CHAPTER 4

THEORY OF NETWORKS

by F. LANGFORD-SMITH, B.Sc., B.E.

For ease of reference

Mathematics	see Chapter 6
Mathematical symbols	see Chapter 38 Sect. 6
Electrical and magnetic units	see Chapter 38 Sect. 1
Electrical and magnetic symbols and abbreviations	see Chapter 38 Sect. 6
Standard graphical symbols	see Chapter 38 Sect. 7
Charts for calculation of reactance and impedance	see Chapter 38 Sect. 9
Greek alphabet	see Chapter 38 Sect. 14
Definitions	see Chapter 38 Sect. 15
Trigonometrical and hyperbolic tables	see Chapter 38 Sect. 21

SECTION 1 : CURRENT AND VOLTAGE

(i) Direct current (ii) Alternating current (iii) Indications of polarity and current flow.

(i) Direct current

We speak of the flow of an electric current in more or less the same way that we speak of the flow of water, but we should remember that the conventional direction of current is opposite to the actual flow of electrons. In most electrical circuit theory however, it is sufficient to consider only the conventional direction of current flow. In Fig. 4.1 there is a battery connected to a load ; the current flows from the positive (+) terminal, through the load, to the negative (−) terminal, and then through the battery to the positive terminal.

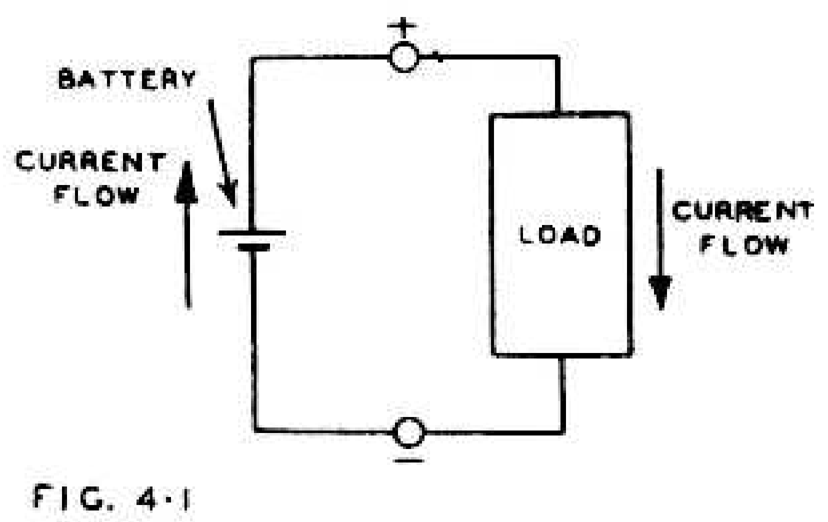


Fig. 4.1. *Flow of current with battery and load.*

Batteries (or cells) may be connected in series as in Fig. 4.2 and the total voltage is then equal to the sum of the voltages of the individual batteries (or cells). When calculating the voltage of any intermediate point with respect to (say) the negative terminal, count the number of cells passed through from the negative terminal to the tapping point, and multiply by the voltage per cell. When batteries are connected in series, each has to supply the full load current.

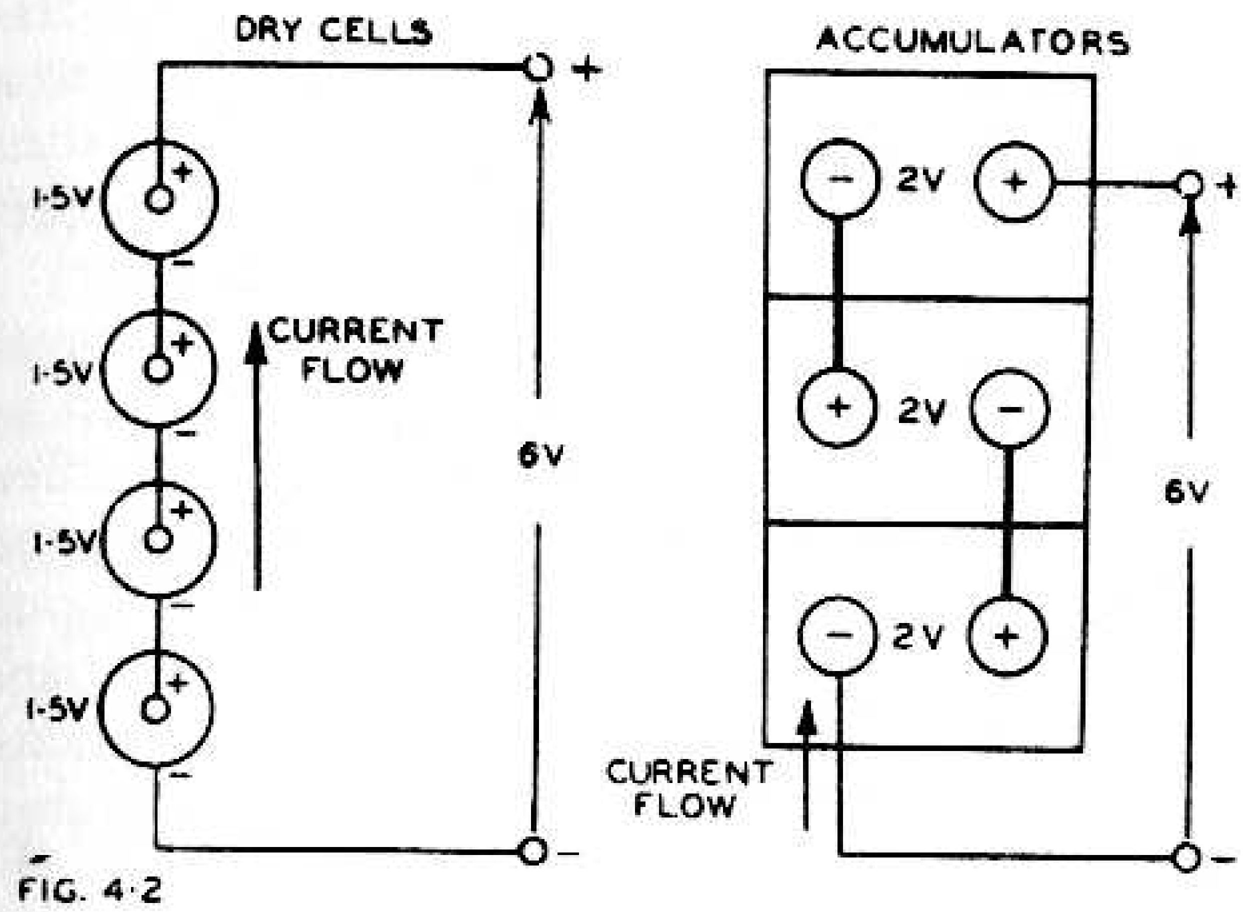


Fig. 4.2. *Cells in series.*

Batteries (or cells) are occasionally connected in parallel as in Fig. 4.3. In this case the terminal voltage is the same as the voltage per battery (or cell). The current does not necessarily divide uniformly between the cells, unless these all have identical voltages and internal resistances.

Direct current may also be obtained from a d.c. generator, or from rectified and filtered a.c. supply. In all such cases there is a certain degree of ripple or hum which prevents it from being pure d.c. ; when the a.c. component is appreciable, the supply may be spoken of as " d.c. with superimposed ripple (or hum) " and must be treated as having the characteristics of both d.c. and a.c. When we speak of d.c. in a theoretical treatise, it is intended to imply pure, steady d.c.

(ii) Alternating current

The ordinary form of d.c. generator actually generates a.c., which is converted to d.c. by the commutator. If a loop of wire is rotated about its axis in a uniform magnetic field, an alternating voltage is generated across its terminals. Thus a.c. is just as fundamental as d.c. The usual power-house generates 3 phase a.c., but in radio receivers we are only concerned with one of these phases. A " sine wave " alternating current is illustrated in Fig. 4.4, where the vertical scale may represent voltage or current, and the horizontal scale represents time.* A **cycle** is the alternation from A to E, or from B to F, or from C to G.

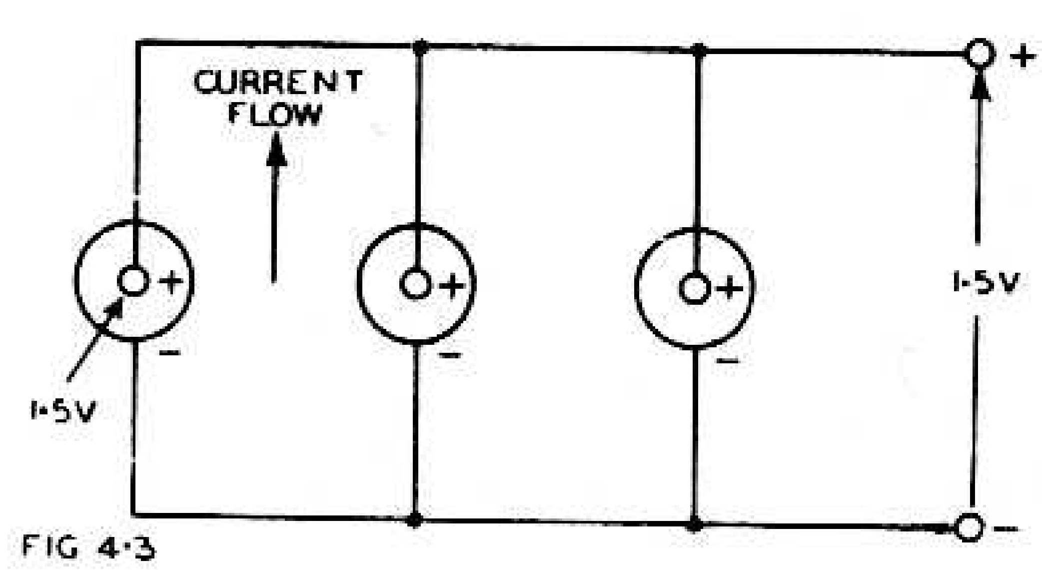


Fig. 4.3. *Cells in parallel.*

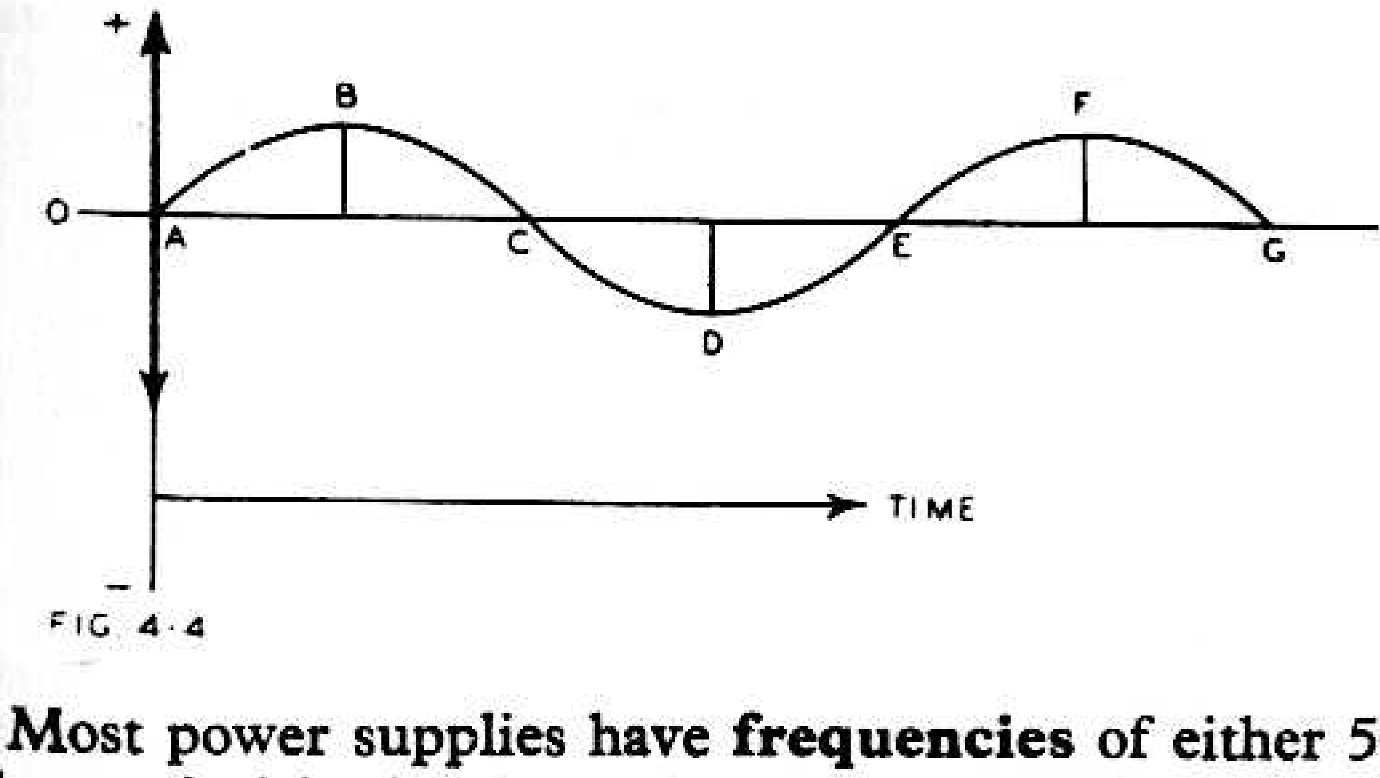


Fig. 4.4. *Form of sine-wave alternating current.*

Most power supplies have **frequencies** of either 50 or 60 cycles per second (c/s). The **period** is the time taken by one cycle, which is 1/50 or 1/60 second, in these two cases.

*For mathematical treatment of periodic phenomena see Chapter 6 Sect. 4.

The precise shape of the wave is very important, and the sine wave has been adopted as the standard a.c. **waveform,** since this is the only one which always has the current waveform of the same shape as the voltage, when applied to a resistance, inductance or capacitance.† In practice we have to deal with various waveforms, some of which may be considered as imperfect (" distorted ") sine waves, while others are of special shapes such as square waves, saw-tooth, or pulse types, or rectified sine waves. However, when we speak of a.c. in any theoretical treatise, it is intended to imply a distortionless sine wave. Other waveforms may be resolved by Fourier Analysis into a fundamental sine wave and a number of harmonic frequency sine waves, the latter having frequencies which are multiples of the fundamental frequency. This subject is treated mathematically in Chapter 6 Sect. 8.

When deriving the characteristics of any circuit, amplifier or network, it is usual to assume the application of a pure sine-wave voltage to the input terminals, then to calculate the currents and voltages in the circuit. In the case of a valve amplifier (or any other non-linear component) the distortion may be either calculated, or measured at the output terminals. If the device is to operate with a special input waveform (e.g. square wave), it is usual to resolve this into its fundamental and harmonic frequencies, and then to calculate the performance with the lowest (fundamental) frequency, an approximate middle** frequency, and the highest harmonic frequency—all these being sine waves.

(iii) Indications of polarity and current flow

In circuit diagrams the polarity of any battery or other d.c. voltage source is usually indicated by + and − signs ; alternatively it may be indicated by an arrow, the head of the arrow indicating positive potential (e.g. Fig. 4.2). A similar convention may be used for the voltage between any points in the circuit (e.g. Fig. 4.14A). The direction of d.c. current flow is indicated by an arrow.

In the case of a.c. circuits a similar convention may be used, except that an arbitrary instantaneous condition is represented (Fig. 4.18A).

SECTION 2 : RESISTANCE

(i) Ohm's Law for d.c. (ii) Ohm's Law for a.c. (iii) Resistances in series (iv) Resistances in parallel (v) Conductance in resistive circuits.

(i) Ohm's Law for direct current

All substances offer some obstruction to the flow of electric current. Ohm's Law states that the current which flows is proportional to the applied voltage, in accordance with the equation

$$I = E/R \qquad (1)$$

where R is the total **resistance** of the circuit. For example in Fig. 4.5 an ideal battery, having zero internal resistance, and giving a constant voltage E under all conditions, is connected across a resistance R. The current which flows is given by eqn. (1) above, provided that

I is expressed in amperes,
E is expressed in volts,
and R is expressed in ohms.*

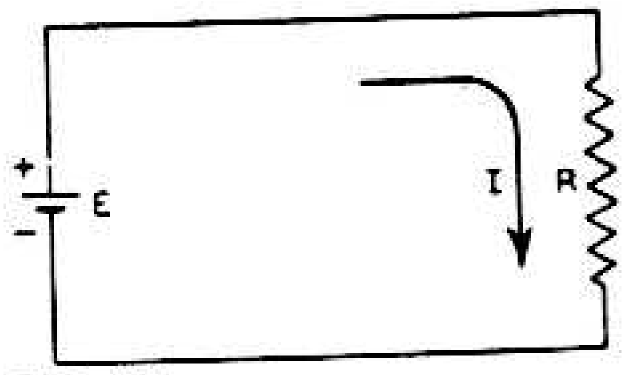


Fig. 4.5. Circuit illustrating Ohm's Law for d.c.

Ohm's Law may also be arranged, for convenience, in the alternative forms

$$E = IR \qquad (2)$$

$$\text{and } R = E/I \qquad (3)$$

†A sinewave has its derivative and integral of the same form as itself.

**Preferably the geometrical mean frequency which is given by $\sqrt{f_1 f_2}$ where f_1 and f_2 are the lowest and highest frequencies.

*It is assumed that the resistance remains constant under the conditions of operation. For other cases see Sect. 7(i).

In a circuit containing more than one battery (or other source of direct voltage), the effective voltage is determined by adding together those voltages which are in the same direction as the current, and subtracting any opposing voltages.

Ohm's Law also holds for any single resistor or combination of resistances. The voltage drop across any resistance R_1, no matter what the external circuit may be, is given by

$$\text{Voltage drop} = IR_1 \tag{4}$$

where I is the current flowing through R_1.

(ii) Ohm's Law for alternating current

Ohm's Law holds also for alternating voltages and currents, except that in this case the voltage (E) and the current (I) must be expressed in their **effective** or **root-mean-square** values of volts and amperes.

