive component of the dynamic impedance of Z_L.

Some additional selectivity is given by the input i-f transformer, and for more exact results the attenuation for a particular bandwidth would be added to that found for the crystal circuit. This additional selectivity is usually negligible around the "nose" of the resonance curve, but can be determined from the universal resonance curves of Chap 9 Sect. 10, evaluating D and b for unequal primary and secondary Q's (Q_1 and Q_2) and noting that Q in these expressions is Q_2.

The resonance curve for the complete circuit is seldom necessary for a preliminary design. The procedure is rather lengthy but not very difficult. First it is necessary to find the resistive component of Z_L (i.e. R_L) at frequencies off resonance. This is carried out from a knowledge of $|Z_L| (= Z_L)$ at resonance, and by determining the reduction factors from the universal selectivity curves. Multiplying $|Z_L|$ by the

indicated attenuation factors gives the required magnitudes of load impedance $|Z'_L|$. To find the resistive component, values of θ corresponding to the various bandwidths (and values of $|Z'_L|$) are read from the universal phase-shift curves of Chapt. 9 Sect. 10 Then

$$R_L = |Z'_L| \cos \theta. \tag{39}$$

The resistive component of Z'_s can be found in a similar way from

$$R'_s = Z'_s \cos \phi$$

but since the impedance of Z'_s is generally small, and the circuit relatively unselective, R'_s can often be taken as equal to Z'_s for a limited range of frequencies near resonance. When Z'_s is very low, it can be neglected in comparison with $R_L + R$.

Then, since the resistance (R) and the inductance (L) of the crystal are known, eqn. (37) can be applied to determine the values of Q_x corresponding to the various bandwidths. (Strictly f_0 would be replaced by the actual operating frequency off resonance, but this is hardly necessary).

Knowing the different values of Q_x, and since the bandwidths corresponding to each Q_x value are known, the various points for the complete selectivity curve can be found from the universal resonance curve for a single tuned circuit (it is unimportant that this is for a parallel tuned circuit rather than a series tuned circuit, since the conditions for parallel and series resonance are practically the same provided the Q is not less than about 10 ; there is, however, a reversal in sign of the phase angles for the two cases). Additional selectivity due to the input transformer can be taken into account if this degree of accuracy is thought to be necessary.

(E) Crystal constants

Before a complete design can be carried out, some data on the equivalent electrical constants of the quartz crystal must be available. In most cases details can be obtained from the crystal manufacturer.

Typical values for the electrical constants of 455 Kc/s quartz crystals widely used in Australian communications receivers are :

     $R = 1500\Omega$; $C = 0.018$ $\mu\mu$F ; $C_0 = 17$ $\mu\mu$F.

The crystals are a special type of X cut bar and have no subsidiary resonances for a range of at least $\pm$ 30 Kc/s from f_r. They are approximately 20 mils (0.02 in.) thick, $\frac{1}{4}$ in. wide, $\frac{5}{8}$ in. long and are mounted between flat electrodes with an air gap not greater than 1 mil. (0.001 in.).
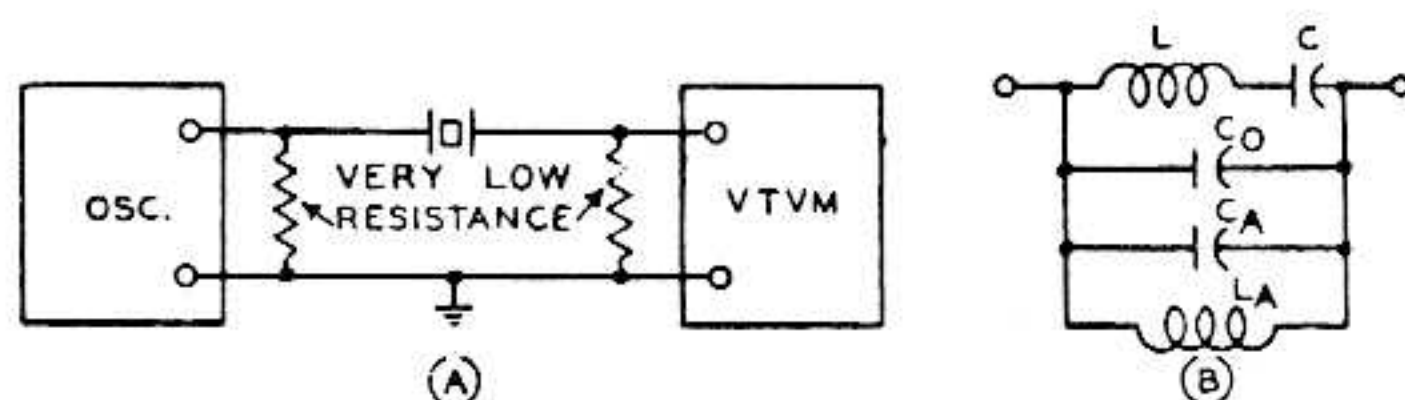

FIG. 26·15 APPROXIMATE METHODS FOR DETERMINING EQUIVALENT
ELECTRICAL CONSTANTS OF QUARTZ CRYSTALS

If the required data cannot be obtained, then details of methods for measuring, firstly, the value of Q for the crystal, can be found in Ref. 65. The experimental set-up is shown in Fig. 26.15(A) and Q is found from

$$Q = \frac{f_r}{2(f_p - f_r)}\sqrt{\frac{E_r}{E_p}} \tag{40}$$

where f_r = series resonance frequency
        f_p = parallel resonance (or antiresonance) frequency
        E_r = voltage across terminating resistance at series resonance
and E_p = voltage across terminating resistance at parallel resonance.

A knowledge of Q will allow the filter circuit to be designed, but if values for C and C_0 are required these can be readily measured (Ref. 64) by using the arrangement of Fig. 26.15(B) and the relations

$$\frac{C}{C_A + C_0} = \left(\frac{f_{p_2} - f_{p_1}}{f_r}\right)^2 \tag{41}$$

and $\qquad f_{pA} = \dfrac{1}{2\pi\sqrt{L_A C_A}}$ $\qquad\qquad\qquad\qquad$ (42)

where $\qquad C$ = series capacitance of crystal

$\qquad\qquad C_A$ = parallel capacitance of circuit $L_A C_A$

$\qquad\qquad L_A$ = parallel inductance of circuit $L_A C_A$

$\qquad\qquad C_0$ = shunt capacitance across crystal due to holder etc.

$\qquad f_{p_1}$ and f_{p_2} = parallel resonance frequencies of combined circuits. The combination has two parallel resonance frequencies, one above and one below f_r.

$\qquad\qquad f_r$ = series resonance frequency, measured with combined circuit.

and $\qquad f_{pA}$ = parallel resonance frequency of circuit $L_A C_A$ alone.

C_0 is determined by disconnecting the crystal and adding capacitance to $L_A C_A$ to retune the circuit to the frequency f_r.

(F) Position of filter in receiver

In most receivers using crystal filters at least two i-f stages are included, since the gain of the crystal stage is usually well below that for a normal i-f stage. Low stage gain immediately after the converter valve may have an adverse effect on signal-to-noise ratio, and it is sometimes preferable to place the filter between the first and second i-f voltage amplifier valves ; however, it is fairly common practice to place the selective crystal stage immediately after the converter to reduce the effects of spurious responses.

It would not be satisfactory to place the filter between the last i-f valve and the detector because of the low impedance of the load in this case.

Since it is common practice to incorporate at least one r-f stage in receivers of the type mentioned, it is unlikely that effects such as possible deterioration in signal-to-noise ratio would be of any consequence. However, good design suggests at least two i-f stages plus one or more r-f stages, depending on performance requirements.

(G) Other types of crystal filters

There are several types of crystal filters suitable for use in radio receivers other than the simple bridge circuit, to which attention has been confined. Typical examples are double-crystal and bridged T filters. These circuits are characterized by having two rejection frequencies, usually placed with geometric symmetry about f_r. Details can be found in Refs. 49, 54, 55 and 61.