(iii) Resistances in series

When two or more resistors are connected so that the current through one is compelled to flow through the others, they are said to be in series, and the total resistance is the sum of their individual resistances. For example, in Fig. 4.6 the total resistance of the circuit is given by

$$R = R_1 + R_2 + R_3 \tag{5}$$

and the current is given by

$$I = E/R = E/(R_1 + R_2 + R_3).$$

It is interesting to note that R_1, R_2 and R_3 form a **voltage divider,** across the battery E. Using eqn. (4):

$$\begin{aligned}
\text{voltage drop across } R_1 &= IR_1 = [R_1/(R_1 + R_2 + R_3)] \times E \\
\text{voltage drop across } R_2 &= IR_2 = [R_2/(R_1 + R_2 + R_3)] \times E \\
\text{voltage drop across } R_3 &= IR_3 = [R_3/(R_1 + R_2 + R_3)] \times E \\
\text{total voltage drop} &= IR_1 + IR_2 + IR_3 \\
&= I\,(R_1 + R_2 + R_3) = E.
\end{aligned}$$

For example, if $E = 6$ volts, $R_1 = 10$ ohms, $R_2 = 10$ ohms and $R_3 = 10$ ohms, then

$$\begin{aligned}
I &= E/(R_1 + R_2 + R_3) = 6/(10 + 10 + 10) \\
&= 6/30 = 0.2 \text{ ampere.}
\end{aligned}$$

$$\begin{aligned}
\text{Voltage between points } C \text{ and } D &= 10 \times 0.2 = 2 \text{ volts} \\
B \text{ and } D &= 20 \times 0.2 = 4 \text{ volts} \\
A \text{ and } D &= 30 \times 0.2 = 6 \text{ volts.}
\end{aligned}$$

The voltage across any section of the voltage divider is proportional to its resistance (it is assumed that no current is drawn from the tapping points B or C).

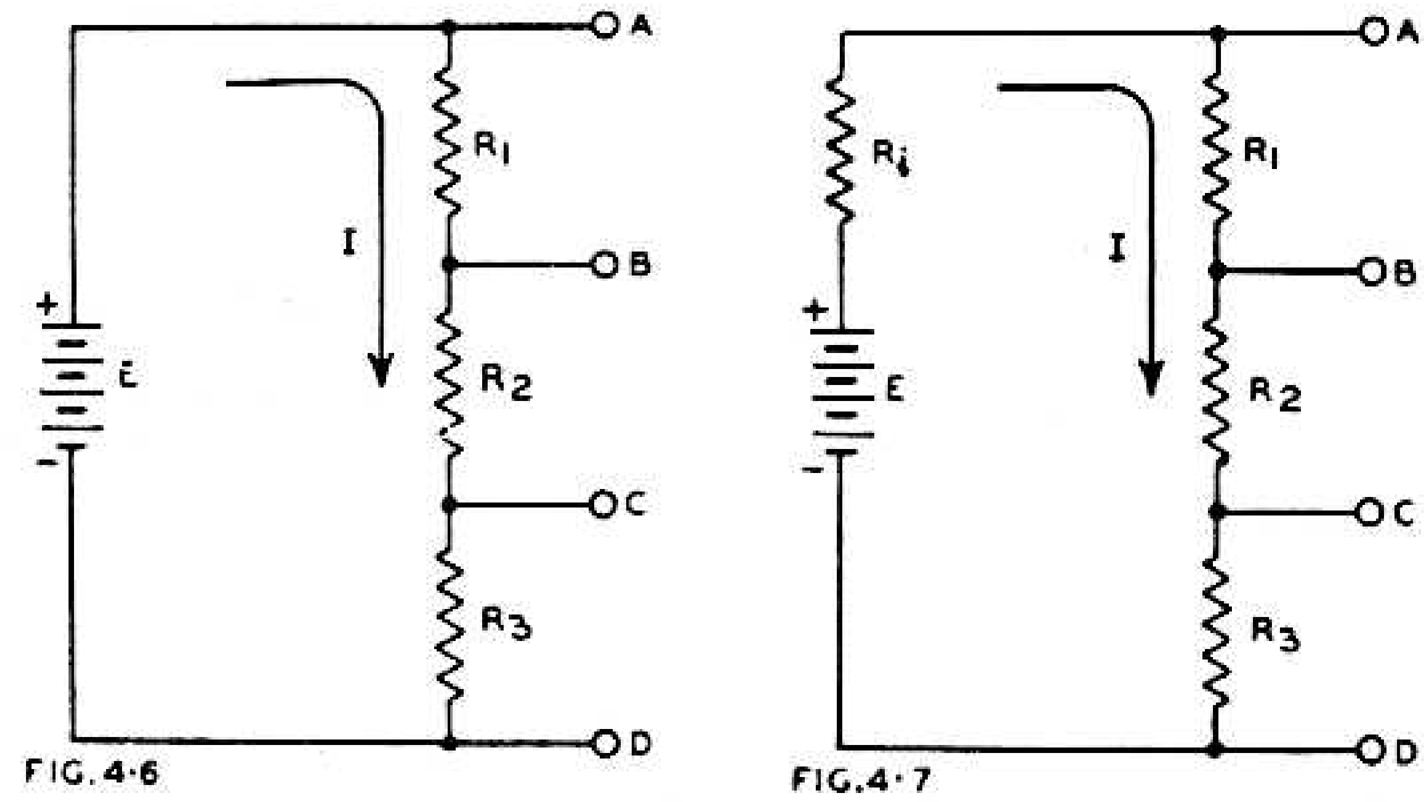


Fig. 4.6. Resistances in series.

Fig. 4.7. Resistances in series, allowing for internal resistance of battery.

If the battery has any appreciable internal resistance the circuit must be modified to the form of Fig. 4.7 where R_i is the equivalent internal resistance. Here we have four resistances effectively in series and $I = E/(R_i + R_1 + R_2 + R_3)$. The voltages between any of the points A, B, C or D will be less than in the corresponding case for zero internal resistance, the actual values being

$$\frac{R_1 + R_2 + R_3}{R_i + R_1 + R_2 + R_3} \times \text{voltage for } R_i = 0.$$

If R_i is less than 1% of $(R_1 + R_2 + R_3)$, then its effect on voltages is less than 1%.

(iv) Resistances in parallel

When two resistances are in parallel (Fig. 4.8) the effective total resistance is given by

$$R = \frac{R_1 R_2}{R_1 + R_2} \qquad (6)$$

When $R_1 = R_2$, $R = R_1/2 = R_2/2$.
When any number of resistances are in parallel (Fig. 4.9) the effective total resistance is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots . \qquad (7)$$

When two or more resistors are in parallel, the total effective resistance may be determined by the graphical method of Fig. 4.10. This method* only requires a piece of ordinary graph paper (or alternatively a scale and set square). As an example, to find the total resistance of two resistors, 50 000 and 30 000 ohms, in parallel take any convenient base AB with verticals AC and BD at the two ends. Take 50 000 ohms on BD and draw the straight line AD; take 30 000 ohms on AC and draw CB; draw the line XY from their junction perpendicular to AB. The height of XY gives the required result, on the same scale.

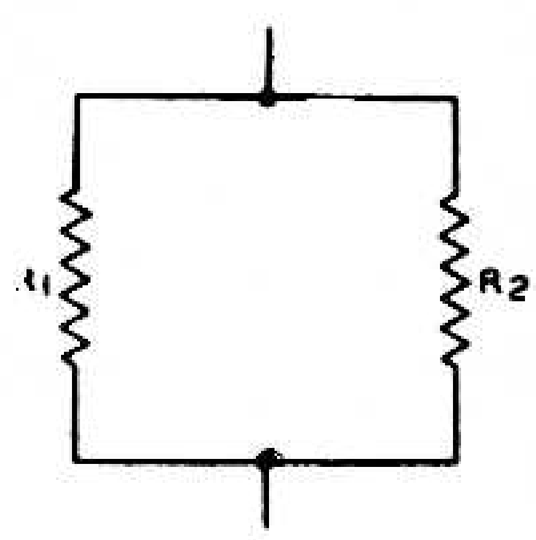


FIG. 4-8

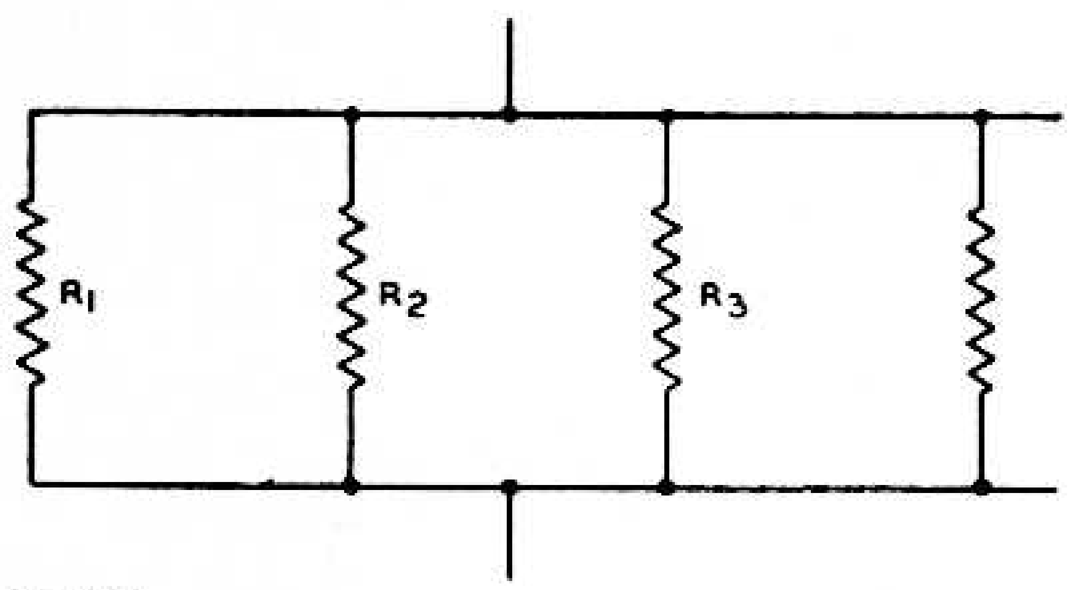


FIG. 4-9

Fig. 4.8. *Two resistances in parallel.* *Fig.* 4.9. *Several resistors in parallel.*

If it is required to determine the resistance of three resistors in parallel, the third being say 20 000 ohms, proceed further to join points E and Y, and the desired result is given by the height PQ. This may be continued indefinitely.

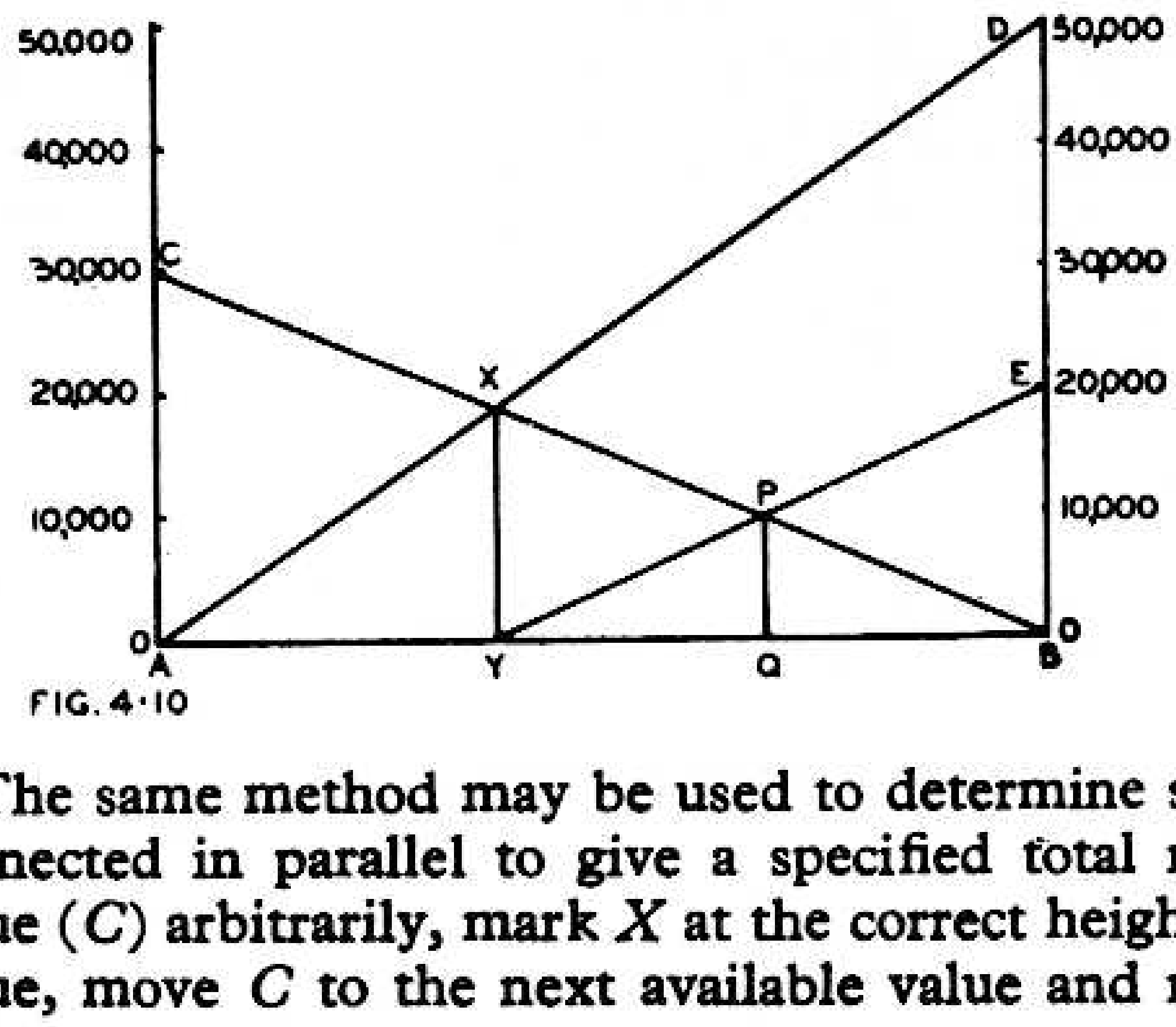


FIG. 4·10

Fig. 4.10. *Graphical method for the determination of the effective resistance of two or more resistors in parallel (after Wireless World).*

The same method may be used to determine suitable values of two resistors to be connected in parallel to give a specified total resistance. In this case, select one value (C) arbitrarily, mark X at the correct height and then find D; if not a suitable value, move C to the next available value and repeat the process until satisfactory.

*"Resistances in Parallel—Capacitances in Series," W.W. 48.9 (Sept. 1942) 205.
"Diallist" "Series C and Parallel R," W.W. 51.4 (April 1945) 126.

When a network includes a number of resistors, some in series and some in parallel, firstly convert all groups in parallel to their effective total resistances, then proceed with the series chain.

(v) Conductance in resistive circuits

The conductance (G) of any resistor is its ability to conduct current, and this is obviously the reciprocal of the resistance—

$$G = 1/R \tag{8}$$

Applying Ohm's Law, we derive

$$I = EG \tag{9}$$

The unit of conductance is the mho (i.e. the reciprocal ohm).

When resistances are in parallel, their effective total conductance is the sum of their individual conductances—

$$G = G_1 + G_2 + G_3 + \ldots . \tag{10}$$

When a number of resistors are in parallel, the current through each is proportional to its conductance. Also,

$$\frac{I_1}{I_{total}} = \frac{G_1}{G_{total}} \tag{11}$$

SECTION 3 : POWER

(i) Power in d.c. circuits (ii) Power in resistive a.c. circuits.

(i) Power in d.c. circuits

The power converted into heat in a resistance is directly proportional to the product of the voltage and the current—

$$P = E \times I \tag{1}$$

where P is expressed in watts, E in volts and I in amperes. This equation may be rearranged, by using Ohm's Law, into the alternative forms—

$$P = I^2R = E^2/R \tag{2}$$

where R is expressed in ohms.

The total **energy** developed is the product of the power and the time. Units of energy are

(1) the **watt-second** or **joule**
(2) the **kilowatt-hour** (i.e. 1000 watts for 1 hour).

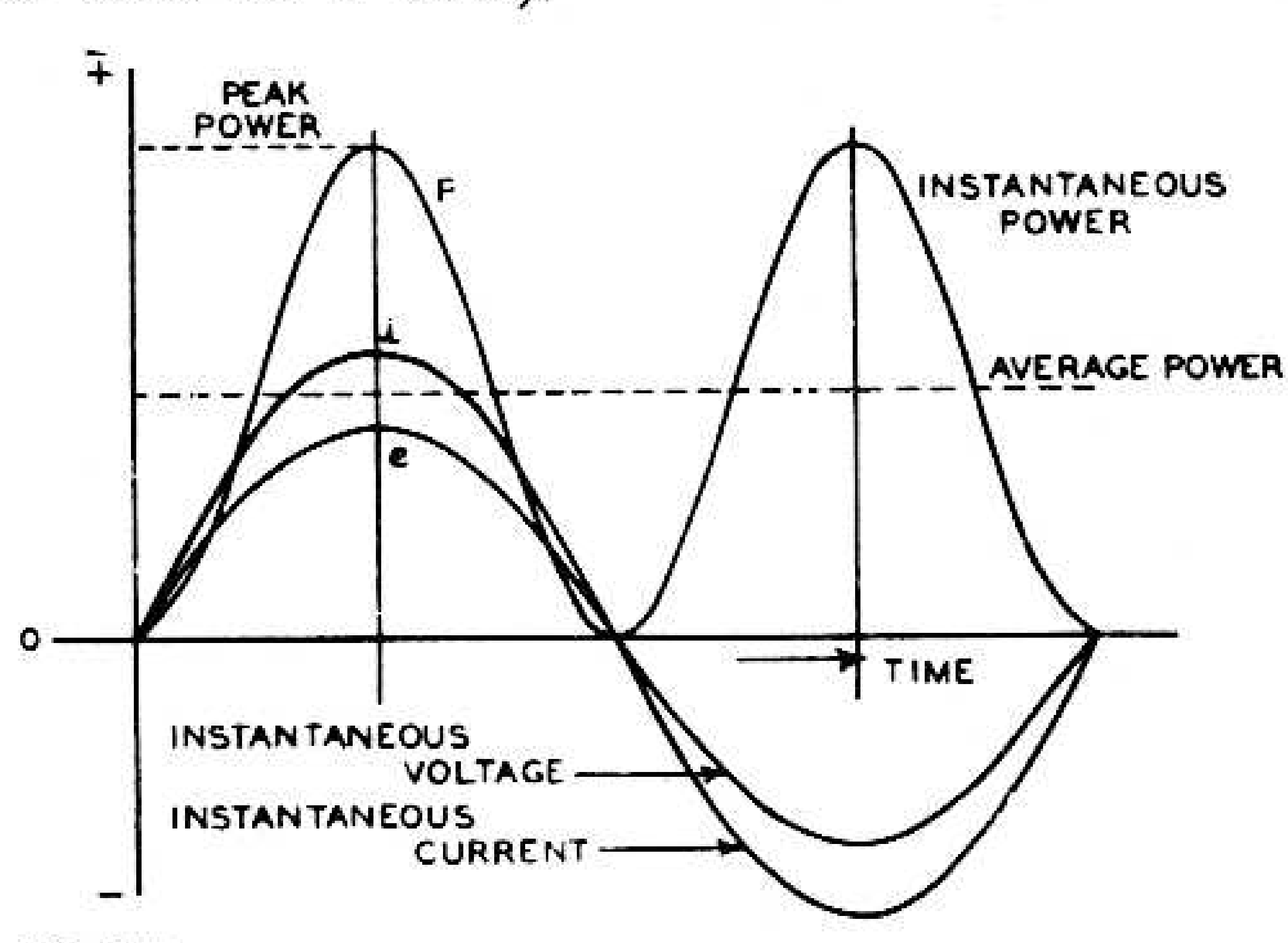


Fig. 4.11. One cycle of sine-wave voltage (e) and current (i) with zero phase angle. The instantaneous power (P) is always positive; the average power is half the peak power.