Resistance balancing is sometimes applied to the complex types of crystal filters to increase attenuation at the rejection frequencies. This method has also been applied to wave filters using LC circuits only. Details are given in Refs. 59, 60 and 61.

(iv) Design example

Using the constants for the 455 Kc/s crystal given in (E) above, it is required to design a crystal filter circuit to have a total bandwidth $(2\Delta f)$ for 3 db attenuation,
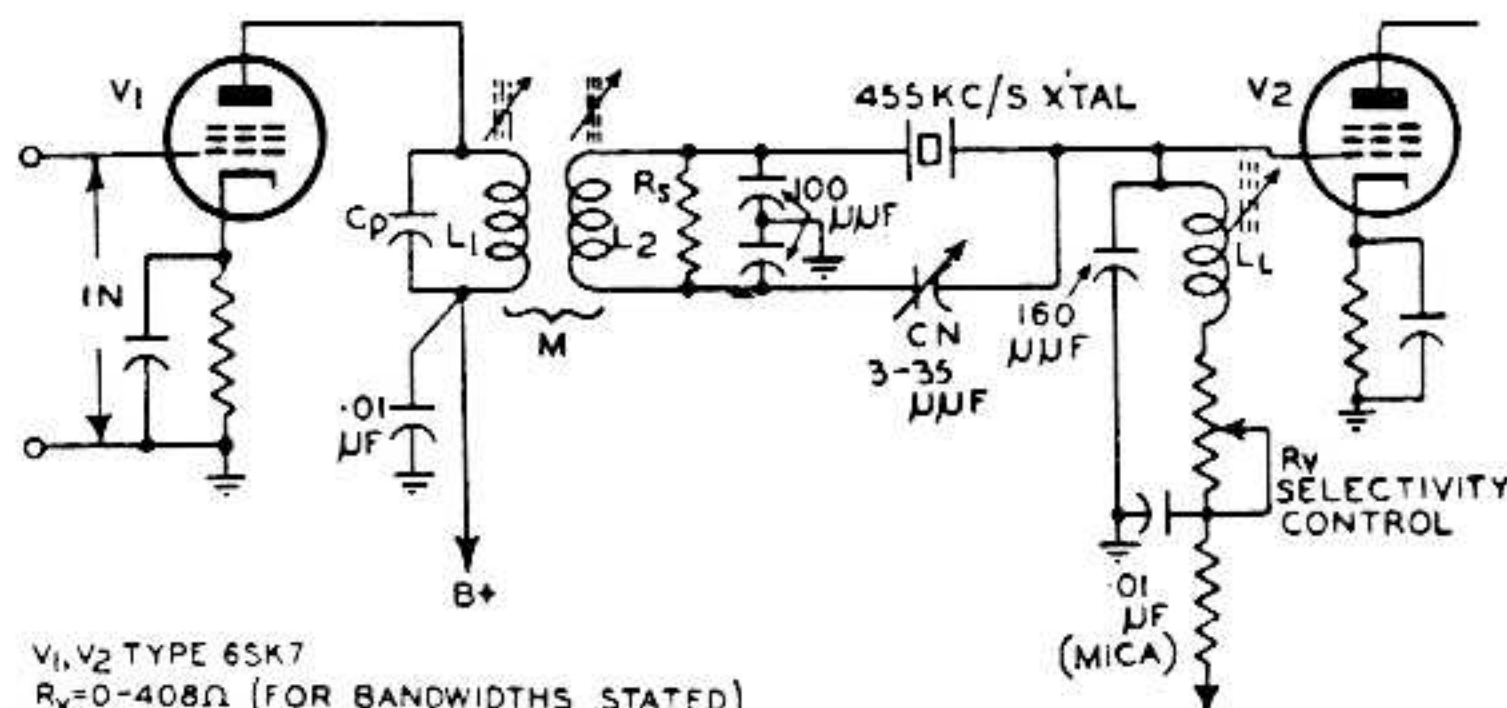
V1, V2 TYPE 6SK7
$R_V = 0-408\,\Omega$ (FOR BANDWIDTHS STATED)
$L_L = 0.586$ mH $\qquad Q_L = 140$
$L_1 = 4.9$ mH $\qquad Q_1 = 140$
$L_2 = 0.32$ mH $\qquad Q_2 = 140$
$k_c = 0.234$ $\qquad R_s = 2460\,\Omega$
$C_p = 25\,\mu\mu F$ (INCLUDING STRAYS)

FIG. 26.16 CRYSTAL FILTER CIRCUIT DESIGNED FROM ILLUSTRATIVE EXAMPLE

which can be varied from 200 c/s to 3 Kc/s. The variation in maximum stage gain should not exceed about 2 db ($\pm$ 1 db about the average gain) but it is desirable to

keep the stage gain as high as possible, consistent with stable operation with varying signal input voltages (i.e. large detuning of the i-f circuits should not occur when the signal voltages vary over a wide range).

To make the problem complete, it will be assumed that the filter is connected between two type 6SK7 pentode voltage amplifier valves. The complete circuit is shown in Fig. 26.16.

(1′) Since $Z_1 = Z_2$ and $C_0 \approx 17$ $\mu\mu$F, let us select a suitable capacitance range for C_N. The smallest residual capacitance for C_N will be about 3 $\mu\mu$F. From this, we have to increase C_N a further 14 $\mu\mu$F to neutralize C_0. In addition, it is desired to move the rejection frequency (f_p) below f_r, so that it is reasonable to allow C_N to increase at least a further 14 $\mu\mu$F. The total increment in C_N is thus 28 $\mu\mu$F ; which gives a range of 3 to 31 $\mu\mu$F ; for convenience this is made, say, 3 to 35 $\mu\mu$F (or whatever is the nearest standard capacitance range).

(2′) From eqn. (38) :

for 200 c/s total bandwidth, and 3 db attenuation,

$$Q_{x_1} = 455/0.2 = 2275 ;$$

for 3 Kc/s,

$$Q_{x_2} = 455/3 = 151.6.$$

(3′) From eqn. (37) (and C = 0.018 $\mu\mu$F)

$$R'_{T_1} = \frac{10^{12}}{2275 \times 2\pi \times 455 \times 10^2 \times 0.018} = 8550 \ \Omega$$

and $R'_{T_2} = \dfrac{8550 \times 2275}{151.6} = 0.128$ MΩ.

So that (since $R = 1500 \ \Omega$)

$$R_{L_1} + Z'_s = 8550 - 1500 = 7050 \ \Omega$$
$$R_{L_2} + Z'_s = 0.128 - 15 \times 10^{-3} = 0.126 \ \text{M}\Omega.$$

For narrow bands of frequencies, $R_{L_1} \approx Z_{L_1}$, and we may write

$$R'_{T_1} = R_{T_1} \text{ and } Z_{L_1} + Z'_s \approx 7050 \ \Omega.$$

(4′) To find Z_{L_2}. It should be clear, from the values just given (Z'_s remains unchanged) that for fairly large bandwidths

$$R_{L_2} \gg Z'_s.$$

Assume that, for the load circuit, $Q_L = 140$. Then by calculation, or from the universal resonance curves, we have for a single tuned circuit and a frequency of 455 Kc/s,

$$\frac{Z_{L_2} \text{ at resonance}}{|Z'_{L_2}| \text{ at 3 Kc/s bandwidth}} = 1.37.$$

Also, the phase shift $\theta = 42° 43'$.

Using these factors in conjunction with eqn. (39),

$$|Z'_{L_2}| = \frac{R_L}{\cos \theta} = \frac{0.126}{0.735}$$

and so $Z_{L_2} = \dfrac{1.37 \times 0.126}{0.735} = 0.235$ MΩ.

(If the maximum bandwidth required is too large it will be found that Z_{L_2} cannot be obtained with ordinary circuit components).

From this, since $Z_{L_2} = Q_L \omega_0 L_L$,

$$L_L = \frac{0.235 \times 10^3}{140 \times 2\pi \times 0.455} = 0.586 \ \text{mH}$$

and $C_L = \dfrac{25\ 330}{0.455^2 \times 586} = 208$ $\mu\mu$F (see below).