(ii) Power in resistive a.c. circuits

The same general principles hold as for d.c., except that the voltage, current and power are varying. Fig. 4.11 shows one cycle of a sinewave voltage and current. The instantaneous power is equal to the product of the instantaneous voltage and instantaneous current at any point in the cycle. The curve of instantaneous power (P) may be plotted point by point, and is always positive.

The heating of a resistor is obviously the result of the **average** or **effective power,** which is exactly half the peak power.

$$P_{peak} = E_{max}\, I_{max} \tag{3}$$

$$P_{av} = \frac{E_{max}\, I_{max}}{2} = \frac{E_{max}}{\sqrt{2}} \times \frac{I_{max}}{\sqrt{2}} \tag{4}$$

$E_{max}/\sqrt{2}$ is called the **effective** or **r.m.s. voltage**

$I_{max}/\sqrt{2}$ is called the **effective** or **r.m.s. current.**

The effective values of voltage and current are the values which have the same heating effect as with d.c. The initials r.m.s. stand for root mean square, indicating that it is the square root of the average of the squares over the cycle.

In a.c. practice, any reference to voltages or currents without specifying which value is intended, should always be interpreted as being r.m.s. (or effective) values. Measuring instruments are usually calibrated in r.m.s. values of currents and voltages.

The **form factor** is the ratio of the r.m.s. to the average value of the positive half-cycle. The following table summarizes the principal characteristics of several waveforms, over the positive half-cycle in each case (see Chapter 30 for rectified waveforms) :

	Sine wave	Square wave	Triangular wave (isosceles)
Form factor (= r.m.s./average)	$\pi/2\sqrt{2} = 1.11$	1.00	$2/\sqrt{3} = 1.15$
Peak/r.m.s.	$\sqrt{2} = 1.414$	1.00	$\sqrt{3} = 1.73$
R.M.S./peak	$1/\sqrt{2} = 0.707$	1.00	$1/\sqrt{3} = 0.58$
Peak/average	$\pi/2 = 1.57$	1.00	2.0
Average/Peak	$2/\pi = 0.64$	1.00	0.5

SECTION 4 : CAPACITANCE

(i) Introduction to capacitance (ii) Condensers in parallel and series (iii) Calculation of capacitance (iv) Condensers in d.c. circuits (v) Condensers in a.c. circuits.

(i) Introduction to capacitance

A capacitor* (or condenser) in its simplest form, consists of two plates separated by an insulator (dielectric).

Any such condenser has a characteristic known as capacitance† whereby it is able to hold an electric charge. When a voltage difference (E) is applied between the plates, current flows instantaneously through the leads connecting the battery to the condenser (Fig. 4.12) until the latter has built up its charge, the current dropping gradually to zero. If the battery is removed, the condenser will hold its charge indefinitely (in practice there is a gradual loss of charge through leakage). If a conducting path is connected across the condenser plates, a current will flow through the conductor but will gradually fall to zero as the condenser loses its charge.

It is found that the charge (i.e. the amount of electricity) which a condenser will hold is proportional to the applied voltage and to the capacitance.

This may be put into the form of an equation :

$$Q = CE \tag{1}$$

where Q = quantity of electricity (the charge) in coulombs,
C = capacitance in farads,
and E = applied voltage.

*The American standard term is "Capacitor."
†It is assumed here that the condenser is ideal, without series resistance, leakage, or dielectric lag.

The unit of capacitance—the Farad (F)—is too large for convenience, so it is usual to specify capacitance as so many microfarads (μF) or micro-microfarads* ($\mu\mu$F). Any capacitance must be converted into its equivalent value in farads before being used in any fundamental equation such as (1) above—

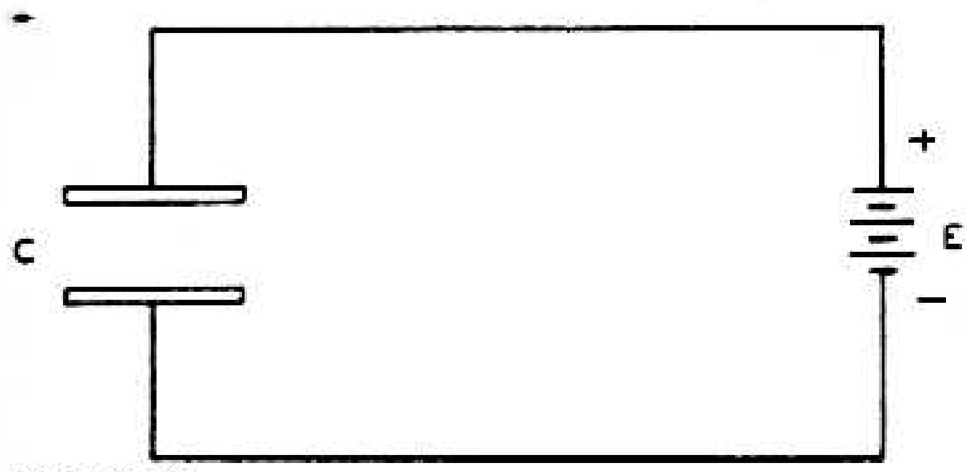
$$1\,\mu F = 1 \times 10^{-6} \text{ farad}$$
$$1\,\mu\mu F = 1 \times 10^{-12} \text{ farad}$$

Note: The abbreviations mF or mmF should not be used under any circumstances to indicate microfarads or micro-microfarads, because mF is the symbol for milli-farads (1 × 10^{-3} farad). Some reasonable latitude is allowable with most symbols, but here there is danger of serious error and misunderstanding.

The **energy stored** in placing a charge on a condenser is

$$W = \tfrac{1}{2}(QE) = \tfrac{1}{2}(CE^2) = Q^2/2C \qquad (2)$$

where W = energy, expressed in joules (watt-seconds)
Q = charge in coulombs
C = capacitance in farads
and E = applied voltage.



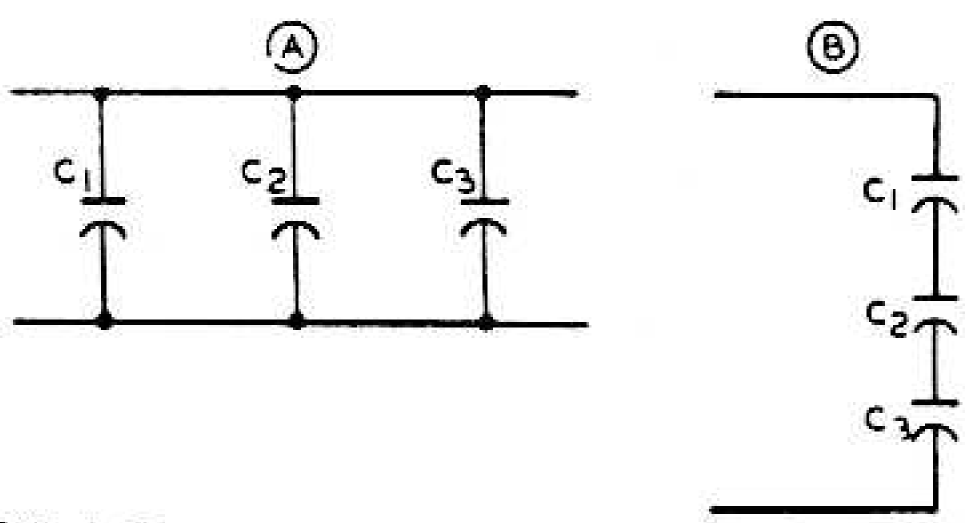


Fig. 4.12. *Condenser connected to a battery.*
Fig. 4.13. *(A) Condensers in parallel. (B) Condensers in series.*

(ii) Condensers in parallel and series

When two or more condensers are connected in parallel (Fig. 4.13A) the total capacitance is the sum of their individual capacitances:

$$C = C_1 + C_2 + C_3 + \ldots \qquad (3)$$

When two or more condensers are connected in series (Fig. 4.13B) the total capacitance is given by:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \qquad (4)$$

When only two condensers are connected in series:

$$C = \frac{C_1 C_2}{C_1 + C_2}. \qquad (5)$$

When two or more **equal** condensers (C_1) are connected in series, the total capacitance is

$$C = C_1/2 \text{ for 2 condensers}$$

and

$$C = C_1/n \text{ for } n \text{ condensers.}$$

Note: The curved plate of the symbol used for a condenser indicates the earthed (outer) plate of an electrolytic or circular paper condenser; when this is not applicable the curved plate is regarded as the one more nearly at earth potential.

(iii) Calculation of capacitance

Parallel plate condenser

When there are two plates, close together, the capacitance is approximately:

$$C = \frac{AK}{11.31d} \quad \mu\mu F \text{ when dimensions are in centimetres}$$

*The name picofarad (pF) is also used as an alternative.

or $C = \dfrac{AK}{4.45d}$ $\mu\mu$F when dimensions are in inches

where A = useful area of one plate in square centimetres (or inches). The useful area is approximately equal to the area of the smaller plate when the square root of the area is large compared with the gap.

K = dielectric constant (for values of common materials see Chapter 38 Sect. 8. For air, $K = 1$.

d = gap between plates in centimetres (or inches).

Capacitance with air dielectric, plates 1 mm. apart

$C = 0.884$ $\mu\mu$F per sq. cm. area of one plate.

Capacitance with air dielectric, plates 0.10 inch apart

$C = 2.244$ $\mu\mu$F per sq. inch area of one plate.

When there are more than two plates, the " useful area " should be interpreted as the total useful area.

Cylindrical condenser (concentric cable)

$$C = \frac{7.354K}{\log_{10} D/d} \quad \mu\mu\text{F per foot length}$$

where D = inside diameter of outside cylinder (inches)

d = outside diameter of inner cylinder (inches)

and K = dielectric constant of material in gap.

(iv) Condensers in d.c. circuits

An **ideal condenser** is one which has no resistance, no leakage, and no inductance. In practice, every condenser has some resistance, leakage and inductance, although these may be neglected under certain conditions of operation.

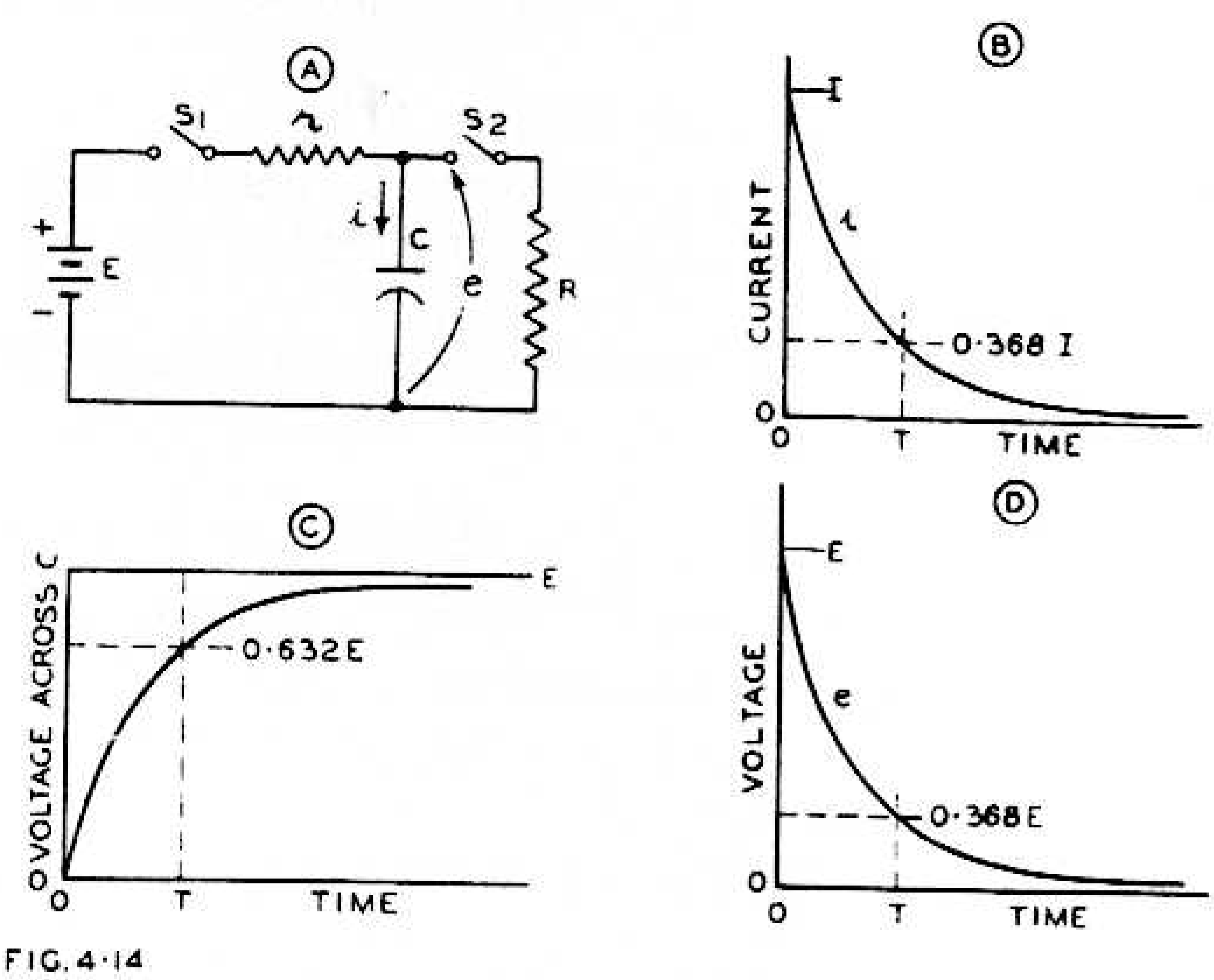


Fig. 4.14. Condenser charge and discharge (A) Circuit (B) Discharge current characteristic (C) Charge voltage characteristic (D) Discharge voltage characteristic.

In Fig. 4.14A, C is an ideal condenser which may be charged by closing switch S_1; r represents the combined internal resistance of the battery E and the resistance of the leads in the circuit. When S_1 is closed, current (i) will flow as indicated in diagram B, the peak current being $I = E/r$ at time $t = 0$. The time for C to become fully charged is infinite—in other words the current never quite reaches zero, although it comes very close to zero after a short period. The equation for the current is of logarithmic form :

$$i = \frac{E}{r}\,\epsilon^{-t/rC} \qquad (6)$$

where ϵ = base of natural logarithms ($\approx$ 2.718)
t = time in seconds after closing switch S_1
E = battery voltage
r = resistance in ohms
and C = capacitance in farads.

The voltage (e) across the condenser is

$$e = E - ri = E\,(1 - \epsilon^{-t/rC}) \qquad (7a)$$

and the curve (diagram C) is of the same shape as the current curve except that it is upside down. The voltage never quite reaches the value E, although it approaches it very closely.

The charge on the condenser is given by

$$q = Q\,(1 - \epsilon^{-t/rC}) \qquad (7b)$$

where q = instantaneous charge on condenser
and $Q = EC$ = final charge on condenser
which follows the same law as the voltage (eqn. 7a).

If we now assume that the condenser C is fully charged, switch S_1 is opened, and switch S_2 is closed, the **discharge characteristic** will be given by

$$i = -\frac{E}{R}\,\epsilon^{-t/RC} \qquad (8)$$

which is of the same form as diagram B, except that the current is in the opposite direction.

The curve of voltage (and also charge) against time for a discharging condenser is in diagram D, and is of the same shape as for current, since $e = Ri$. These charge and discharge characteristics are called **transients**.

In order to make a convenient measure of the time taken to discharge a condenser, we adopt the **time constant** which is the time taken to discharge a condenser on the assumption that the current remains constant throughout the process at its initial value. In practice, as explained above, the discharge current steadily falls with time, and under these conditions the time constant is the time taken to discharge the condenser to the point where the voltage or charge drops to $1/\epsilon$ or 36.8% of its initial value. The same applies also to the time taken by a condenser in process of being charged, to reach a voltage or charge of $(1 - 1/\epsilon)$ or 63.2% of its final value.

The time constant (T) is equal to

$$T = RC \qquad (9)$$

where T is the time constant in seconds,
R is the total resistance in the circuit, either for charge or discharge, in ohms,
and C is the capacitance in farads.

This also holds when R is in megohms and C in microfarads.

(v) Condensers in a.c. circuits

When a condenser is connected to an a.c. line, current flows in the circuit, as may be checked by inserting an a.c. ammeter in series with the condenser. This does not mean that electrons flow through the condenser from one plate to the other ; they are insulated from one another.

Suppose that a condenser of capacitance C farads is connected directly across an a.c. line, the voltage of which is given by the equation $e = E_m \sin \omega t$. The condenser will take sufficient charge to make the potential difference of its plates at every instant equal to the voltage of the line. As the impressed voltage continually varies in magnitude and direction, electrons must be continually passing in and out of the condenser to maintain its plates at the correct potential difference. This continual charging and discharging of the condenser constitutes the current read by the ammeter.