(5′) If it is possible to make Z_L higher than 0.235 megohm then a voltage step-up is possible using the arrangement of Fig. 26.17, which also reduces detuning effects

when C_N is varied. Good circuit stability (which requires a large value for C_L) is most important since the circuit bandwidth and gain are very critical to detuning. The main causes of detuning are capacitance changes at the input of V_2 due to grid bias variations, and resetting of C_N; this latter capacitance change is offset, to some extent, in the alternative circuit since it appears across only part of the tuned circuit capacitance. Any form of tapping down is helpful in reducing detuning variations due to C_N, but this is limited by the impedance required for the filter load circuit.
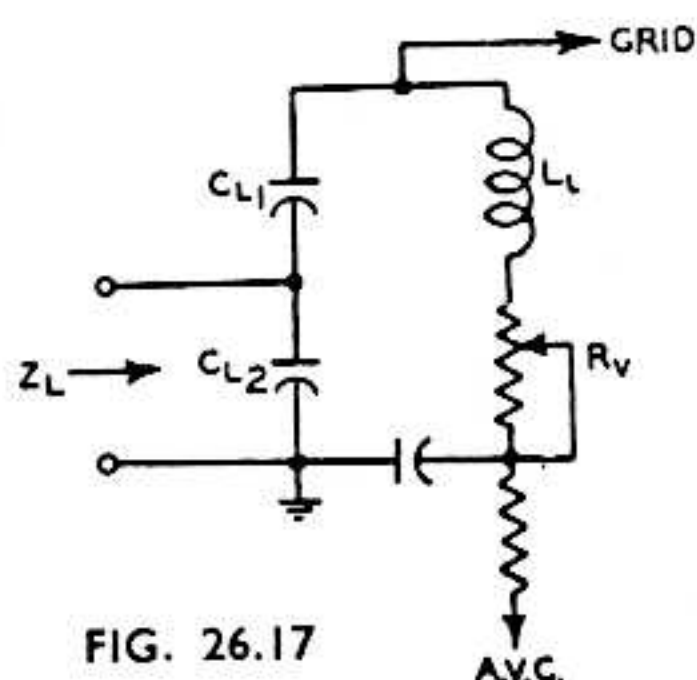

FIG. 26.17

Fig. 26.17. Alternative load circuit for crystal filter stage.

Methods of reducing detuning effects are discussed in Sect. 7(ii), and could be applied here to offset changes in valve input-capacitance. Connecting the grid to a tap on the tuned load is an obvious means of reducing the effects of valve input-capacitance variation. This method involves a loss in gain, but this is not necessarily serious as in some receivers the total gain is more than can usefully be employed.

The actual fixed capacitance for C_L is found approximately as follows :

Valve input capacitance	$=$	6.5 $\mu\mu$F
Strays across coil + wiring etc.	$=$	8.5 $\mu\mu$F
$C_N + C_0 = 2C_0 = 2 \times 17$	$=$	$\underline{34\ \mu\mu\text{F}}$ (when C_0 is neutralized by C_N)
Total strays	$=$	49 $\mu\mu$F

Fixed capacitance req. $= 208 - 49 = 159\ \mu\mu$F (say 160 $\mu\mu$F).

(6') From eqn. (34) ; and since $R_T = 8550\ \Omega$ and
$R_{T_2} = 0.237$ MΩ (approximately ; i.e. $Z_{L_2} + R$),

$$\alpha_0 = \frac{0.237\ (7050)}{8550\ (0.235)} = 0.83$$

(which shows that the gain variation is practicable).

Then, since $\alpha = 2$ db down $= 0.794$, we have from eqn. (35A)

$$Z'_s = \frac{\dfrac{0.83}{7050} - \dfrac{0.794}{0.235 \times 10^6}}{\dfrac{0.83}{} - \dfrac{0.794}{}} \approx \frac{0.036 \times 7050}{0.83} = 306\ \Omega,$$

so that $Z_s = 4Z'_s = 1224\ \Omega$.

(7') To determine the approximate range of the selectivity control R_v. For maximum bandwidth $R_v = 0$. For minimum bandwidth, since $Z_{L_1}\ (= R_{L_1}) = 7050 - 306 = 6744\ \Omega$,

then $Q_{L_1} = \dfrac{Z_{L_1}}{\omega_0 L_L} = \dfrac{6744}{1680} = 4$ [This is an approximation only ; see end of step (8')]

and $R'_v = \dfrac{\omega_0 L_L}{Q_L} = \dfrac{1680}{4} = 420\ \Omega.$

Resistance already in circuit when Q is 140, is
$R_L = 1680/140 = 12\ \Omega.$

Range of R_v required is from $0 - 408\ \Omega$.

This is the range if R_v is connected in series with L_L. If R_v is to be connected in series with the capacitive arm of Z_L it should be realized that it would actually be in

series with the fixed capacitance of $160\ \mu\mu F$ only, and not the total capacitance, so that new values for the range of R_v would have to be calculated in this case.

(8′) The design of the input transformer is the next step. For maximum gain the transformer will be critically-coupled (this term is hardly a correct one in the design which follows and for this particular case it might be preferable to use other methods).

Consider first the secondary circuit. The total secondary capacitance, if we select $100\ \mu\mu F$ capacitors (connected in series) for the ratio arms of the bridge circuit, will be $100/2 + 14.5 = 64.5\ \mu\mu F$. The $14.5\ \mu\mu F$ represents the approximate total for

$$\frac{C_N}{4} + \frac{C_0}{4} = \frac{C_0}{2} \text{ (since } C_N = C_0) = 8.5\ \mu\mu F \text{ plus an allowance of } 6\ \mu\mu F$$

for coil and circuit strays.

Then the apparent secondary inductance required is

$$L_2 = \frac{25.33}{0.455^2 \times 64.5} = 1.9 \text{ mH.}$$

From step (6′) above, $Z_s = 1224\ \Omega$. Since critical-coupling (see remarks above) will halve the actual value of Q_2, we must use an uncoupled value for Z_s of $2448\ \Omega$. This allows us to determine the uncoupled secondary magnification factor

$$Q_2 = \frac{2448}{\omega_0 L_2} = \frac{2448}{5440} = 0.45.$$

Because of the low value for Q_2, the condition of $\omega_0^2 L_2 C_2 = 1$ is no longer sufficiently accurate. For cases such as this, where Q is less than about 10, proceed exactly as before but modify the value of L_2 by a factor $Q^2/(1 + Q^2)$. This gives the true condition for resonance (unity power factor) if Q is assumed constant and L_2 is variable as it will be in most i-f transformers of the type being considered. The actual value required for L_2 is now

$$L_2 \text{ (actual)} = 1.9\left[\frac{0.45^2}{1 + 0.45^2}\right] = 0.32 \text{ mH.}$$

(9′) Since the secondary Q is very low, and L_2 is fixed by other considerations, it should be clear that if we require reasonably high stage gain the primary $Q (= Q_1)$ and the primary inductance L_1 should be as high as possible ; since this will allow $QL = \sqrt{Q_1 Q_2 L_1 L_2}$ to be increased.

As the minimum capacitance across L_1 will be about $25\ \mu\mu F$ (valve output + strays, which will be fairly high for a large winding), then it is possible to make

$$L_1 = \frac{25.33}{0.455^2 \times 25} = 4.9 \text{ mH.}$$

Of course, it may not be advisable to resonate the primary with stray capacitances only, but there are practically no detuning effects present (except those due to temperature and humidity variations). The possibility of instability must not be overlooked, however, when the grid to plate gain is high, and for the case given it would probably be necessary to neutralize the i-f stage or to reduce the gain by increasing the capacitance value above $25\ \mu\mu F$ (and so reducing L_1). However, L_1 will be made large here to illustrate the design procedure to be adopted in cases such as this, where L_1 is not equal to L_2 and also to bring out an additional useful point which could be overlooked in connection with the design of a.v.c. systems ; see Chapter 27 Sect. 3(xv).

(10′) Assume that unloaded Q values of 140 for the primary and secondary can be obtained. The plate resistance (r_p) for type 6SK7 is $0.8\ M\Omega$ for a set of typical operating conditions. Then the actual value of Q_1 is

$$Q_1 = \frac{Q_u r_p}{Q_u \omega_0 L_1 + r_p} = \frac{140 \times 0.8}{1.96 + 0.8} = 40.6.$$

(11′) In order that the actual value of Z_s shall be $1224\ \Omega$ (and so $Z'_s = 306\ \Omega$) the secondary circuit must be loaded with a resistance R_s given by

$$R_s = \frac{Q_u Q_2 \omega_0 L_2}{Q_{u_2} - Q_2} = \frac{140 \times 2448}{140 - 0.45} = 2460\ \Omega$$

where L_2 has the apparent value of 1.9 mH, and not its actual value, so as to make allowance for the use of approximate expressions which are normally only applicable

when Q exceeds about 10.

(12′) $Q = \sqrt{Q_1 Q_2} = \sqrt{40.6 \times 0.45} = 4.28$.

The coefficient of coupling $= k_c = 1/4.28 = 0.234$.

$L = \sqrt{L_1 L_2} = \sqrt{4.9 \times 0.32} = 3.06$ mH.

The actual value of L_2 is required here.

(13′) It is now proposed to estimate the gain. To find the true gain for the conditions selected it would be necessary to solve the complete equivalent circuit. It should also be noted that the condition for secondary circuit series resonance does not coincide with the condition for parallel resonance. Correct circuit matching (i.e. that the value of Z'_s is the required one) can be readily checked from the equivalent secondary circuit. This has been done to ensure that the previous modifications to the transformer constants are quite satisfactory. Here the original gain equation (33) will be used, and then a suitable correction factor (namely X_{c_2}/X_{L_2}) will be applied for this special case where $X_{c_2} \neq X_{L_2}$. The results show an appreciable error, about $+ 20\%$ in the worst practical cases, but the additional labour involved in a more exact analysis is hardly justified.

For the valve type selected $g_m = 2000$ μmhos (2 mA/volt). Gain from grid of V_1 to grid of V_2, using eqn. (33), is

$$\text{Gain (max. bandwidth)} = \left[\frac{0.235}{0.235 + 306 \times 10^{-6} + 1500 \times 10^{-6}}\right] \times$$

$$\left[\frac{2 \times 4.28 \times 2\pi \times 0.455 \times 1.25}{4}\right]$$

$$= 0.992 \times 7.7 = 7.6 \text{ times (17.6 db)}.$$

Gain (min. bandwidth) $= 6744/8550 \times 7.7 = 6.07$ times (15.6 db).

The factor $X_{C2}/X_{L2} = 5470/917 = 5.95$ and so the maximum gain is about 45 times (33 db) and the minimum gain about 36 times (31 db). The calculated value from the equivalent circuit is 38 times.

Thus the maximum total gain variation (including all approximations) is about 2 db as specified i.e. ± 1 db about the average gain. Larger variations in gain, if permissible, would also allow increased overall gain ; the disadvantages have been discussed previously.

(14′) For some purposes a standard i-f transformer using fixed capacitances of about 100 μμF, and the coupling increased to critical when the secondary and primary are correctly loaded, would give satisfactory results. The gain variation, and maximum gain, are largely controlled by the value of the damping resistor R_s connected across the secondary of the transformer.

To control bandwidth it is necessary to increase Z_L to increase the maximum bandwidth, R_v is increased to decrease the minimum bandwidth. If switched steps are required for bandwidth control, R_v can be calculated for each step ; the remainder of the design is exactly as before.

The arrangement used for the input transformer is only one of many possible circuits. For cases such as the one given here (where the total gain is not a prime requirement) it may be preferable to leave the secondary circuit untuned and to use a tapped resistor to provide the ratio arms for the bridge circuit. The design procedure is readily developed from the usual coupled circuit theory.

SECTION 7 : DETUNING DUE TO A.V.C.

(i) Causes of detuning   (ii) Reduction of detuning effects (A) General (B) Circuits.

(i) Causes of detuning

Detuning of i-f (and r-f) circuits is largely due to changes in valve input capacitance. The capacitance change is brought about when the grid bias on the valve is

altered, the bias change being due to the a.v.c. voltage altering with different signal inputs. This alteration in bias also leads to variation in the valve input conductance which will have to be considered in some circuits, since the conductance increases by about 2 : 1, from cut-off to normal operating conditions, with typical valve types operating at 100 Mc/s (see Ref. 68 ; also Chapter 2 Sect. 8 and Chapter 23, Sect. 5).

If all the factors which cause valve input capacitance and conductance changes are considered the problem is rather involved as can be seen from the discussions given in Chapter 2, Sect. 8 and Chapter 23, Sect. 5. We will confine attention here, mainly, to input capacitance changes which are most troublesome in receiver design.

One of the principal causes of change in valve input capacitance is the variation in position and density of the space charge distribution between grid and cathode, brought about by the change in grid bias.

Variation in grid-plate transconductance (g_m) is another principal cause of input capacitance change as will be seen later.

Short-circuit input capacitance changes are not very much affected by frequency, although the change in capacitance with transconductance is slightly greater at low frequencies than at high. The slight variation with frequency will not usually be important when compared with the changes due to alteration of transconductance, so that the data given in Chapter 2, Sect. 8 can be used directly. From this data, it is seen that the short-circuit input capacitance alters by about 1 to 2 $\mu\mu$F in typical cases. It is interesting to compare these changes with that due to grid-to-plate coupling i.e. Miller effect (see Chapter 2, Sect. 8).

Suppose, for simplicity, that the plate load acts as a pure resistance. Then the input capacitance change is given by

$$\Delta C = C_g - C_{gp} = C_{gp}\left(\frac{g_m r_p R_L}{r_p + R_L}\right) \tag{43}$$

Taking a type 6SK7, with a plate load of 0.25 megohm, as a typical example. Then since $C_{gp} = 0.003$ $\mu\mu$F, $r_p = 0.8$ MΩ (this value is not constant, but it has been assumed so for simplicity) and g_m changes from 0-2000 μmhos, the input capacitance change is

$$\Delta C = 0.003 \frac{2 \times 0.8 \times 0.25 \times 10^3}{1.05} = 1.14 \ \mu\mu\text{F}.$$

If C_{gp} is augmented by stray capacitances, such as those inevitably present due to the socket, wiring etc., this figure for ΔC will be very appreciably increased. The corresponding change in short-circuit input capacitance is 1.18 $\mu\mu$F (see table Chapter 2 Sect. 8).

The effects are additive and cause an increase in input capacitance as the grid bias voltage becomes more positive.

(ii) Reduction of detuning effects
(A) General

There are a number of methods available for reducing the effects of valve input capacitance variations, and these methods (in most cases) also reduce input conductance variations. Usually the reduction of these effects will lead to a loss in stage gain unless additional circuit changes are made.

In the case of the ordinary broadcast receiver special circuits are not often used. The only precautions taken are :

(1) The total capacitance tuning the i-f transformer secondary, in particular, is made as large as is practicable consistent with other circuit requirements. Values of 200 $\mu\mu$F, or so, should be used if possible, although values of the order of 100 $\mu\mu$F, are generally used, since it is unlikely that the change in input signal will be such as to cause the maximum input capacitance change. Large values of tuning capacitance for the transformer are also helpful in increasing the attenuation at frequencies far removed from resonance. Also, with some types of converter valves (e.g. 6A8-G) a large tuning capacitance for the primary of the first i-f transformer can be of assistance in reducing degeneration in the signal input circuit connected to the converter ; this applies particularly to signal frequencies which approach the intermediate frequency.

(2) The receiver is aligned on small input signal voltages, since detuning effects will be less serious when the input signal is large.

(3) The circuit layout is such as to minimize stray grid-to-plate capacitance.

The degree of circuit mistuning in terms of frequency shift can be determined by comparing the input capacitance change with the total capacitance across the valve input circuit. A number of helpful practical examples of circuit mistuning are given in Ref. 69.

There are several additional factors which are of importance.