At any instant, $q = Ce$, where q is the instantaneous charge on the condenser. The current (i) is the rate of change (or differential* with respect to time) of the charge,

*See Chapter 6, Section 7.

i.e., $i = dq/dt$

But $q = Ce = CE_m \sin \omega t$

Therefore $i = \frac{d}{dt}(CE_m \sin \omega t)$

Therefore $i = \omega CE_m \cos \omega t$ (10)

Eqn. (10) is the equation of the current flowing through the condenser, from which we may derive the following facts :

1. It has a peak value of ωCE_m ; the current is therefore proportional to the applied voltage, also to the capacitance and to the frequency (since $f = \omega/2\pi$).
2. It has the same angular velocity (ω) and hence the same frequency as the applied voltage.
3. It follows a cosine waveform whereas the applied voltage has a sine waveform. This is the same as a sinewave advanced 90° in phase—we say that the current leads the voltage by 90° (Fig. 4.15).

Considering only the magnitude of the condenser charging current,

$I_m = \omega CE_m$ (peak values)

Therefore $I_{rms} = \omega CE_{rms}$ (effective values)

Where I and E occur in a.c. theory, they should be understood as being the same as I_{rms} and E_{rms}.

This should be compared with the equivalent expression when the condenser is replaced by a resistance (R) :

$$I_{rms} = \frac{E_{rms}}{R}.$$

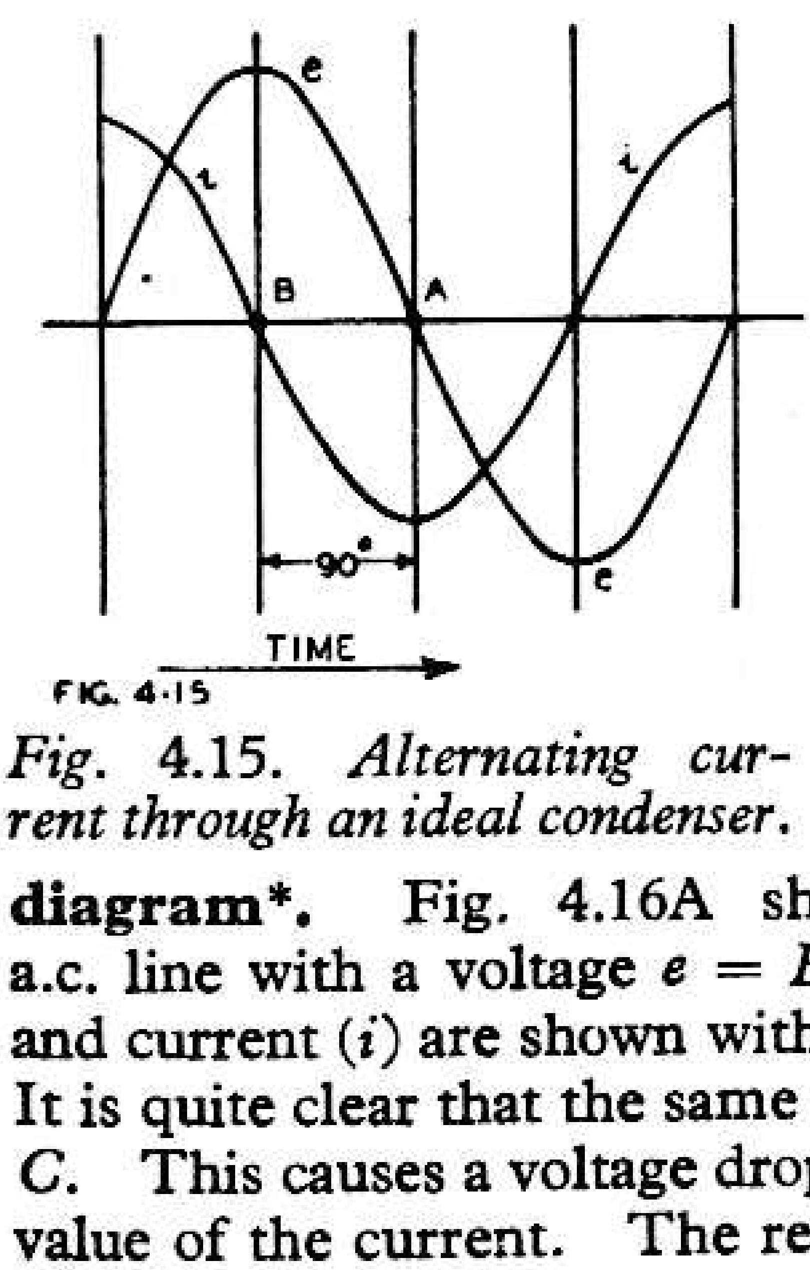


Fig. 4.15. *Alternating current through an ideal condenser.*

It will be seen that R in the resistance case, and $(1/\omega C)$ in the capacitance case, have a similar effect in limiting the current. We call $(-1/\omega C)$ the **capacitive reactance**† (X_c) of the condenser, since it has the additional effect of advancing the phase of the current. We here adopt the convention of making the capacitive reactance negative, and the inductive reactance positive ; the two types of reactance are vectorially 180° out of phase.

The relationships between the various voltages and currents are well illustrated by a **vector diagram***. Fig. 4.16A shows a circuit with R and C in series across an a.c. line with a voltage $e = E_m \sin \omega t$. The instantaneous** values of voltage (e) and current (i) are shown with arrows to indicate the convention of positive direction. It is quite clear that the same current which passes through R must also pass through C. This causes a voltage drop RI across R and $(I/\omega C)$ across C where I is the r.m.s. value of the current. The relative phase relationships are given by the angular displacements in the vector diagram. These are determined trigonometrically by the peak values I_m, RI_m and $I_m/\omega C$; for convenience, the lengths of the vectors are marked in Fig. 4.16B according to the effective values I, RI and $I/\omega C$. The current vector is distinguished by a solid arrowhead ; it may be to any convenient scale since there is no connection between the voltage and current scales. Since R is purely resistive, RI must be in phase with I, but the current through C must lead the voltage drop across C by 90°. This is shown on the vector diagram 4.16B where the direction of I is taken as the starting point ; RI is drawn in phase with I and of length equal to the voltage drop on any convenient scale ; $I/\omega C$ is drawn so that I leads it by 90° ; and the resultant (E) of RI and $I/\omega C$ is determined by completing the parallelogram The resultant E is the vector sum of the voltage drops across R and C, which must

†A table of capacitive reactances is given in Chapter 38 Sect. 9 Table 42.

*See Chapter 6 Section 5(iv).

**It is illogical to show directions or polarities on the r.m.s. values of current (I) or voltage (E) since these are the effective values of alternating currents.

therefore be the applied (line) voltage. It will be seen that the current I leads the voltage E by an angle ϕ where

$$\tan \phi = \frac{I}{\omega C} \cdot \frac{1}{RI} = \frac{1}{\omega CR} = \frac{|X_c|}{R} \quad \text{(10a)}$$

If $R = 0$, then $\tan \phi = \infty$, and $\phi = 90°$.

Since ϕ is the phase angle of the current with respect to the voltage, the angle ϕ in the circuit of Fig. 4.16A is positive.

The instantaneous current flowing through the circuit of Fig. 4.16A is therefore given by

$$i = I_m \sin(\omega t + \phi) \quad \text{(10b)}$$

where $\phi = \tan^{-1}(1/\omega CR)$.

The value of I_m in eqn. (10b) is given by

$$I_m = E_m/\mathbf{Z} \quad \text{(10c)}$$

where $\mathbf{Z}$ is called the impedance.

Similarly in terms of effective values,

$$I = E/\mathbf{Z} \quad \text{(10d)}$$

Obviously $\mathbf{Z}$ is a vector (or complex) quantity having phase relationship as well as magnitude, and is printed in bold face to indicate this fact. This may be developed further by the use of the j notation.

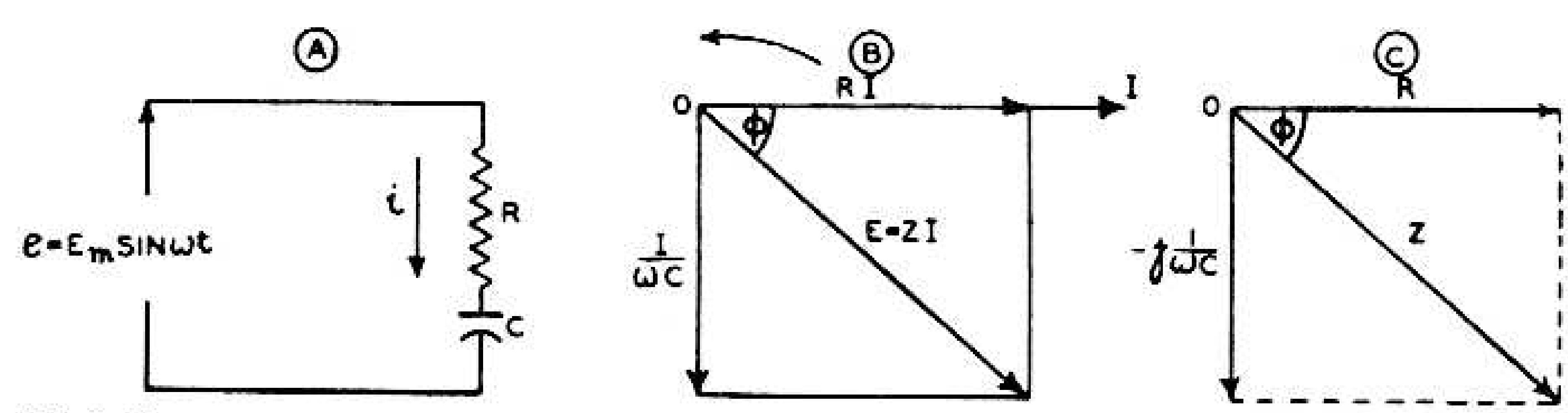


FIG. 4·16

Fig. 4.16. *(A) Resistance and capacitance across a.c. line (B) Vector diagram of voltage relationships (C) Vector diagram of impedance (Z) with its real component (R) and reactive component —j (1/ωC).*

Using the j Notation

The operator j* signifies a positive vector rotation of 90°, while $-j$ signifies a negative rotation of 90°. Instead of working out a detailed vector diagram, it is possible to treat a circuit problem very much more simply by using the j notation. For example in Fig. 4.16, we may equate the applied voltage E to the sum of the voltage drops across R and C:

$$E = RI - j\frac{I}{\omega C} \quad \text{(11)}$$

the $-j$ indicating 90° vector rotation in a negative direction, which is exactly what we have in diagram B.

It is sometimes more convenient to put $-jI/\omega C$ into the alternative form $+I/j\omega C$ which may be derived by multiplying both numerator and denominator by j (since $j^2 = -1$). Thus

$$E = RI + I/j\omega C \quad \text{(12)}$$

From (11) we can derive:

$$\mathbf{Z} = E/I = R - j(1/\omega C) \quad \text{(13)}$$

For example, in Fig. 4.16A let $R = 100$ ohms and $C = 10\,\mu$F, both connected in series, and let the frequency be 1000 c/s.

Then $\omega = 2\pi \times 1000 = 6280$

$1/\omega C = 1/(6280 \times 10 \times 10^{-6}) = 15.9$ ohms.

and $\mathbf{Z} = R - j(1/\omega C)$

$= 100 - j\,15.9 \quad$ (13a)

Eqn. (13a) indicates at a glance a resistance of 100 ohms in series with a negative reactance (i.e. a capacitive reactance) of 15.9 ohms. Values of capacitive reactances

*See Chapter 6 Sect. 5(iv).

for selected capacitances and frequencies are given in Chapter 38 Sect. 9 Table 42. Even if $R = 0$, we still write the impedance in the same form,

$$\mathbf{Z} = 0 - j\,15.9 \tag{13b}$$

Thus **Z** is a complex* quantity, that is to say it has a "real" part (R) and an "imaginary" part ($1/\omega C$) at 90° to R, as shown in Fig. 4.16C. The absolute magnitude (modulus) of **Z** is :

$$|Z| = \sqrt{R^2 + (1/\omega C)^2} \tag{14}$$

and its phase angle ϕ is given by

$$\tan\phi = -\frac{1}{\omega CR} \tag{15}$$

which is the same as we derived above from the vector diagram, except for the sign. The negative sign in eqn. (15) is because ϕ is the phase shift of **Z** with respect to R.

In a practical condenser, we may regard R in Fig. 4.16A as the equivalent series loss resistance of the condenser itself.

It is obvious from Fig. 4.16B that

$$\cos\phi = R/\mathbf{Z} \tag{16a}$$

where **Z** is given by equation (13) above.
Cos ϕ is called the "Power factor."

If ϕ is nearly 90°, X is nearly equal to Z, apart from sign, and so

$$\cos\phi \approx R/|X| \tag{16b}$$

where $|X|$ indicates that the value of the reactance is taken apart from sign. With this approximation we obtain

$$\text{Power factor} = \cos\phi \approx \omega CR \tag{17}$$

Note that the power factor of any resistive component is always positive. A negative power factor indicates generation of power.

The total power dissipated in the circuit (Fig. 4.16A) will be

$$P = EI\cos\phi = EI \times \text{power factor} \tag{17a}$$

$$= I^2R = \frac{E^2}{R[1 + X_c^2/R^2]} \tag{17b}$$

where P = power in watts
$X_c = 1/\omega C$
and $\cos\phi$ is defined by eqn. (16a) or the approximation (16b).

The Q factor of a condenser is the ratio of its reactance to its resistance—

$$Q = 1/\omega CR = \tan\phi \approx 1/(\text{power factor}) \tag{18}$$

SECTION 5 : INDUCTANCE

(i) Introduction to inductance (ii) Inductances in d.c. circuits (iii) Inductances in series and parallel (iv) Mutual inductance (v) Inductances in a.c. circuits (vi) Power in inductive circuits.

(i) Introduction to inductance

An inductor, in its simplest form, consists of a coil of wire with an air core as commonly used in r-f tuning circuits. Any inductor has a characteristic known as inductance whereby it sets up an electro-magnetic field when a current is passed through it. When the current is varied, the strength of the field varies ; as a result, an electromotive force is induced in the coil. This may be expressed by the equation :

$$e = -N\frac{d\phi}{dt} \times 10^{-8} \text{ volts} \tag{1}$$

where e = e.m.f. induced at any instant
N = number of turns in the coil
ϕ = flux through the coil

*See Chapter 6, Sect. 6(i).

and $\frac{d\phi}{dt}$ = rate of change (differential with respect to time) of the flux.

The direction of the induced e.m.f. is always such as to oppose the change of current which is producing the induced voltage. In other words, the effect of the induced e.m.f. is to assist in maintaining constant both current and field.

We may also express the relationship in the form :

$$e = -L\frac{di}{dt} \quad (2)$$

where all the values are expressed in practical units—

e = e.m.f. induced at any instant, in volts
L = inductance, in henrys
i = current in amperes

and $\frac{di}{dt}$ = rate of change of current, in amperes per second.

The inductance L varies approximately* with the square of the number of turns in the coil, and may be increased considerably by using iron cores** (for low frequencies) or powdered iron (for radio frequencies). With iron cores the value of L is not constant, so that eqn. (2) cannot be used accurately in such cases.

The **energy stored in a magnetic field** is

$$\text{energy} = \tfrac{1}{2} LI^2 \quad (2a)$$

where energy is measured in joules (or watt seconds)
L is measured in henrys
and I is measured in amperes.

(ii) Inductances in d.c. circuits

When an inductance (L henrys) with a total circuit resistance (R ohms) is connected to a d.c. source of voltage (E), the current rises gradually to the steady value $I = E/R$. During the gradual rise, the current follows the logarithmic law

$$i = \frac{E}{R}(1 - \epsilon^{-Rt/L}) \quad (3)$$

where i = current in amperes at time t,
ϵ = base of natural logarithms $\approx$ 2.718,
and t = time in seconds after switch is closed.

The **time constant** (T) is the time in seconds from the time that the switch is closed until the current has risen to $(1 - 1/\epsilon)$ or 63.2% of its final value :

$$T = L/R \quad (4)$$

where L = inductance in henrys
and R = resistance in ohms.

The decay of current follows the law

$$i = \frac{E}{R}(\epsilon^{-Rt/L}) \quad (5)$$

The rise and decay of current are called **transients**.

(iii) Inductances in series and parallel

Inductances in series

The total inductance is equal to the sum of the individual inductances, provided that there is no coupling between them :

$$L = L_1 + L_2 + L_3 + \ldots \quad (6)$$

Inductances in parallel

The total inductance is given by eqn. (7), provided that there is no coupling between them :

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots \quad (7)$$

*See Chapter 10 for formulae for calculating the inductance of coils.
**See Chapter 5 for iron cored inductances.

(iv) Mutual inductance

When two coils are placed near to one another, there tends to be coupling between them, which reaches a maximum when they are placed co-axially and with their centres as close together as possible.

If one such coil is supplied with varying current, it will set up a varying magnetic field, which in turn will induce an e.m.f. in the second coil. This induced e.m.f. in the secondary is proportional to the rate of current change in the first coil (primary) and to the mutual inductance of the two coils :

$$e_2 = -M\frac{di_1}{dt} \tag{8}$$

where e_2 = voltage induced in the secondary,

$\frac{di_1}{dt}$ = rate of change of current in the primary in amperes per second,

and M = mutual inductance of the two coils, in henrys.

(Compare Equations 2 and 8.)

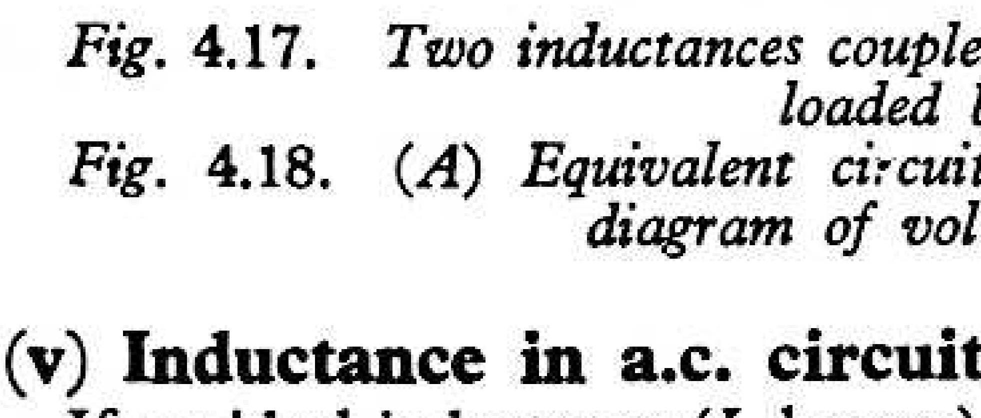
M may be either positive or negative, depending on the rotation of, or connections to, the secondary. M is regarded as positive if the secondary voltage (e_2) has the same polarity as the induced voltage in a single coil.