(1) Detuning will cause sideband asymmetry and so lead to the possibility of increased distortion. This is of particular importance in high fidelity receivers.

When the carrier is received on the side of the i-f selectivity curve, one set of sidebands is almost completely eliminated (or at least substantially reduced in amplitude), and the amplitudes of the carrier and low frequency components of the other set of sideband frequencies are reduced. The tuned circuits will introduce phase and frequency distortion and the diode detector will give rise to non-linear distortion because of the absence of one set of sidebands. Experiment has shown that with the usual type of receiver the amount of detuning that can be tolerated before distortion becomes noticeable to a critical listener is about ± 1 Kc/s.

(2) Detuning of the r-f and converter input circuits can lead to a reduction in signal-to-noise ratio. This effect is not often very serious, when compared with the deterioration in signal-to-noise ratio caused by the reduction in gain of the r-f stages as the a.v.c. bias is increased ; further discussion of this point will be given in connection with the design of a.v.c. systems in Chapter 27.

(3) Detuning can result in considerable loss in adjacent channel selectivity. This is not particularly serious if the receiver has been aligned on small signal input, since the deterioration occurs when the desired signal is large.

(4) The i-f stages lend themselves more readily to compensation methods than the r-f circuits ; in the case of the r-f circuits exact compensation can usually be obtained at one frequency only. Since the r-f stages are often relatively unselective, detuning is not often a very serious factor and it is usual to compensate only in the i-f circuits.

(5) Even although exact compensation for input capacitance changes can be obtained at the resonant frequency, the compensation is not complete at frequencies removed from the i-f centre frequency, and there will always be some departure from symmetry on the " skirts " of the overall response curve. However, since compensation is required mainly at the centre frequency, this is not serious.

(6) The neutralization methods to be described in Section 8, in connection with stability, will affect input capacitance variations which are due to grid-to-plate coupling. If the amplifier is neutralized, then the input capacitance changes with grid bias will be due, mainly, to space charge effects.

(B) Circuits

Attention will be confined to the circuits of Fig. 26.18 (some discussion of which has already been given in Chapter 23, Sect. 5), since these allow satisfactory results to be obtained with a small number of components. Other types of compensating circuits can be found in Refs. 67, 71, 72.

In Figs. 26.18A and B, C_b is a by-pass capacitor and may be neglected in the discussion which follows. It is possible to show that, for complete compensation of the input capacitance variation, the condition required is

$$R_k g_k = \Delta C / C_{gk} \qquad (44)$$

where R_k = unbypassed cathode resistance

g_k = grid-to-cathode transconductance ; which is given by $g_m(I_k/I_b)$ in which g_m is the mutual conductance ; I_k is the total d.c. cathode current (for a pentode this is usually the sum of the plate and screen-grid currents) ; I_b is the d.c. plate current

ΔC = valve input capacitance change

and C_{gk} = grid-to-cathode capacitance.

For a typical case suppose $\Delta C = 3$ $\mu\mu$F, $C_{gk} = 6$ $\mu\mu$F and $g_k = 2000$ (11.8/9.2) = 2560 μmhos, so that

$$R_k = (3/6) \times (10^6/2560) = 195 \ \Omega.$$

Since $\Delta C/C_{gk}$ determines the value of R_k for a particular valve, it is sometimes convenient to increase C_{gk} artificially to obtain a more suitable value for R_k. This is done by adding a small capacitance between the grid and cathode terminals. However, too large a total value for C_{gk} can have an undesirable effect on the valve input conductance, particularly at high frequencies, and so it is not advisable to increase C_{gk} to more than about twice its usual value or even less, at high frequencies (say 10 Mc/s or so). For typical results see Ref. 68. For 455 Kc/s i-f circuits it is often permissible to connect the whole of C_2 in parallel with C_{gk}.

Because of the presence of the unbypassed cathode resistor there will be a change in the effective mutual conductance (and consequently a loss in gain) given by

$$g_{m(effective)} = \frac{g_m}{1 + R_k g_k}. \tag{45}$$

In our example :

$$g_{m(effective)} = \frac{2000}{1 + 195 \times 2.56 \times 10^{-3}} = 1330 \ \mu\text{mhos}.$$

If C_{gk} is increased to 20 $\mu\mu$F then $R_k = 58.5 \ \Omega$ and the effective $g_m = 1740 \ \mu$mhos. The actual cathode bias resistor required is approximately 260 Ω so that about 200 Ω ($= R'_k$) would be used and this is then by-passed in the usual manner by C_k, as shown in Fig. 26.18B. The increased value of C_{gk} would need some corresponding reduction in C_2 since the total capacitance tuning the i-f transformer secondary includes the resultant capacitive reactance due to C_{gk} and R_k in series if the reactances of C_b and C_k are neglected.
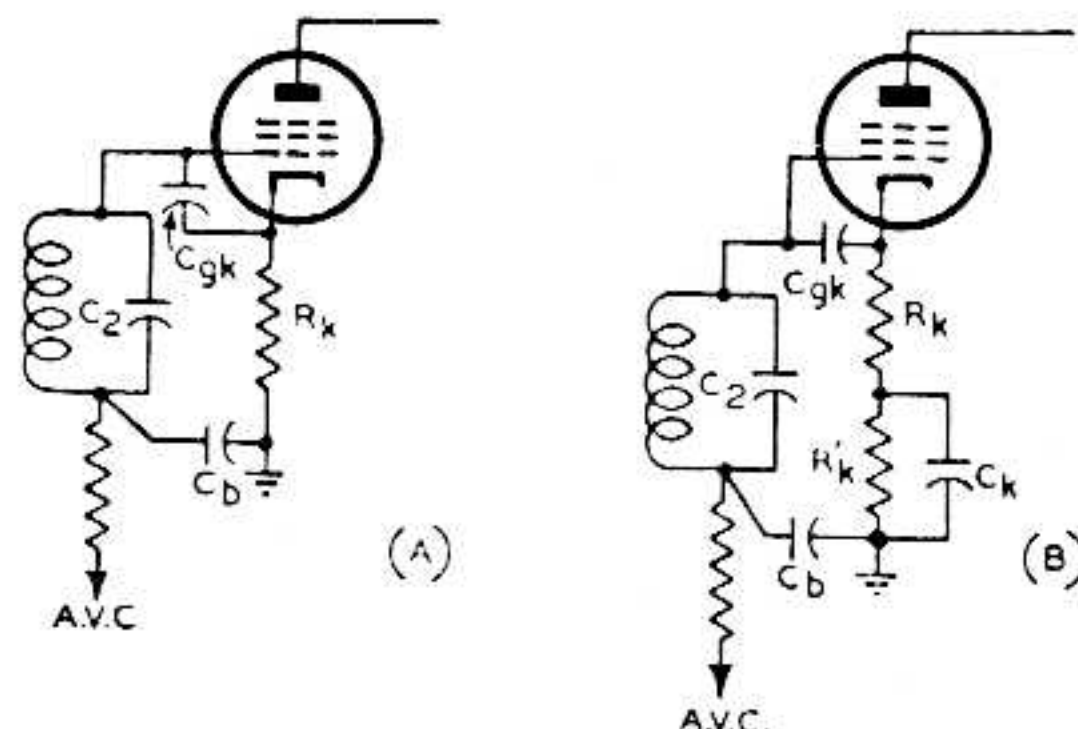

FIG. 26·18 METHOD FOR REDUCING VARIATIONS OF INPUT ADMITTANCE

The use of an unbypassed cathode resistor results in an increase of the short-circuit feedback admittance, and this effect can be used to increase the stage gain so as to offset the loss in g_m and, also, the reduction in Q of the input transformer due to the cathode resistor forming part of the tuned circuit. Two methods are available for increasing the feedback admittance ; the first is to connect the plate and screen by-pass capacitors back to the cathode ; the second is to add a small capacitance between plate and cathode, the plate and screen by-pass capacitors being connected to ground in the usual manner. These methods of increasing gain need to be treated with some care as they can easily lead to oscillation and particular attention is needed if instability is to be avoided. With some valve types (e.g. 6SG7, 6SH7, 9001, 9003, 6AG5, 6AK5) the plate-to-cathode capacitance is already relatively high because of the internal connections between the suppressor grid, beam confining electrodes, and the cathode ; this can lead to oscillation difficulties without external capacitance being added.

In the majority of cases it is preferable to return plate and screen by-pass capacitors (also the suppressor grid) to ground, and add a capacitance from plate to cathode if this is thought to be desirable.

With any circuit the capacitances can be measured and the value for R_k accurately calculated, but in a practical case it is generally much simpler to determine an approximate value for R_k and then find the exact value experimentally.

SECTION 8 : STABILITY

(i) Design data   (ii) Neutralizing circuits.

(i) Design data

The discussion of Chapter 23, Sect. 7 should be used in conjunction with the data to be given here ; also, the detuning effects discussed in the previous section are closely bound up with the data which follow.

For stages using single tuned circuits the expression given in Chapter 23 is more conveniently arranged as

$$\text{Max. stable impedance} = \sqrt{\frac{2}{g_m \omega_0 C_{gp}}} \tag{46}$$

from which it immediately follows that the

$$\text{Max. stable gain} = \sqrt{\frac{2g_m}{\omega_0 C_{gp}}} \tag{47}$$

For double tuned circuits which are critically-coupled—the usual case—Jaffe (Ref. 74) has shown that

$$\text{Max. stable impedance} = \sqrt{\frac{0.79}{g_m \omega_0 C_{gp}}} \tag{48}$$

$$\text{and max. stable gain} = \sqrt{\frac{0.79 \, g_m}{\omega_0 C_{gp}}} \tag{49}$$

where g_m = mutual conductance of amplifier valve
$\omega_0 = 2\pi \times f_0$ (and f_0 is operating frequency ; i-f in this case)
C_{gp} = total grid-to-plate capacitance made up from that due to the valve, wiring and valve socket etc.

Additional data are given in Ref. 75 for cases where the input and output circuits are not identical. This reference also shows that greater gain is possible when the impedance of the input circuit is less than that of the output circuit ; a condition which is fulfilled only in special cases in practical i-f circuits.

The equations given above, and in the references, are useful in forming an estimate of the possible maximum gain obtainable and in indicating some of the causes of instability ; but detailed calculations for various circuit possibilities hardly seem to be justified.

It may be helpful to note that when selecting a particular type of valve for maximum gain combined with stability, the most suitable is that having the greatest value of g_m/C_{gp} consistent with other circuit requirements (and cost). For an i-f valve, g_m/C_{gp} is of considerable importance but for a r-f valve operating at v-h-f it is also very important to consider the factor $g_m\sqrt{R_i}$, as pointed out in Chapter 23, Sect. 5.

(ii) Neutralizing circuits (See Supplement page 1498)

A simple neutralizing circuit, which requires only the addition of one capacitor, is shown in Fig. 26.19(A). The equivalent capacitance bridge is shown in Fig. 26.19(B). It is assumed that the cathode by-pass capacitance is large.

The condition for the effects of the grid-to-plate capacitance to be neutralized is

$$C_N = C\frac{C_{gp}}{C_{gk}} \tag{50}$$

where C_N = neutralizing capacitance required
C = a.v.c. by-pass capacitance (usually about 0.01 μF)
C_{gp} = total grid-to-plate capacitance including strays
and C_{gk} = total grid-to-cathode capacitance including strays.

For a typical case $C = 0.01 \ \mu$F, $C_{gp} = 0.01 \ \mu\mu$F and $C_{gk} = 15 \ \mu\mu$F.
Then $C_N = 0.01 \times 10^{-6} (0.01/15) = 6.7 \ \mu\mu$F.

The exact value for C_N is determined experimentally. For receiver production it is generally sufficient to use a fixed value for C_N which is reasonably close to the
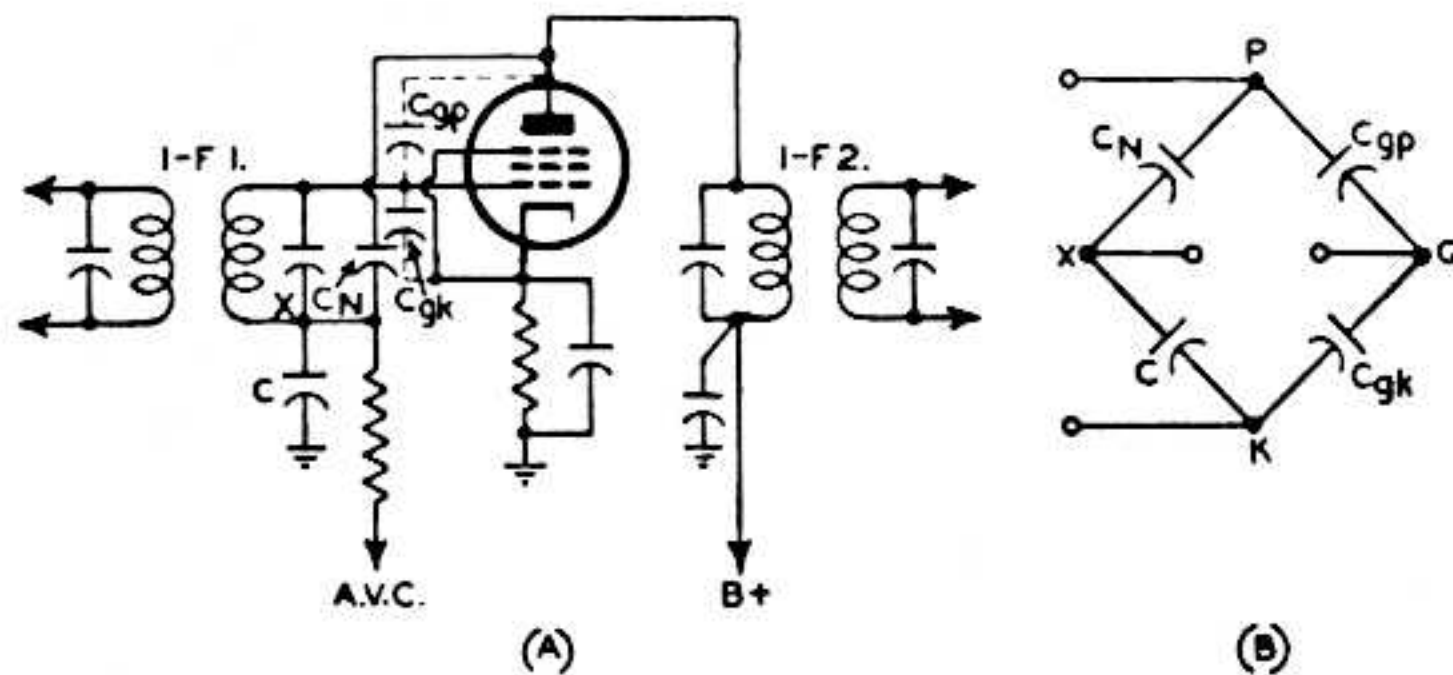
FIG. 26·19   I-F NEUTRALIZATION CIRCUIT

value required. The neutralization is generally not sufficiently critical to require different capacitance values in different receivers of the same type, as even partial neutralization is helpful and often sufficient.

Other circuits using various arrangements of inductance and capacitance for neutralization and stabilization can be found in the references. Two interesting alternatives (Refs. 79 and 80) to that of Fig. 26.19 are shown in Fig. 26.20.

Circuit (1) will be briefly discussed ; circuit (2) should be self-explanatory. In both cases all the components have their usual values except C_3 in circuit (1), and L_a is an added inductance (in i-f and some r-f circuits) in circuit (2) ; the latter arrangement is of interest when the cathode is grounded for d.c. To obtain complete neutralization in circuit (1) it is required to make

$$C_3 = C_1(C_{pk}/C_{gp}) \tag{51}$$

where C_1 = i-f tuning capacitor

        C_{pk} = total plate-cathode capacitance including all strays

and C_{gp} = total grid-plate capacitance including all strays.