The maximum possible (theoretical) value of M is when $M = \sqrt{L_1L_2}$, being the condition of unity coupling, but in practice this cannot be achieved. The coefficient of coupling (k) is given by

$$k = M/\sqrt{L_1L_2} \tag{9}$$

so that k is always less than unity.

If the secondary is loaded by a resistance R_2 (Fig. 4.17), current will flow through the secondary circuit.



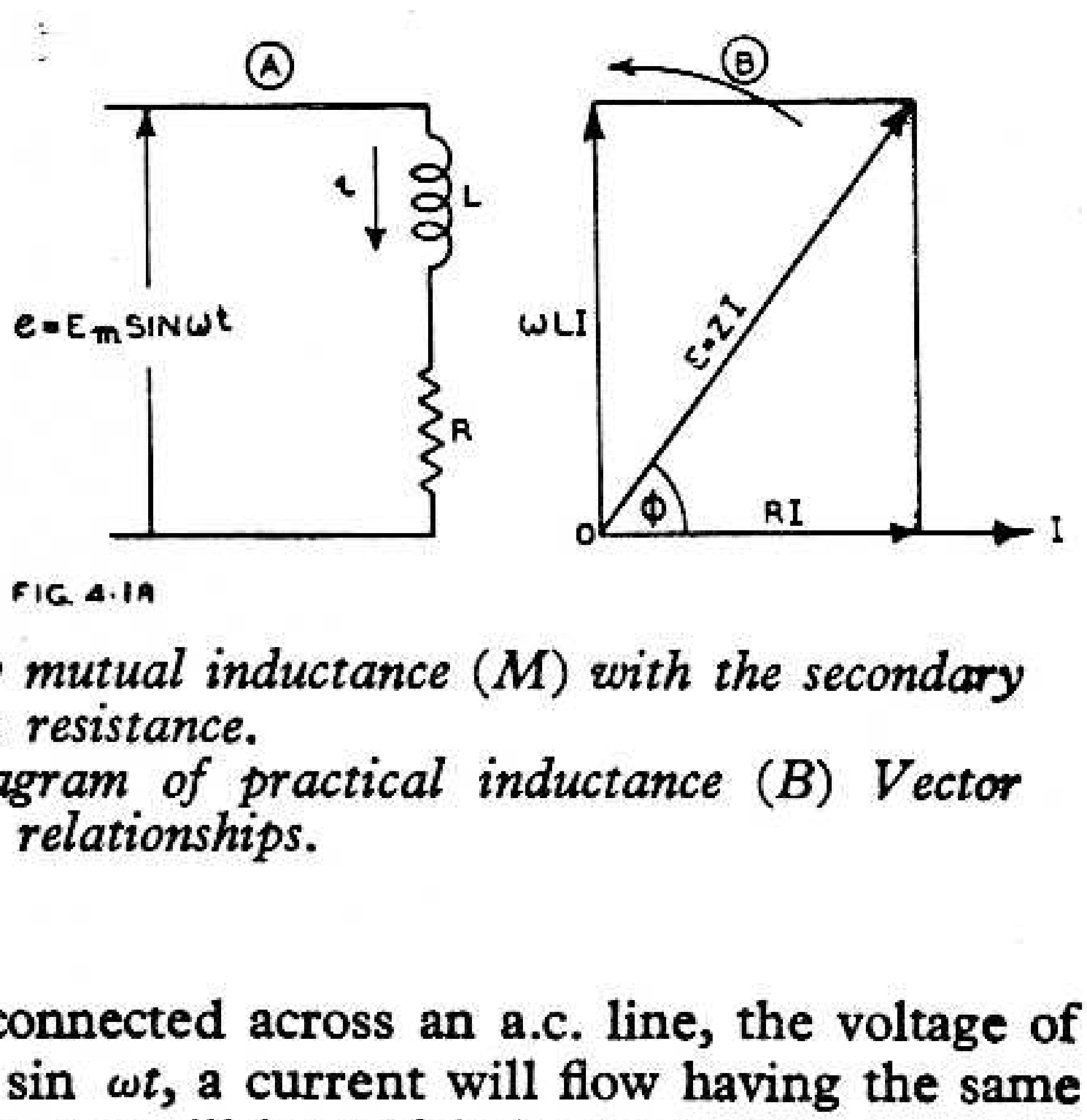


Fig. 4.17. *Two inductances coupled by mutual inductance (M) with the secondary loaded by a resistance.*

Fig. 4.18. *(A) Equivalent circuit diagram of practical inductance (B) Vector diagram of voltage relationships.*

(v) Inductance in a.c. circuits

If an ideal inductance (L henrys) is connected across an a.c. line, the voltage of which is given by the equation $e = E_m \sin \omega t$, a current will flow having the same waveform as the line voltage, but the current will lag 90° behind the voltage. The inductance is said to have an **inductive reactance*** (X_L) equal to ωL, and the current will be

$$I_{rms} = E_{rms}/\omega L \tag{10}$$

We may helpfully compare this with the case of a condenser :

	Current	Reactance	Phase shift†
Capacitance :	$I_{rms} = \omega CE_{rms}$	$X_c = -1/\omega C$	+90°
Inductance :	$I_{rms} = E_{rms}/\omega L$	$X_L = \omega L$	−90°

Every practical inductance has appreciable resistance, and we may draw the equivalent circuit of any normal inductor as an ideal inductance in series with a resistance

*A table of inductive reactances is given in Chapter 38 Sect. 9 Table 41.
†Of current with respect to voltage.

(Fig. 4.18A). If there is any other resistance in the circuit, it may be added to the resistance of the inductor to give the total resistance R.

If an alternating voltage is applied across L and R in series (Fig. 4.18A) the vector diagram may be drawn as in (B). The current vector I is first drawn to any convenient scale ; the vector of voltage drop across R is then drawn as RI in phase with I ; the vector of voltage drop across L is then drawn as ωLI so that I will lag behind it by 90°—hence ωLI is drawn as shown ; the parallelogram is then completed to give the resultant $E = \mathbf{Z}I$ with a phase angle such that $\tan \phi = \omega L/R$.

Using the j notation we may write :

$$E = RI + j\omega LI \tag{11}$$

where $+j$ indicates 90° vector rotation in a positive direction.

From (11) we can derive :

$$\mathbf{Z} = E/I = R + j\omega L \tag{12}$$

For example, if $R = 150$ ohms, $L = 20$ henrys and $f = 1000$ c/s, then

$$\omega = 2\pi \times 1000 = 6280$$

$$\omega L = 6280 \times 20 = 125\,600 \text{ ohms}$$

and

$$\mathbf{Z} = 150 + j\,125\,600 \tag{12a}$$

Eqn. (12a) indicates at a glance a resistance of 150 ohms in series with a positive reactance (i.e. an inductive reactance) of 125 600 ohms. Values of inductive reactances are given in Chapter 38 Sect. 9 Table 41.

The magnitude (modulus) of $\mathbf{Z}$ is :

$$|Z| = \sqrt{R^2 + (\omega L)^2} \tag{13}$$

and its phase angle is given by

$$\tan \phi = \omega L/R \tag{14}$$

as also shown by the vector diagram.

(vi) Power in inductive circuits

The power drawn from the line in Fig. 4.18A is the integral over one cycle of the instantaneous values of $e \times i$. As shown by Fig. 4.19, during parts of each cycle energy is being taken by the circuit, while during other parts of the cycle energy is being given back by the circuit. The latter may be regarded as negative power being taken by the circuit, and is so drawn. The expression for the power is

$$P = E_m \sin \omega t \times I_m \sin(\omega t - \phi)$$
$$= E_m I_m \sin \omega t \,.\, \sin(\omega t - \phi)$$
$$= E_m I_m \sin \omega t (\sin \omega t \cos \phi - \cos \omega t \sin \phi)$$
$$= E_m I_m (\sin^2 \omega t \cos \phi - \sin \omega t \cos \omega t \sin \phi).$$

Now the average value of ($\sin \omega t \cos \omega t$) over one cycle is zero*,

Therefore $P = E_m I_m (\sin^2 \omega t \cos \phi)$

$$= \tfrac{1}{2} E_m I_m (1 - \cos 2\omega t)(\cos \phi)$$
$$= E_{rms} I_{rms} \cos \phi \tag{15}$$

(since the average value of $\int_0^{2\pi} \cos 2\omega t = 0$*)

where P is expressed in watts, E and I in volts and amperes.

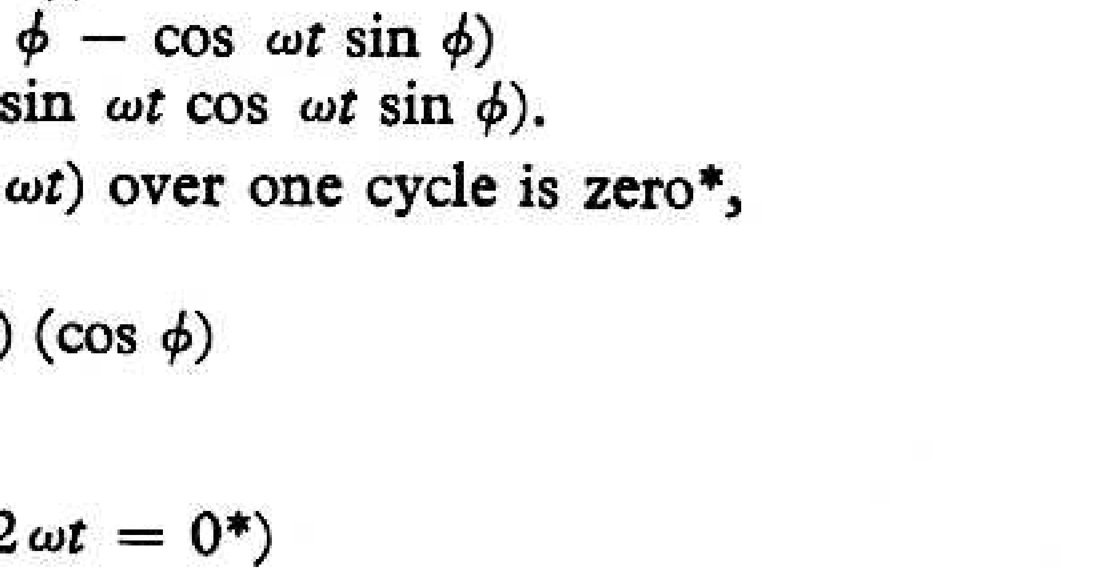


Fig. 4.19. Power in an inductive circuit with applied sine-wave voltage (e).

*See Chapter 6 Sect. 7(iii) (Definite Integrals).

From eqn. (15) the power is equal to the product of the effective voltage and current multiplied by $\cos \phi$, which is called the **Power Factor**, its value being given by $\cos \phi = R/\sqrt{R^2 + (\omega L)^2}$.

When the load is purely resistive, $L = 0$ and the power factor $= 1$; the power is therefore $P = E_{rms} I_{rms}$.

From (15) we may derive for the general case :

$$P = EI \cos \phi = EI \, . \, R/\sqrt{R^2 + (\omega L)^2}$$

Therefore $P = I \, . \, I\sqrt{R^2 + (\omega L)^2} \times R/\sqrt{R^2 + (\omega L)^2} = I^2R$ (16a)

$$\text{or } P = \frac{E^2}{R[1 + (\omega L)^2/R^2]} \quad (16b)$$

Therefore $R = P/I^2$ (17)

where P = power in watts

ωL = inductive reactance in ohms

R = resistance in ohms

and I = current in amperes.

In other words, the total power taken from the line in the circuit of Fig. 4.18A is the power dissipated by the effective total resistance R. There is no loss of power in an ideal inductance with zero resistance, although it draws current from an a.c. line, because the power factor is zero ($\phi = 90°$, therefore $\cos \phi = 0$). The product of $I \times E$ in this case is called the **wattless power** or **reactive power,** or more correctly the **reactive volt-amperes.**

The power factor at any given frequency gives the ratio of the resistance of a coil to its impedance and may be used as a figure of merit for the coil. A good coil should have a very small power factor.

The power factor is almost identical with the inverse of the coil magnification factor Q (Chapter 9), and the error is less than 1% for values of Q greater than 7 :

$$Q = \omega L/R = \tan \phi$$

$$\text{Power factor} = \cos \phi = R/Z$$

$$\tan \phi \approx 1/\cos \phi \text{ (error } < 1\% \text{ for } \phi > 82°)$$

Therefore Power Factor $\approx 1/Q$ for $Q > 7$.

SECTION 6 : IMPEDANCE AND ADMITTANCE

(i) Impedance a complex quantity (ii) Series circuits with L, C and R (iii) Parallel combinations of L, C and R (iv) Series-parallel combinations of L, C and R (v) Conductance, susceptance and admittance (vi) Conversion from series to parallel impedance.

(i) Impedance, a complex quantity

Impedance has already been introduced in Sections 4 and 5, in connection with series circuits of C and R or L and R. Impedance is a complex quantity, having both a resistive and a reactive component. We are therefore concerned, not only with its magnitude, but also with its phase angle.

(ii) Series circuits with L, C and R

When a resistance, an inductance and a capacitance are connected in series across an a.c. line (Fig. 4.20A), the same current will flow through each.

In using the j notation, remember that j simply means 90° positive vector rotation (the voltage drop for an inductance) and $-j$ means 90° negative vector rotation (the voltage drop for a capacitance).

In terms of the effective values

where E = applied voltage

and I = current through circuit, we have :

Voltage drop through $R = RI$
" " " $L = j\omega LI$
" " " $C = (-j/\omega C)I$
The total voltage drop is equal to the applied voltage,
therefore $E = [R + j(\omega L - 1/\omega C)]\, I$
Hence $\mathbf{Z} = R + j(\omega L - 1/\omega C)$ (1)

Here R is the resistive component of $\mathbf{Z}$, while $(\omega L - 1/\omega C)$ is the reactive component.

It may therefore be written as

$$\mathbf{Z} = R + jX \text{ where } X = (\omega L - 1/\omega C).$$

For example, let $R = 500$ ohms, $L = 20$ henrys, and $C = 1$ μF, all connected in series across an a.c. line with a frequency of 50 c/s.
Then

$$\omega = 2\pi \times 50 = 314$$
$$1/\omega C = 1/(314 \times 10^{-6}) = 3180 \text{ ohms}$$
$$X = \omega L - 1/\omega C = 6280 - 3180 = 3100 \text{ ohms}$$

(the positive sign indicates that this is inductively reactive) and

$$\mathbf{Z} = R + jX = 500 + j\,3100.$$

If L had been 5 henrys, then X would have been

$$1570 - 3180 = -\ 1610 \text{ ohms}$$

which is capacitively reactive.

The phase angle is given by $\tan\phi = X/R$. The magnitude of the impedance is given by $|Z| = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$ (1a)

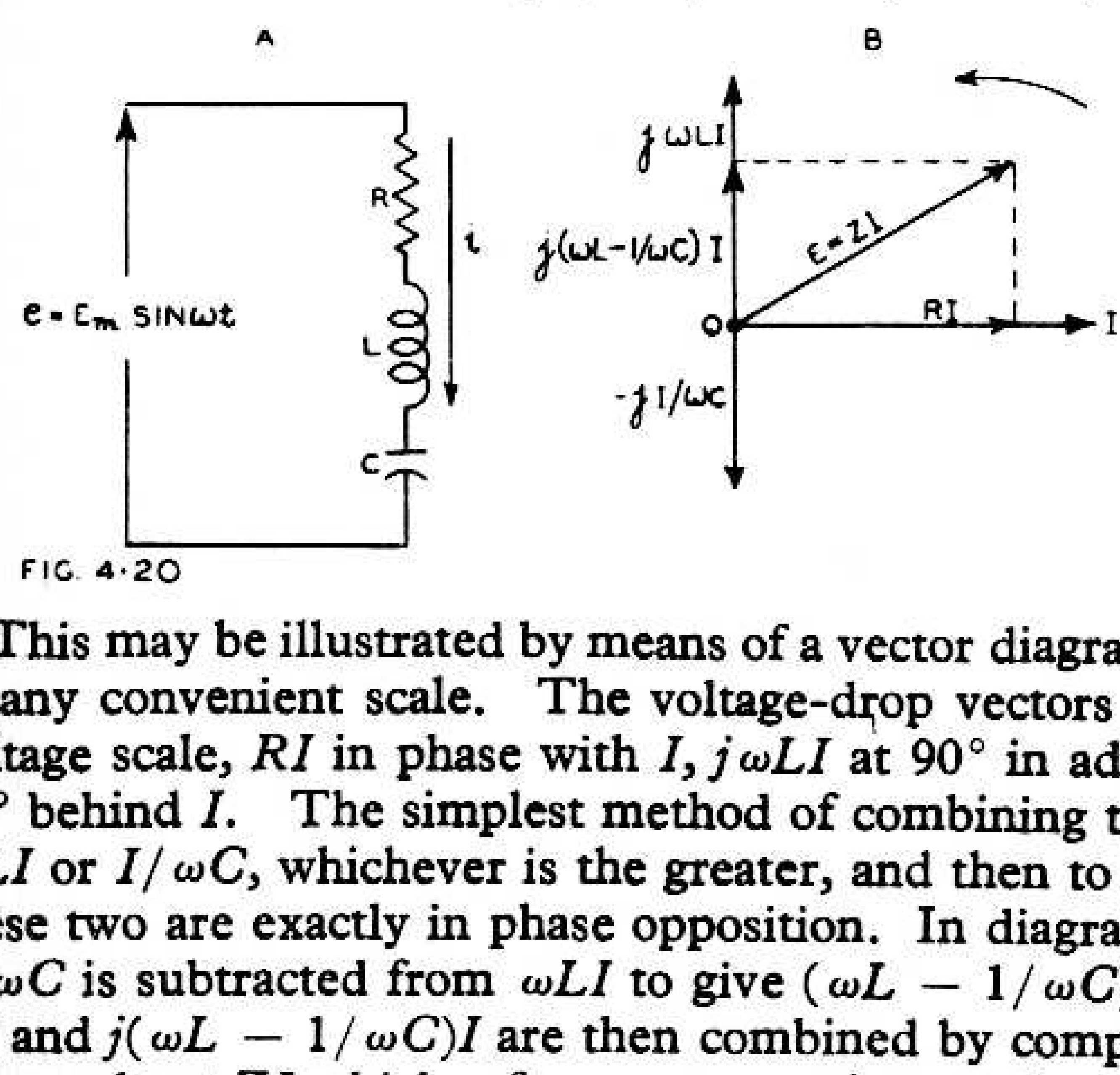


Fig. 4.20. (A) Circuit of R, L and C in series across a.c. line (B) Vector diagram of voltage relationships.