In a typical case for a 455 Kc/s i-f transformer $C_1 = 100$ $\mu\mu$F, $C_{pk} = 10$ $\mu\mu$F, $C_{gp} = 0.01$ $\mu\mu$F.

Then $C_3 = 100 \times (10/0.01) = 0.1$ μF, which is used as the by-pass capacitor across the cathode resistor. Also, remember that exact neutralization may not be necessary.

$$C_3 = C_1 \frac{C_{pk}}{C_{gp}}$$

$$L_a = L_1 \frac{C_{gp}}{C_{pk}}$$

(1)   (2)
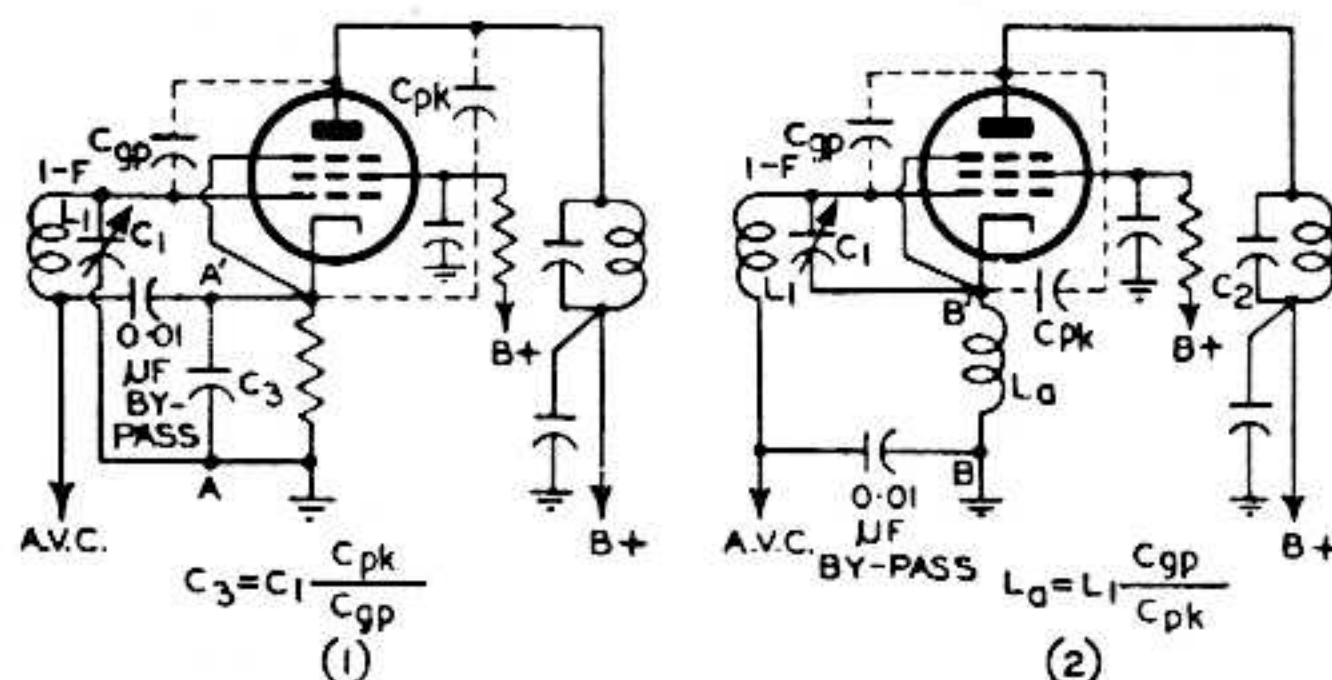
BOTH CIRCUITS ARE DEGENERATIVE, BUT ARE RE-GENERATIVE WHEN AA', BB' ARE REVERSED

FIG. 26·20 METHODS FOR STABILISING R-F AND I-F AMPLIFIERS

This circuit requires no additional components and C_3 is set to the value required for neutralization. It will be noted that the capacitance of C_3 is somewhat larger than is generally used for a by-pass capacitor. The only disadvantage is that an additional lug is required on the i-f transformer base and care is necessary to keep critical leads as short as possible. The advantage of the circuit in a r-f stage should be apparent.

Receivers which combine the detector diode in the i-f amplifier valve sometimes require additional neutralization because of feedback (which may be either regenerative or degenerative) caused by the grid-to-diode capacitance. A typical circuit is shown in Fig. 26.21. This requires two capacitors, C_{N1} and C_{N2}, in addition to the usual circuit components.

Analysis of this circuit (Ref. 77) shows that sufficient accuracy can be obtained if two bridge circuits are used to find the values of C_{N1} and C_{N2}. Although there is interaction between the two circuits, which affects the values selected for C_{N1} and C_{N2}, the calculated values are sufficiently close to allow the practical circuit to be satisfactorily neutralized. The values of C_{N1} and C_{N2} are adjusted in the circuit until complete neutralization is obtained. Fixed values can then be selected, since the circuit is not very critical to small changes in the values of the neutralizing capacitors.
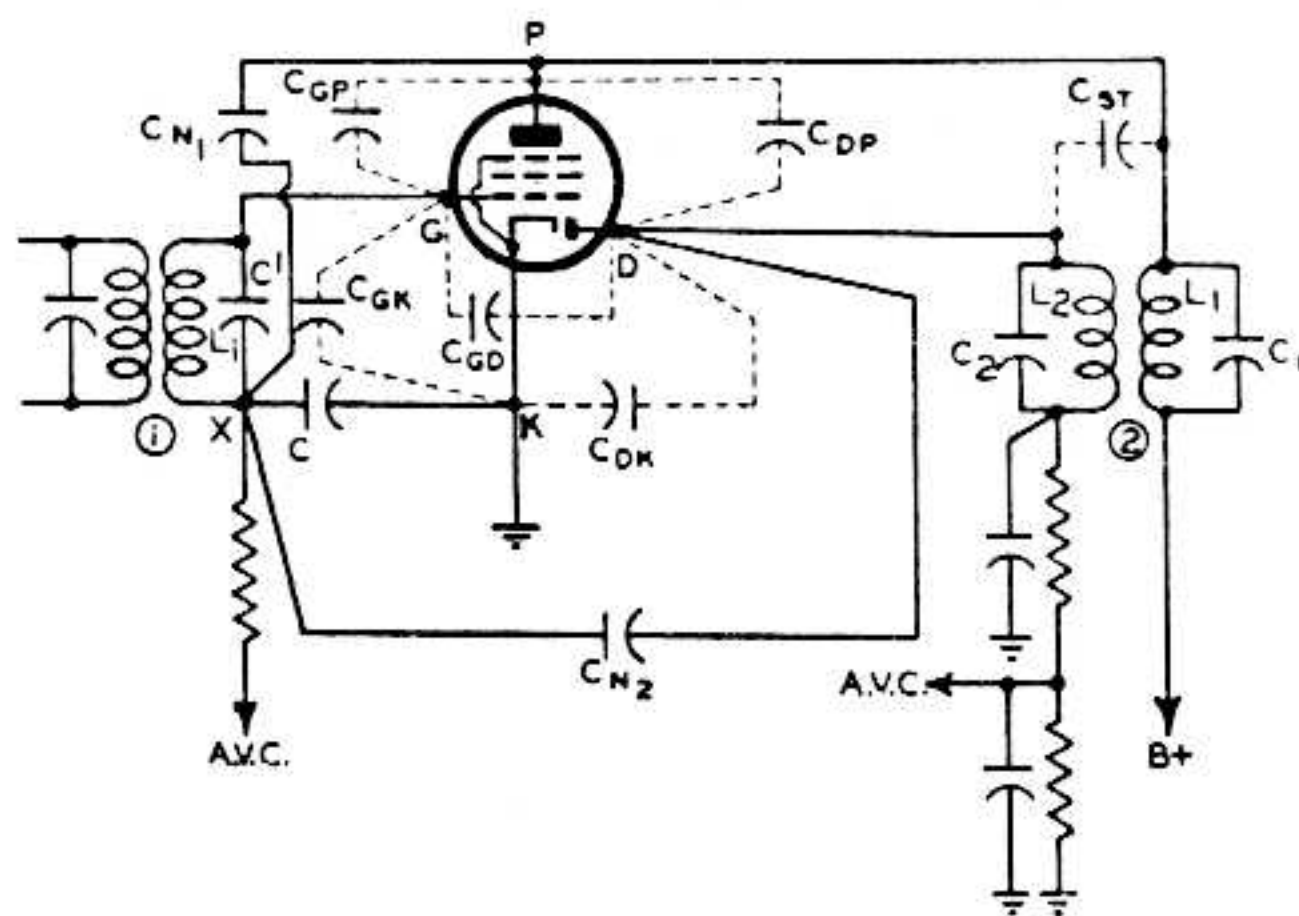

FIG. 26·21   NEUTRALIZATION IN DIODE PENTODE

The equations required are (50) above, which is used to determine C_{N1}, just as previously, and

$$C_{N2} = C(C_{gd}/C_{gk}) \tag{52}$$

where C = a.v.c. by-pass capacitance (as before)

C_{gd} = grid-to-diode capacitance including strays

and C_{gk} = grid to cathode capacitance (as before).