This may be illustrated by means of a vector diagram (Fig. 4.20 B) where I is drawn to any convenient scale. The voltage-drop vectors are then drawn, to the selected voltage scale, RI in phase with I, $j\omega LI$ at 90° in advance of I and $-jI/\omega C$ lagging 90° behind I. The simplest method of combining the three vectors is to take either ωLI or $I/\omega C$, whichever is the greater, and then to subtract the other from it, since these two are exactly in phase opposition. In diagram B, ωLI is the greater, so that $I/\omega C$ is subtracted from ωLI to give $(\omega L - 1/\omega C)I$. The two remaining vectors, RI and $j(\omega L - 1/\omega C)I$ are then combined by completing the parallelogram, to give the resultant $\mathbf{Z}I$ which, of course, must be equal to the applied voltage E.

A special case arises when $\omega L = 1/\omega C$ in eqn. (1); the reactive component becomes zero and $\mathbf{Z} = R$. This is the phenomenon of **series resonance** which is considered in greater detail in Chapter 9.

Special cases of equation (1)

R and L only:	Put $1/\omega C = 0$,	$\mathbf{Z} = R + j\omega L$
R and C only:	Put $\omega L = 0$,	$\mathbf{Z} = R - j/\omega C = R + 1/j\omega C$
L and C only:	Put $R = 0$,	$\mathbf{Z} = j(\omega L - 1/\omega C)$
R only:		$\mathbf{Z} = R + j0$
L only:		$\mathbf{Z} = 0 + j\omega L$
C only:		$\mathbf{Z} = 0 - j/\omega C = 1/j\omega C$

Note that $Q = |\tan\phi| = |X|/R$.

A table of inductive reactances is given in Chapter 38 Sect. 9(i) Table 41.

A table of capacitive reactances is given in Chapter 38 Sect. 9(ii) Table 42.

A table and two charts to find X, R or Z, when a reactance and resistance are connected in series, any two values being known, is given in Chap. 38 Sect 9(iv) Table 44.

Series Combinations of L, C and R

Series combination	Impedance $\mathbf{Z} = R + jX$	Magnitude of impedance $\lvert Z\rvert = \sqrt{R^2 + X^2}$	Phase angle $\phi = \tan^{-1}(X/R)$	Admittance* $\mathbf{Y} = 1/\mathbf{Z}$
R	ohms R	ohms R	radians 0	mhos $1/R$
L	$+j\omega L$	ωL	$+\pi/2$	$-j(1/\omega L)$
C	$-j(1/\omega C)$	$1/\omega C$	$-\pi/2$	$j\omega C$
$R_1 + R_2$	$R_1 + R_2$	$R_1 + R_2$	0	$1/(R_1 + R_2)$
$L_1(M)L_2$	$+j\omega(L_1 + L_2 \pm 2M)$	$\omega(L_1 + L_2 \pm 2M)$	$+\pi/2$	$-j/\omega(L_1 + L_2 \pm 2M)$
$C_1 + C_2$	$-j\frac{1}{\omega}\left(\frac{C_1 + C_2}{C_1C_2}\right)$	$\frac{1}{\omega}\left(\frac{C_1 + C_2}{C_1C_2}\right)$	$-\frac{\pi}{2}$	$j\omega\left(\frac{C_1C_2}{C_1 + C_2}\right)$
$R + L$	$R + j\omega L$	$\sqrt{R^2 + \omega^2L^2}$	$\tan^{-1}\frac{\omega L}{R}$	$\frac{R - j\omega L}{R^2 + \omega^2L^2}$
$R + C$	$R - j\frac{1}{\omega C}$	$\sqrt{\frac{\omega^2C^2R^2 + 1}{\omega^2C^2}}$	$-\tan^{-1}\frac{1}{\omega RC}$	$\frac{\omega^2C^2R + j\omega C}{\omega^2C^2R^2 + 1}$
$L + C$	$+j\left(\omega L - \frac{1}{\omega C}\right)$	$\left(\omega L - \frac{1}{\omega C}\right)$	$\pm\frac{\pi}{2}$	$-\frac{j\omega C}{\omega^2LC - 1}$
$R + L + C$	$R + j\left(\omega L - \frac{1}{\omega C}\right)$	$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$	$\tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)$	$\frac{R - j(\omega L - 1/\omega C)}{R^2 + (\omega L - 1/\omega C)^2}$

*See Sect. 6(v) below.

General case with a number of arms connected in series, each arm being of the form $R + X$

Arm (1) : $\mathbf{Z}_1 = R_1 + jX_1$ (i.e. R_1 in series with X_1)

Arm (2) : $\mathbf{Z}_2 = R_2 + jX_2$

Arm (3) : $\mathbf{Z}_3 = R_3 + jX_3$ etc.

then the total impedance is given by

$$\mathbf{Z} = (R_1 + R_2 + R_3 + \ldots) + j\,(X_1 + X_2 + X_3 + \ldots)$$

(iii) Parallel combinations of L, C and R

When a number of resistance and reactive elements are connected in parallel across an a.c. line, the same voltage is applied across each. For example in Fig. 4.20C there are three parallel paths across the a.c. line, and the current through each may be determined separately.

Let E_o = r.m.s. value of line voltage.

Then r.m.s. current through $L = I_1 = E_o/\omega L$

" " " " $C = I_2 = E_o\omega C$

" " " " $R = I_3 = E_o/R$.

The phase relationships between these currents are shown in Fig. 4.20D. There is 180° phase angle between the current through L and that through C, so that the resultant of these two currents is $(I_2 - I_1)$. The vector resultant of $(I_2 - I_1)$ and I_3 is given by I_o in Fig. 4.20D. Thus the current through L and C may be considerably greater than the total line current I_o.

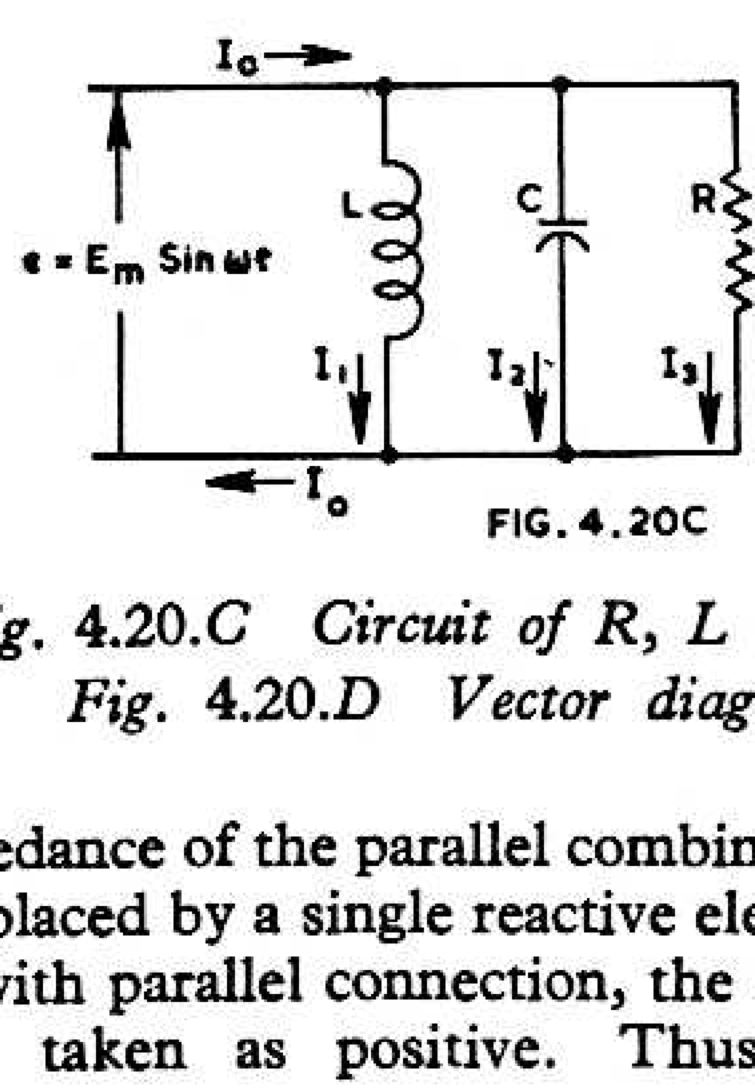


FIG. 4.20C

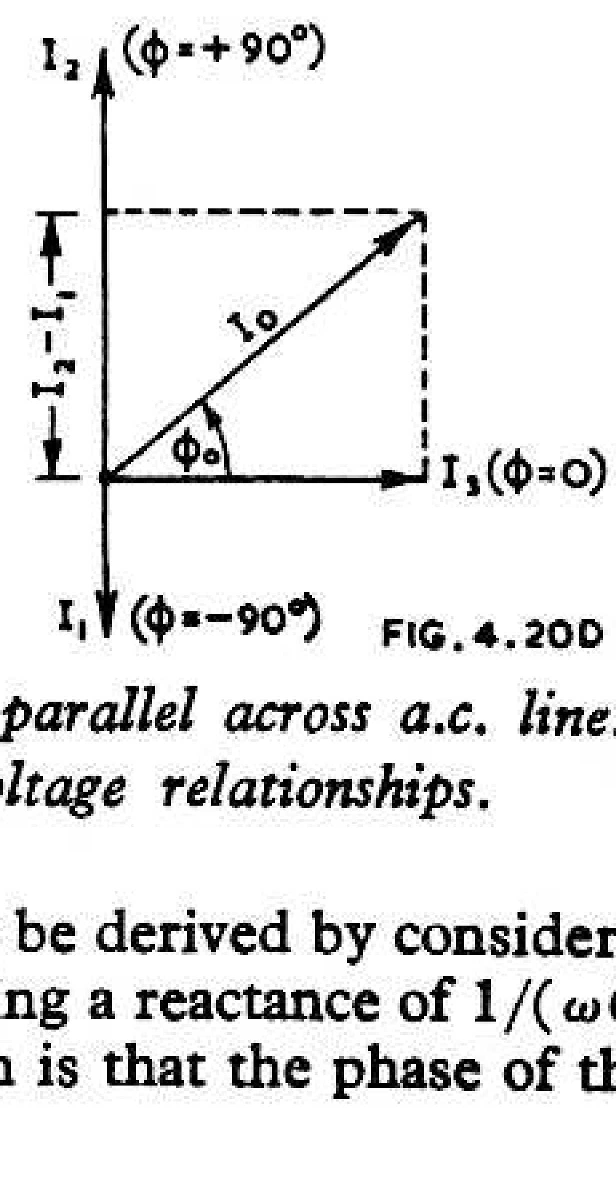


FIG. 4.20D

Fig. 4.20.C Circuit of R, L and C in parallel across a.c. line.
Fig. 4.20.D Vector diagram of voltage relationships.

The impedance of the parallel combination may be derived by considering L and C as being replaced by a single reactive element having a reactance of $1/(\omega C - 1/\omega L,$. Note that with parallel connection, the convention is that the phase of the capacitive element is taken as positive. Thus

$$I_o = I_3 + j\,(I_2 - I_1)$$
$$= \frac{E_o}{R} + j\,E_o\left(\omega C - \frac{1}{\omega L}\right)$$

Therefore $$\mathbf{Z} = \frac{E_o}{I_o} = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)} \times \frac{\frac{1}{R} - j\left(\omega C - \frac{1}{\omega L}\right)}{\frac{1}{R} - j\left(\omega C - \frac{1}{\omega L}\right)}$$

$$= \frac{\frac{1}{R} - j\left(\omega C - \frac{1}{\omega L}\right)}{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2} \qquad 2)$$

Parallel combinations of L, C and R

Parallel combination	Impedance $\mathbf{Z} = R + jX$	Magnitude of impedance $\lvert Z\rvert = \sqrt{R^2 + X^2}$	Phase angle $\phi = \tan^{-1}(X/R)$	Admittance* $\mathbf{Y} = 1/\mathbf{Z}$
	ohms	ohms	radians	mhos
R_1, R_2	$\frac{R_1R_2}{R_1+R_2}$	$\frac{R_1R_2}{R_1+R_2}$	0	$\frac{R_1+R_2}{R_1R_2}$
C_1, C_2	$-j\frac{1}{\omega(C_1+C_2)}$	$\frac{1}{\omega(C_1+C_2)}$	$-\frac{\pi}{2}$	$+j\omega(C_1+C_2)$
L, R	$\frac{\omega^2L^2R + j\omega LR^2}{\omega^2L^2+R^2}$	$\frac{\omega LR}{\sqrt{\omega^2L^2+R^2}}$	$\tan^{-1}\frac{R}{\omega L}$	$\frac{1}{R} - \frac{j}{\omega L}$
R, C	$\frac{R - j\omega R^2C}{1+\omega^2R^2C^2}$	$\frac{R}{\sqrt{1+\omega^2R^2C^2}}$	$\tan^{-1}(-\omega RC)$	$\frac{1}{R} + j\omega C$
L, C	$+j\frac{\omega L}{1-\omega^2LC}$	$\frac{\omega L}{1-\omega^2LC}$	$\pm\frac{\pi}{2}$	$j\left(\omega C - \frac{1}{\omega L}\right)$
$L_1(M)L_2$	$+j\omega\frac{L_1L_2 - M^2}{L_1+L_2\mp 2M}$	$\omega\frac{L_1L_2-M^2}{L_1+L_2\mp 2M}$	$\pm\frac{\pi}{2}$	$-j\frac{1}{\omega}\left(\frac{L_1+L_2\mp 2M}{L_1L_2-M^2}\right)$
L, C, R	$\frac{\frac{1}{R} - j\left(\omega C - \frac{1}{\omega L}\right)}{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$	$\frac{R}{\sqrt{1 + R^2\left(\omega C - \frac{1}{\omega L}\right)^2}}$	$\tan^{-1} - R\left(\omega C - \frac{1}{\omega L}\right)$	$\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$

*See Sect. 6(v) below.

Note that $Q = \lvert\tan\varphi\rvert$.

The magnitude of the impedance of a reactance X in parallel with a resistance R is given by

$$|Z| = \frac{RX}{\sqrt{X^2 + R^2}} \tag{3a}$$

$$= \frac{\omega LR}{\sqrt{\omega^2 L^2 + R^2}} \quad \text{when } X = \omega L \tag{3b}$$

$$= \frac{R}{\sqrt{1 + \omega^2 R^2 C^2}} \quad \text{when } X = 1/\omega C \tag{3c}$$

$$= \frac{R}{\sqrt{1 + R^2(\omega C - 1/\omega L)^2}} \quad \left\{\begin{array}{l}\text{when } L \text{ and } C \\ \text{are in parallel}\end{array}\right\} \tag{3d}$$

A table and a chart to find X, R or Z when a reactance and resistance are connected in parallel, is given in Chapter 38 Sect. 9(iii) Table 43.

A simple graphical method for determining the resultant impedance of a reactance and a resistance in parallel is given in Fig. 4.20E where OA and OB represent R and X respectively and OD is drawn at right angles to the hypotenuse. The length OD represents the scalar value of the resultant impedance, while the angle DOA is equal to the phase angle ϕ between the applied voltage and the resultant current.

For other graphical methods for impedances in parallel see Reed, C.R.G. (letter) W.E. 28.328 (Jan. 1951) 32; Benson, F. A. (letter) W.E. 28.331 (April 1951) 128.

FIG. 4.20 E

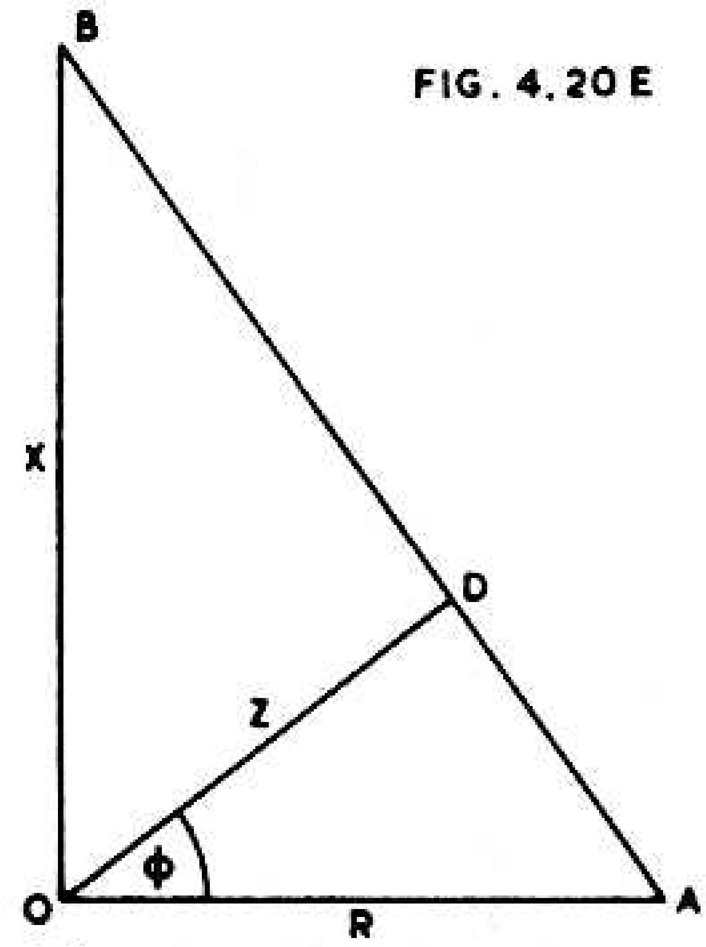


Fig. 4.20E. Impedances in parallel.

(iv) Series-parallel combinations of L, C and R

Some simple combinations of L, C and R are described in (A) to (E) below. The general procedure for other combinations is described in (F) below.

(A) Parallel tuned circuit with losses in both L and C (Fig. 4.21A)

Impedance of branch (1) : $\mathbf{Z}_1 = R_1 + j\omega L$ (4a)

" " " (2) : $\mathbf{Z}_2 = R_2 - j/\omega C$ (4b)

Let $\mathbf{Z}$ = total impedance of $\mathbf{Z}_1$ and $\mathbf{Z}_2$ in parallel.

$$\text{Then } \mathbf{Z} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(R_1 + j\omega L)(R_2 - j/\omega C)}{(R_1 + R_2) + j(\omega L - 1/\omega C)} \tag{4c}$$

Multiplying both numerator and denominator by $(R_1 + R_2) - j(\omega L - 1/\omega C)$, we have

$$\mathbf{Z} = \frac{(R_1R_2 + j\omega LR_2 - jR_1/\omega C + L/C)[R_1 + R_2 - j(\omega L - 1/\omega C)]}{(R_1 + R_2)^2 + (\omega L - 1/\omega C)^2} \tag{5}$$

$$\text{Let } \mathbf{Z} = \frac{A + jB}{(R_1 + R_2)^2 + (\omega L - 1/\omega C)^2} \tag{6}$$

and let us now determine the values of A and B :

$$\begin{aligned}\text{Numerator} &= R_1^2R_2 + jR_1R_2\omega L - jR_1^2/\omega C + R_1L/C + R_1R_2^2 + jR_2^2\omega L \\ &\quad - jR_1R_2/\omega C + R_2L/C - jR_1R_2\omega L + R_2\omega^2L^2 - R_1L/C \\ &\quad - j\omega L^2/C + jR_1R_2/\omega C - R_2L/C + R_1/\omega^2C^2 + jL/\omega C^2 \\ &= R_1^2R_2 + R_1R_2^2 + R_2\omega^2L^2 + R_1/\omega^2C^2 + j[R_2^2\omega L - R_1^2/\omega C \\ &\quad - \omega L^2/C + L/\omega C^2].\end{aligned}$$

$$\text{Therefore } A = R_1R_2(R_1 + R_2) + R_2\omega^2L^2 + R_1/\omega^2C^2 \tag{7}$$

$$\text{and } \quad B = R_2^2\omega L - R_1^2/\omega C - (L/C)(\omega L - 1/\omega C) \tag{8}$$

which values should be used in eqn. (6).

Note that A divided by the denominator (eqn. 6) is the effective resistance of the total circuit, while B divided by the denominator is the effective reactance.

The magnitude of the impedance may be obtained most readily from eqn. (4c) by replacing $\mathbf{Z}_1$ and $\mathbf{Z}_2$ in the numerator only by $Z_1\angle\phi_1$ and $Z_2\angle\phi_2$ respectively.

$$\mathbf{Z} = \frac{(Z_1\angle\phi_1)(Z_2\angle\phi_2)}{(R_1+R_2)+j(\omega L - 1/\omega C)} = \frac{Z_1 Z_2\angle(\phi_1+\phi_2)}{(R_1+R_2)+j(\omega L - 1/\omega C)} \quad (9)$$

where $Z_1 = \sqrt{R_1^2 + \omega^2 L^2}$

and $Z_2 = \sqrt{R_2^2 + 1/\omega^2 C^2}$

$$\text{whence } |Z| = \left[\frac{(R_1^2 + \omega^2 L^2)(R_2^2 + 1/\omega^2 C^2)}{(R_1+R_2)^2 + (\omega L - 1/\omega C)^2}\right]^{\frac{1}{2}} \quad (10)$$

The phase angle is given by $\tan^{-1} B/A$

$$\text{i.e. } \phi = \tan^{-1}\left[\frac{R_2^2\omega L - R_1^2/\omega C - (L/C)(\omega L - 1/\omega C)}{R_1R_2(R_1+R_2) + R_2\omega^2L^2 + R_1/\omega^2C^2}\right] \quad (11)$$

When $LC\omega^2 = 1$, eqn. (10) may be reduced to

$$|Z| = \frac{L}{C}\cdot\frac{[1 + (C/L)(R_1^2 + R_2^2) + (C^2/L^2)R_1^2R_2^2]^{\frac{1}{2}}}{R_1+R_2} \quad (12)$$

or $|Z| \approx (L/C)(R_1 + R_2)$ (13)

with an error $< 1\%$ provided $Q > 10$

where $Q = \omega L/(R_1 + R_2)$.

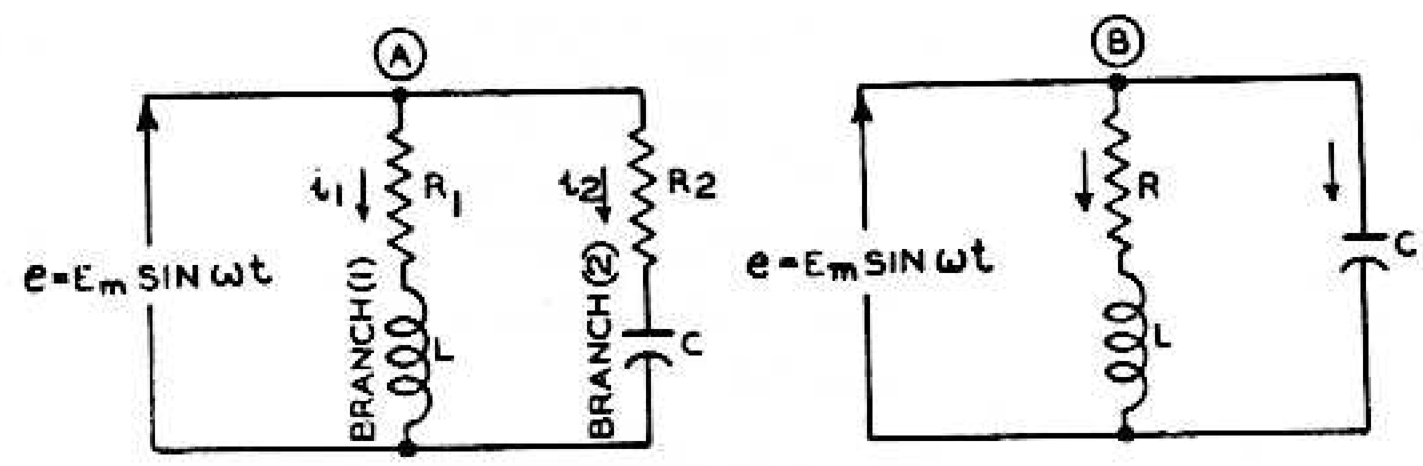


Fig. 4.21. (*A*) *Network incorporating* 4 *elements for impedance calculations* (*B*) *Simplified network with* 3 *elements.*

(B) Parallel tuned circuit with loss in L only (Fig. 4.21B)

This is a special case of (A) in which $R_2 = 0$.

It may be shown that eqn. (5) becomes

$$\mathbf{Z} = \frac{R - j\omega\,[CR^2 + L\,(\omega^2LC - 1)]}{\omega^2C^2R^2 + (\omega^2LC - 1)^2} \quad (14)$$

The magnitude of the impedance derived from eqn. (10) becomes

$$|Z| = \left[\frac{R^2 + \omega^2L^2}{\omega^2C^2R^2 + (\omega^2LC - 1)^2}\right]^{\frac{1}{2}} \quad (15)$$

The phase angle derived from eqn. (11) becomes

$$\phi = \tan^{-1}\frac{\omega[L(1 - \omega^2LC) - CR^2]}{R} \quad (16)$$

(a) The effective reactance of eqn. (14) is zero (i.e. the power factor is unity) when

$CR^2 = L(1 - \omega^2LC)$

i.e. when $\omega^2L^2C = L - CR^2$

$$\text{i.e. when } \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad (17)$$

which can be written in the form

$$\text{when } \omega = \frac{1}{\sqrt{LC}}\sqrt{1 - \frac{CR^2}{L}} = \omega_o\sqrt{1 - \frac{CR^2}{L}} \quad (18)$$

where $\omega_o = 1/\sqrt{LC}$ is the value of ω when the resistance R is zero.

This condition, namely that the effective reactance is zero, which may also be expressed as the condition of unity power factor, is one of several possible definitions of **parallel resonance**. This is the definition used in Chapter 9. It should be noted that the expression giving the value of ω is not independent of the resistance in the circuit as is the case with series resonance.

If $LC\omega^2 = 1$, which is the condition for resonance if the resistance is zero, the impedance is given very closely, when R is small, by

$$Z \approx L/CR \tag{19}$$

The remaining possible definitions of parallel resonance are conditions to give maximum impedance. The condition of maximum impedance is sometimes called *anti-resonance.* Maximum impedance occurs under slightly different conditions for the variables C, L and ω*.

(b) Condition of maximum total impedance when C is the variable

The maximum value of **Z** which can be obtained is

$$|Z|_{max} = \frac{R_1^2 + \omega^2 L^2}{R_1} = R_1(1 + Q^2) \tag{20}$$

where $Q = \omega L/R_1$

$$\text{which occurs when } C = \frac{L}{R_1^2 + \omega^2 L^2} \tag{21}$$

This is also the condition giving unity power factor.

(c) Condition of maximum total impedance when L is the variable

The maximum value of **Z** which can be obtained is

$$|Z|_{max} = \frac{2R_1}{\sqrt{1 + 4\omega^2 C^2 R_1^2} - 1} \tag{22}$$

$$\text{which occurs when } L = \frac{1 + \sqrt{1 + 4\omega^2 C^2 R_1^2}}{2\omega^2 C} \tag{23}$$

$$\text{or when } C = \frac{L}{\omega^2 L^2 - R_1^2} \tag{24}$$

If L is the variable and Q is maintained constant, the maximum value of **Z** which can be obtained is

$$|Z|_{max} = Q/\omega C \tag{25}$$

$$\text{which occurs when } L = \frac{Q^2}{\omega^2 C(1 + Q^2)} \tag{26}$$

(d) Condition of maximum total impedance when the applied frequency is the variable

The maximum value of **Z** which can be obtained is

$$|Z|_{max} = \frac{L}{C\sqrt{R_1^2 - \frac{L}{C}\left(\sqrt{2R_1^2\frac{C}{L} + 1} - 1\right)^2}} \tag{27}$$

which occurs when

$$\omega = 2\pi f = \sqrt{\frac{\sqrt{(2R_1^2 C/L) + 1}}{LC} - \frac{R_1^2}{L^2}} \tag{28}$$

In practice, for all normal values of $Q(= \omega L/R)$ as used in tuned circuits, these four cases are almost identical and the frequency of parallel resonance is approximately the same as that for series resonance.

We may summarise the resonance frequencies for the various conditions given above :

Series resonance $LC\omega^2 = 1$

Parallel resonance (a) $LC\omega^2 = 1 - CR^2/L$

(b) $LC\omega^2 = 1 - CR^2/L$

(c) $LC\omega^2 = 1 + CR^2/L$

(d) $LC\omega^2 = \sqrt{1 + 2CR^2/L} - CR^2/L$

*See R. S. Glasgow (Book) "Principles of Radio Engineering" (McGraw-Hill Book Co., New York and London, 1936) pp. 35-44.

Note that $CR^2/L = 1/Q^2$ under conditions (a) and (b)
$CR^2/L \approx 1/Q^2$ under conditions (c) and (d).

For further information on tuned circuits see Chapter 9.

(C) Circuit of Fig. 4.21C

This is a special case of Fig. 4.21A in which
$(1/\omega C) = 0$. From eqn. (6)

$$\mathbf{Z} = \frac{R_1R_2(R_1 + R_2) + R_2\omega^2L^2 + j\omega LR_2^2}{(R_1 + R_2)^2 + \omega^2L^2} \tag{29}$$

From eqn. (10),

$$|Z| = R_2\left[\frac{R_1^2 + \omega^2L^2}{(R_1 + R_2)^2 + \omega^2L^2}\right]^{\frac{1}{2}} \tag{30}$$

From eqn. (11)

$$\phi = \tan^{-1}\frac{\omega LR_2}{R_1(R_1 + R_2) + \omega^2L^2} \tag{31}$$

(D) Circuit of Fig. 4.21D

Impedance of arm (1) $= \mathbf{Z}_1 = R_1 + j(\omega L_1 - 1/\omega C_1)$
" " " (2) $= \mathbf{Z}_2 = R_2 + j(\omega L_2 - 1/\omega C_2)$

Let $X_1 = \omega L_1 - 1/\omega C_1$
and $X_2 = \omega L_2 - 1/\omega C_2$
then $\mathbf{Z}_1 = R_1 + jX_1$; $\mathbf{Z}_2 = R_2 + jX_2$.

$$\mathbf{Z} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(R_1 + jX_1)(R_2 + jX_2)}{(R_1 + R_2) + j(X_1 + X_2)}$$
$$= \frac{(R_1R_2 - X_1X_2) + j(R_1X_2 + R_2X_1)}{(R_1 + R_2) + j(X_1 + X_2)}$$

which may be put into the form

$$\mathbf{Z} = \frac{R_1(R_2^2 + X_2^2) + R_2(R_1^2 + X_1^2) + j[X_1(R_2^2 + X_2^2) + X_2(R_1^2 + X_1^2)]}{(R_1 + R_2)^2 + (X_1 + X_2)^2} \tag{32}$$

The magnitude of the impedance may be derived by the method used for deriving eqn. (10)—

$$|Z| = \left[\frac{(R_1^2 + X_1^2)(R_2^2 + X_2^2)}{(R_1 + R_2)^2 + (X_1 + X_2)^2}\right]^{\frac{1}{2}} \tag{33}$$

The phase angle may be derived from eqn. (32)—

$$\phi = \tan^{-1}\frac{X_1(R_2^2 + X_2^2) + X_2(R_1^2 + X_1^2)}{R_1(R_2^2 + X_2^2) + R_2(R_1^2 + X_1^2)} \tag{34}$$

(E) Circuit of Fig. 4.21E

1. Determine the impedance ($\mathbf{Z}_1$) of L_1 C_1 and R_1 in series, using eqn. (1)—

$$\mathbf{Z}_1 = R_1 + j(\omega L_1 - 1/\omega C_1) \tag{35}$$

2. Determine the impedance ($\mathbf{Z}_2$) of L_2C_2 and R_2 in parallel using eqn. (2) but separating the resistive and reactive components—

$$\mathbf{Z}_2 = \frac{\dfrac{1}{R_2}}{\left(\dfrac{1}{R_2}\right)^2 + \left(\omega C_2 - \dfrac{1}{\omega L_2}\right)^2} - j\,\frac{\left(\omega C_2 - \dfrac{1}{\omega L_2}\right)}{\left(\dfrac{1}{R_2}\right)^2 + \left(\omega C_2 - \dfrac{1}{\omega L_2}\right)^2} \tag{36}$$

3. Determine the combined impedance ($\mathbf{Z}$) by adding the resistive and reactive components of (14) and (15)—

$$\mathbf{Z} = A + jB \tag{37}$$

$$\text{where } A = R_1 + \frac{\dfrac{1}{R_2}}{\left(\dfrac{1}{R_2}\right)^2 + \left(\omega C_2 - \dfrac{1}{\omega L_2}\right)^2}$$

and $$B = \omega L_1 - \frac{1}{\omega C_1} - \frac{\omega C_2 - \dfrac{1}{\omega L_2}}{\left(\dfrac{1}{R_2}\right)^2 + \left(\omega C_2 - \dfrac{1}{\omega L_2}\right)^2}.$$

The magnitude of the impedance is given by

$$|Z| = \sqrt{A^2 + B^2} \quad (38)$$

The phase angle ϕ is given by

$$\phi = \tan^{-1}(B/A) \quad (39)$$

where A and B have the same values as for eqn. (37).

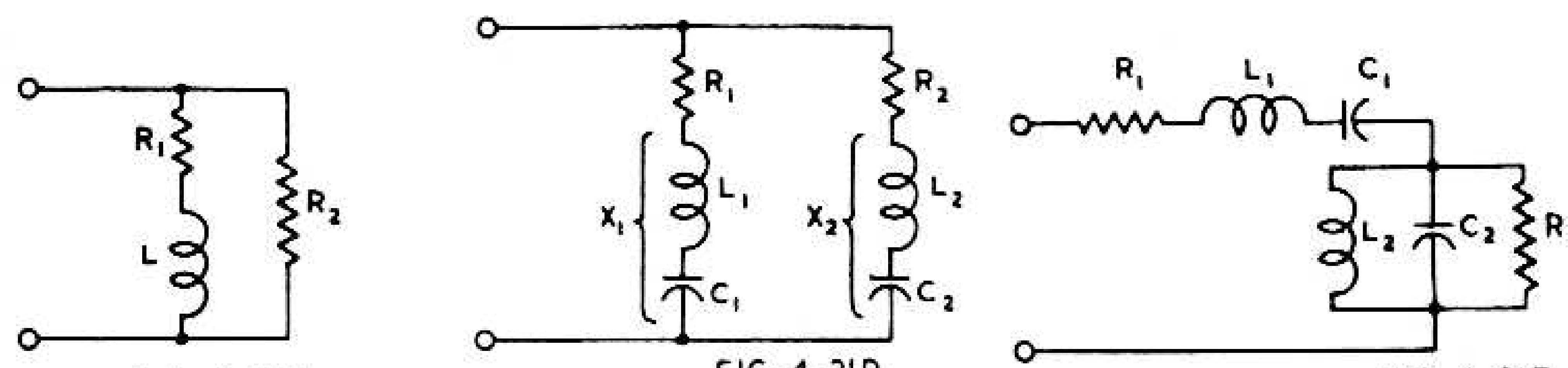


Fig. 4.21. *(C) (D) (E) Series-parallel networks for impedance calculations.*

(F) General procedure to find the impedance of a two-terminal network

1. If possible, divide the network into two or more parallel branches, each of which is connected to the two terminals but has no other connection with any other branch.
2. Find the impedance of each branch, using the methods described in A, B and C above.
3. Determine the impedance (**Z**) of the network from the relation

$$\frac{1}{\mathbf{Z}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \ldots \quad (40)$$

Note : If there are more than two parallel branches it is more convenient to work in terms of admittance—see Sect. 6(v) below.

Alternatively, if the circuit cannot be divided into parallel branches, treat it as a series circuit, firstly determining the resistive and reactive components of each section, and then adding all the resistive and all the reactive values separately, as in (E).

(v) Conductance, susceptance and admittance

In an arm* containing both reactance and resistance in series, the **conductance** (G_1) of the arm is given by

$$G_1 = \frac{R_1}{R_1^2 + X_1^2} \text{ mhos} \quad (41)$$

where $X_1 = (\omega L_1 - 1/\omega C_1)$.

When there are a number of such arms in parallel, the resultant conductance is the sum of their separate conductances, that is

$$G = G_1 + G_2 + G_3 + \ldots$$

The **susceptance** (B_1) of the arm under similar conditions, is given by

$$B_1 = \frac{X_1}{R_1^2 + X_1^2} \text{ mhos} \quad (42)$$

where $X_1 = (\omega L_1 - 1/\omega C_1)$.

[Inductive susceptance is regarded as positive. Capacitive susceptance is regarded as negative.]