For a typical case :

C_{gp} = 0.007 $\mu\mu$F (including strays), C_{gd} = 0.009 $\mu\mu$F (including strays),

C = 0.01 μF, C_{gk} = 15 $\mu\mu$F (including strays),

C_{N1} = 0.01 × 10⁻⁶ × (0.007/15) = 3.3 $\mu\mu$F,

C_{N2} = 0.01 × 10⁻⁶ × (0.009/15) = 6 $\mu\mu$F.

In an actual receiver, using a type 6SF7, the values required were C_{N1} = 4 $\mu\mu$F ; C_{N2} = 7 $\mu\mu$F. With wiring re-arrangement the values required were C_{N1} = 3 $\mu\mu$F and C_{N2} = 5 $\mu\mu$F.

For methods of alignment and a suitable neutralizing procedure see Ref. 77.

SECTION 9 : DISTORTION

(i) *Amplitude modulation i-f stages*   (ii) *Frequency modulation i-f stages.*

(i) Amplitude modulation i-f stages

In Chapter 23, Sect. 8(i) the causes of modulation envelope distortion were briefly discussed. These same factors apply, also, to i-f amplifier stages, but become more serious because of the larger signal voltages which must be handled by the valves.

The most severe conditions for non-linear distortion will be obtained in the voltage amplifier preceding the detector stage, and the worst condition is given when the grid bias, due to a.v.c., is large. To reduce the distortion due to non-linearity of the valve characteristics, the final amplifier valve is sometimes operated with fixed bias, but this also affects the a.v.c. characteristic (see Chapter 27 Sect. 3). As an alternative, partial a.v.c. may be applied to the last stage. If the screen voltage is supplied by means of a series resistor (as is common practice) instead of a voltage divider, appreciable reduction in distortion is possible. The screen voltage rises as the grid bias voltage is increased and, although there is some loss of efficiency in the a.v.c.

system, the distortion is materially reduced ; this can be confirmed experimentally. As a compromise between distortion and a.v.c. action, the screen supply voltage can be obtained by means of a high resistance voltage divider arrangement if so desired. Detailed measurements of distortion have been made on typical i-f voltage amplifier valves for various methods of screen voltage supply and some of the results can be found in Ref. 83.   In cases where the range of bias voltage is limited, a high resistance voltage divider can be arranged to give less distortion than that given by the series feed arrangement.   A total increase in non-linear distortion of the order of 2% for 90% amplitude modulation is to be expected in typical cases using a series screen resistor.   Because of the internal construction of remote cut-off valves, such as those used in i-f and r-f amplifiers, the screen current is often rather variable and this suggests a voltage divider in cases where variations in screen voltage are important (see Ref. 84).

For methods of measuring the signal handling capabilities of i-f amplifier valves and the non-linear distortion from this cause, the reader is referred to Ref. 4 (page 335) and Ref. 83.

Frequency distortion, caused by sideband cutting due to the selectivity of the tuned circuits, is always present to some extent ; this should be obvious from the discussion on the design of i-f transformers and it is not proposed to treat the matter further here.

(ii) Frequency modulation i-f stages

With frequency modulation receivers, distortion of the amplitude (in itself) of the frequency modulated wave is generally not of great importance, provided the amplitude limiting device is capable of smoothing out the variations.   Non-linearity in i-f and r-f valve characteristics are of secondary importance, with this system of reception, and the non-linear distortion appearing in the receiver output from this cause is usually negligible ; this is the reverse of the case for amplitude modulation.

The amount of distortion caused by the non-linearity of phase shift/frequency characteristics of the tuned circuits is of considerable importance, however, and poorly designed i-f transformers will lead to non-linear distortion in the receiver output.   If the criterion for non-linearity given in eqn. 24 [Sect. 4(v) of this chapter] is fulfilled, then the distortion in the receiver output due to the i-f circuits preceding the discriminator (or whatever detection system is used) will be quite small, and in most cases less than that introduced by the detection and audio systems.

When a F-M receiver is accurately tuned to the centre frequency of the carrier, the non-linear distortion of the modulating signal introduced by the tuned circuits will consist mainly of odd order harmonics.   Of these the third harmonic (H_3) is the largest and a method of estimating its magnitude is helpful when i-f transformers are being designed.   The required expression is (Ref. 82),

$$H_3 = \frac{4A^3}{\beta} (1 + A^2)^{\frac{1}{2}} \left[\frac{3Q^2k^2 - 4A^2 - 1}{(1 + Q^2k^2)^3} \right]. \tag{53}$$

For a critically-coupled transformer $Qk = 1$ and so

$$H_3 = \frac{A^3}{\beta} (1 + A^2)^{\frac{1}{2}}(1 - 2A^2) \tag{54}$$

where $A = \dfrac{\Delta F}{f_0} Q$

ΔF = frequency deviation

f_0 = central carrier frequency

Q = magnification factor $(Q_1 = Q_2)$

β = modulation index = $\Delta F/f$

f = a-f modulating frequency

and k = coefficient of coupling (the coupling may be inductive or capacitive and either shunt or series).

For 1 db attenuation it is known that, for a critically-coupled transformer with identical primary and secondary circuits, $Q = f_0/$total bandwidth.   From this we

may write $A = \Delta F/$total bandwidth. If it is taken that the total frequency deviation $(2\Delta F)$ is equal to the total bandwidth for 1 db attenuation, then we obtain

$$A = \tfrac{1}{2} \text{ and } H_3 = \frac{\sqrt{5}}{32\beta} = \frac{7}{\beta}\%. \tag{55}$$

The Q required for this condition is given from the previously stated relationship $Q = f_0/$total bandwidth.

Suppose $Q = 71$ is obtained, for a critically-coupled transformer, with $f_0 = 10.7$ Mc/s, $2\Delta F = 150$ Kc/s and $\beta = 5$. Then the third harmonic audio distortion introduced by each transformer is 1.4% (approx.) ; using either eqns. (54) or (55).

If $Q = 80$ is obtained from the procedure of Sect. 4, then eqn. (54) gives $H_3 = 1.57\%$; eqn. (55) is not applicable in this case.

It should be clear that a simple approximate design procedure can be developed from the conditions leading to eqn. (55).

The distortion given in the examples is somewhat higher than that experienced under normal operating conditions. The third harmonic of 15 Kc/s will hardly trouble the listener, but harmonics of the lower audio frequencies would be serious if their amplitudes are appreciable, and the possibility of intermodulation effects should not be neglected.

Applying eqn. (54) for $Q = 71$, $\Delta F = 50$ Kc/s (more nearly the usual operating condition) and a modulation frequency of 5 Kc/s, then $H_3 = 0.074\%$. (See also Chapter 27, Sect. 2 under heading " Non-linear distortion ").

It should be carefully noted that eqn. (55) applies only for the special conditions under which it was derived. Distortion is normally calculated from eqns. (53) and (54).

Only a few of the significant factors have been mentioned here, and for more detailed information the reader is referred to Refs. 82, 85, 86 and 87.

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