When there are a number of such arms in parallel, the resultant susceptance is the sum of their separate susceptances,

$$B = B_1 + B_2 + B_3 + \ldots \quad (43)$$

When any arm includes only resistance, the conductance of the arm is $1/R_1$ and the susceptance zero,

i.e. $G_1 = 1/R_1$ $\qquad B_1 = 0.$

*An arm is a distinct set of elements in a network, electrically isolated from all other conductors except at two points.

When any arm has only inductance, the conductance of the arm is zero and the susceptance is $1/\omega L_1$,

i.e. $G_1 = 0 \qquad B_1 = 1/\omega L_1$

When any arm has only capacitance, the conductance of the arm is zero, and the susceptance is $-\omega C$,

i.e. $G_1 = 0 \qquad B_1 = -\omega C$

The following relationships hold between R, X, G and B

$$R = \frac{G}{G^2 + B^2}\,;\; X = \frac{B}{G^2 + B^2} \tag{44}$$

The **admittance ($\mathbf{Y}_1$) of any arm containing resistance and reactance in series** is given by

$$\mathbf{Y}_1 = G_1 - jB_1 \tag{45}$$

$$\text{i.e. } \mathbf{Y}_1 = G_1 - jB_1 = \frac{R_1 - jX_1}{R_1{}^2 + X_1{}^2} = \frac{1}{R_1 + jX_1} = \frac{1}{\mathbf{Z}_1}, \tag{46}$$

indicating that **the admittance is the reciprocal of the impedance.**

Thus the value of the admittance may always be derived from the impedance—

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{1}{R + jX}\cdot\frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2} \tag{46a}$$

$$\text{Similarly } \mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1}{G - jB}\cdot\frac{G + jB}{G + jB} = \frac{G + jB}{G^2 + B^2} \tag{46b}$$

The negative sign in front of jB in eqn. (45) deserves special attention—

Admittance of inductive arm (R and L in series):

$$\mathbf{Y} = G - jB = \frac{R - j\omega L}{R^2 + \omega^2 L^2} \tag{47}$$

Admittance of capacitive arm (R and C in series):

$$\mathbf{Y} = G + jB = \frac{R + j/\omega C}{R^2 + 1/\omega^2 C^2} = \frac{\omega^2 C^2 R + j\omega C}{\omega^2 C^2 R^2 + 1} \tag{48}$$

The admittance of any arm containing resistance, capacitance and inductance in series (Fig. 4.20A) is given by

$$\mathbf{Y} = G - jB \tag{49}$$

where $G = \dfrac{R}{R^2 + X^2}$

$X = (\omega L - 1/\omega C)$

and $B = \dfrac{X}{R^2 + X^2} = \dfrac{(\omega L - 1/\omega C)}{R^2 + (\omega L - 1/\omega C)^2}$

$$\text{therefore } \mathbf{Y} = \frac{R - j(\omega L - 1/\omega C)}{R^2 + (\omega L - 1/\omega C)^2} \tag{50}$$

Values of admittance for various series combinations are included in the table in Sect 6(iii).

Admittance of parallel-connected impedances

L and R in parallel: $G = G_1 + G_2 = 0 + 1/R = 1/R$

$B = B_1 + B_2 = 1/\omega L + 0 = 1/\omega L$

therefore $\mathbf{Y} = G - jB = (1/R) - j(1/\omega L)$

Similarly

C and R in parallel: $\mathbf{Y} = G - jB = (1/R) + j\omega C$

L and C in parallel: $\mathbf{Y} = G - jB = j(\omega C - 1/\omega L)$

L, C and R in parallel: $\mathbf{Y} = G - jB = (1/R) + j(\omega C - 1/\omega L)$

See also table of Parallel Combinations of L, C and R—Column 5, in Sect. 6(iii).

Admittance of series-parallel-connected impedances

When there are a number of arms in parallel, each arm including resistance and reactance in series,

Arm (1) : $\mathbf{Z}_1 = R_1 + jX_1$ or $\mathbf{Y}_1 = G_1 - jB_1$
Arm (2) : $\mathbf{Z}_2 = R_2 + jX_2$ or $\mathbf{Z}_2 = G_2 - jB_2$
Arm (3) : $\mathbf{Z}_3 = R_3 + jX_3$ or $\mathbf{Y}_3 = G_3 - jB_3$ etc.
where $X_1 = (\omega L_1 - 1/\omega C_1)$ etc.
then the total admittance is given by the vector sum

$$\mathbf{Y} = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \ldots$$
$$= (G_1 + G_2 + G_3 + \ldots) - j(B_1 + B_2 + B_3 + \ldots) \qquad (51)$$

The following examples may alternatively be handled by the use of eqn. (46a) provided that the value of $\mathbf{Z}$ is known.

Example (A) : Fig. 4.21A

Arm (1) is inductive : $\mathbf{Y}_1 = G_1 - jB_1$

$$G_1 = \frac{R_1}{R_1^2 + X_1^2}; \; B_1 = \frac{X_1}{R_1^2 + X_1^2} \qquad (X_1 = \omega L)$$

Arm (2) is capacitive : $\mathbf{Y}_2 = G_2 + jB_2$

$$G_2 = \frac{R_2}{R_2^2 + X_2^2}; \; B_2 = \frac{X_2}{R_2^2 + X_2^2}, \qquad (X_2 = -1/\omega C)$$

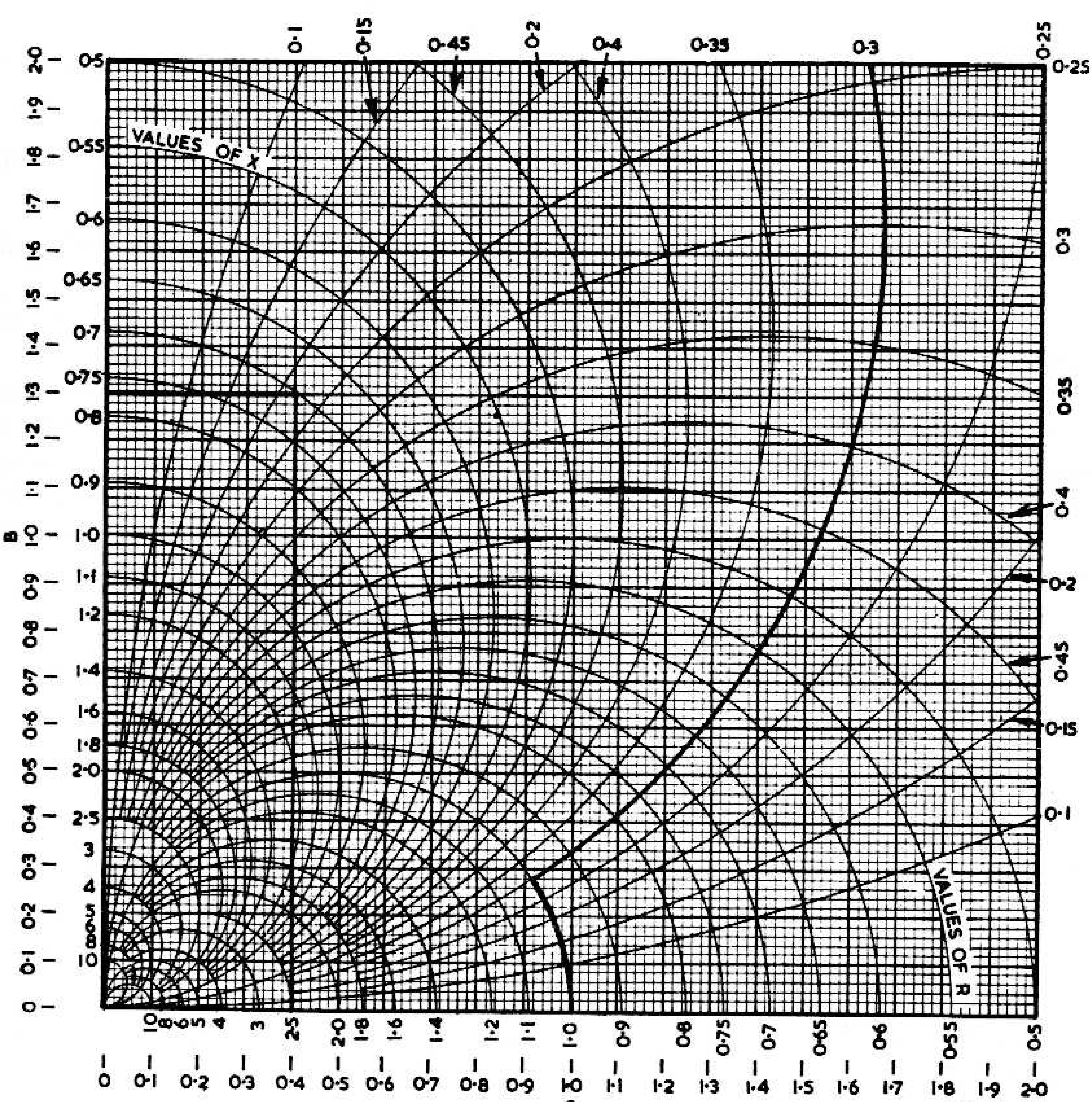


FIG. 4.21F

Fig. 4.21F. *Chart for conversion between resistance and reactance, and conductance and susceptance*

$$Z = (R \pm jX)\,10^n, \quad Y = (G \mp jB)\,10^{-n}.$$

On collecting the terms we get

$$\mathbf{Y} = (G_1 + G_2) - j(B_1 + B_2)$$
$$= \frac{R_1 + \omega^2C^2R_1R_2(R_1 + R_2) + \omega^4L^2C^2R_2}{(R_1^2 + \omega^2L^2)(1 + \omega^2C^2R_2^2)}$$
$$+ j\omega\left[\frac{CR_1^2 - L + \omega^2LC(L - CR_2^2)}{(R_1^2 + \omega^2L^2)(1 + \omega^2C^2R_2^2)}\right] \quad (52)$$

Example (B) : Fig. 4.21B.

This is a special case of (A) in which $R_2 = 0$. From eqn. (52)

$$\mathbf{Y} = \frac{R - j\omega[L(1 - \omega^2LC) - CR^2]}{R^2 + \omega^2L^2} \quad (53)$$

Example (C) : Fig. 4.21C.

This is a special case of Example (A) in which $X_2 = (1/\omega C) = 0$.

$$\mathbf{Y} = \frac{R_1(R_1 + R_2) + \omega^2L^2 - j\omega LR_2}{R_2(R_1^2 + \omega^2L^2)} \quad (54)$$

Example (D) : Fig. 4.21D.

$$\mathbf{Y}_1 = G_1 - jB_1 = \frac{R_1}{R_1^2 + X_1^2} - j\frac{X_1}{R_1^2 + X_1^2}$$
$$\mathbf{Y}_2 = G_2 - jB_2 = \frac{R_2}{R_2^2 + X_2^2} - j\frac{X_2}{R_2^2 + X_2^2}$$
$$\mathbf{Y} = \frac{R_1}{R_1^2 + X_1^2} + \frac{R_2}{R_2^2 + X_2^2} - j\left[\frac{X_1}{R_1^2 + X_1^2} + \frac{X_2}{R_2^2 + X_2^2}\right]$$
$$= \frac{R_1R_2(R_1+R_2)+R_1X_2^2+R_2X_1^2-j[R_1^2X_2+R_2^2X_1+X_1X_2(X_1+X_2)]}{(R_1^2+X_1^2)(R_2^2+X_2^2)} \quad (55)$$

Chart for conversion between resistance and reactance, conductance and susceptance.

Fig. 4.21F can be used in conversions between resistance and reactance, and conductance and susceptance.

Example 1. The impedance of a circuit is $1 + j\,0.3$; determine its admittance.

Method : $\mathbf{Z} = 1.0 + j\,0.3$. Enter the chart on the semi-circle $R = 1.0$ and follow it until it meets the arc $X = 0.3$. The corresponding values of G and B are 0.917 and 0.275. Thus $\mathbf{Y} = 0.917 - j\,0.275$.

Example 2. The admittance of a circuit is $0.000\,004 + j\,0.000\,013$; determine its impedance.

Method : $\mathbf{Y} = 0.000\,004 + j\,0.000\,013 = (0.4 + j\,1.3)\times 10^5$.

Enter the chart on the lines $G = 0.4$ and $B = 1.3$.

The intersection is at $R = 0.22$, $X = 0.7$. Thus

$$\mathbf{Z} = (0.22 - j\,0.7)10^5 = 22\,000 - j\,70\,000.$$

At series resonance, $\omega L = 1/\omega C$ and $X = 0$, so that $Y = G = 1/R$.

The admittance at any frequency is given graphically by the **Admittance Circle Diagram** (Fig. 4.22) in which the vector **OY** represents the admittance, where Y is any point on the circle. The diameter of the circle is equal to $1/R$ and the admittance at series resonance, when the frequency is f_0, is represented by OA.

As the frequency of the voltage applied to the series tuned circuit (Fig. 4.20A) is increased from zero to in-

FIG. 4.22

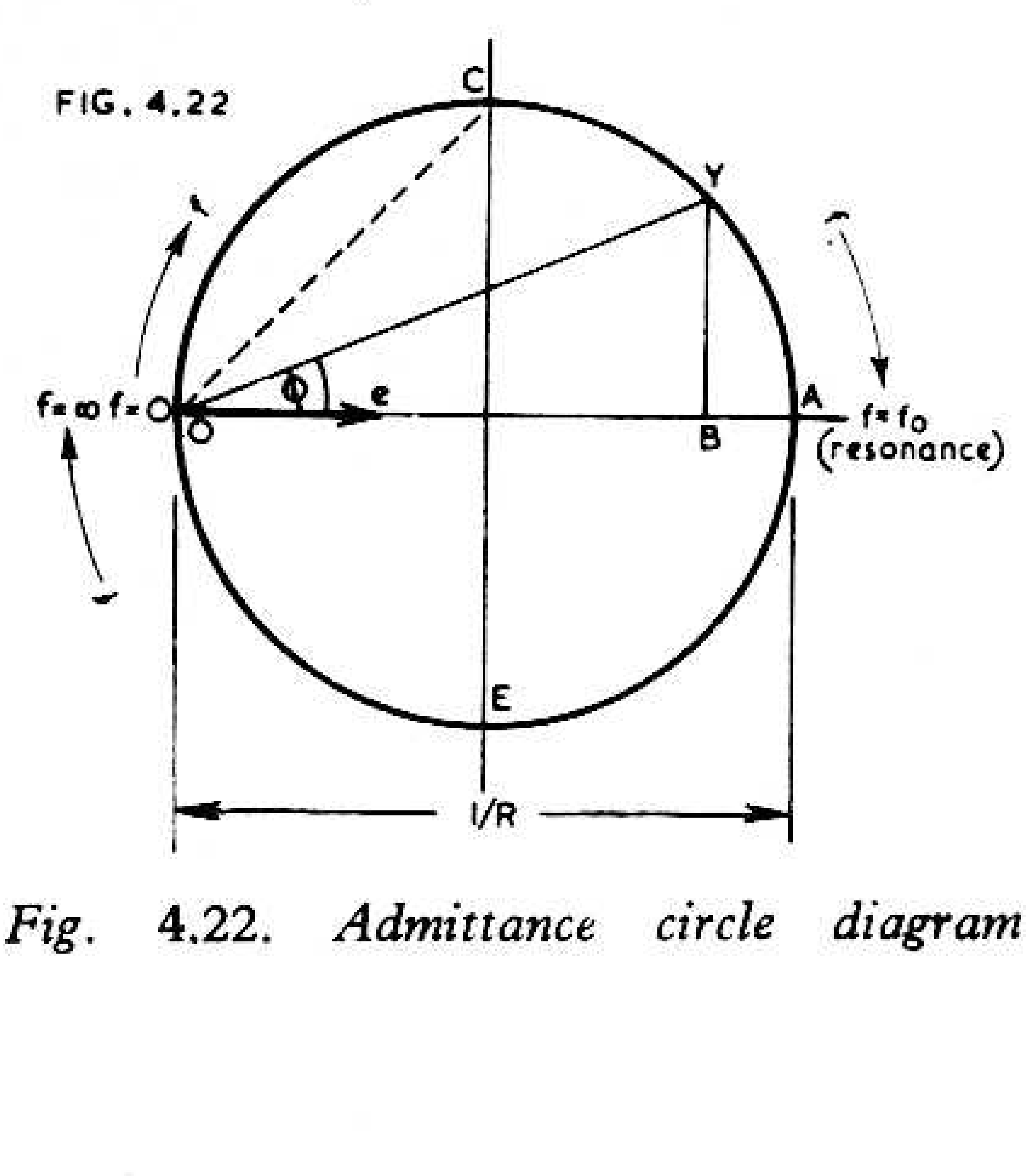


Fig. 4.22. Admittance circle diagram.

finity, so the point Y moves from O (at zero frequency) through C to A (at resonance) and thence through E to O again (at an infinite frequency). The angle ϕ is the angle by which the current i leads the applied voltage e; when ϕ is negative the current lags behind the voltage. Thus when OY lies in the upper portion of the circle the circuit is capacitive, and when it lies in the lower portion of the circle the circuit is inductive.

Fig. 4.22.A Series form of impedance; (B) Equivalent parallel form.

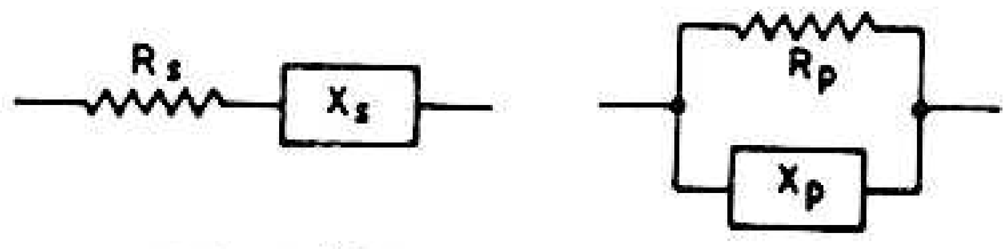


FIG. 4.22A FIG. 4.22B

When Y is at the point C, the admittance will be represented by OC and ϕ will be 45°; the reactive and resistive components of the impedance will therefore be equal. At zero frequency ϕ will be $+90°$ and the admittance will be zero: at infinite frequency ϕ will be $-90°$ and the admittance will also be zero.

(vi) Conversion from series to parallel impedance

It is possible to convert from the series form $\mathbf{Z} = R_s + jX_s$ as shown in Fig. 4.22A

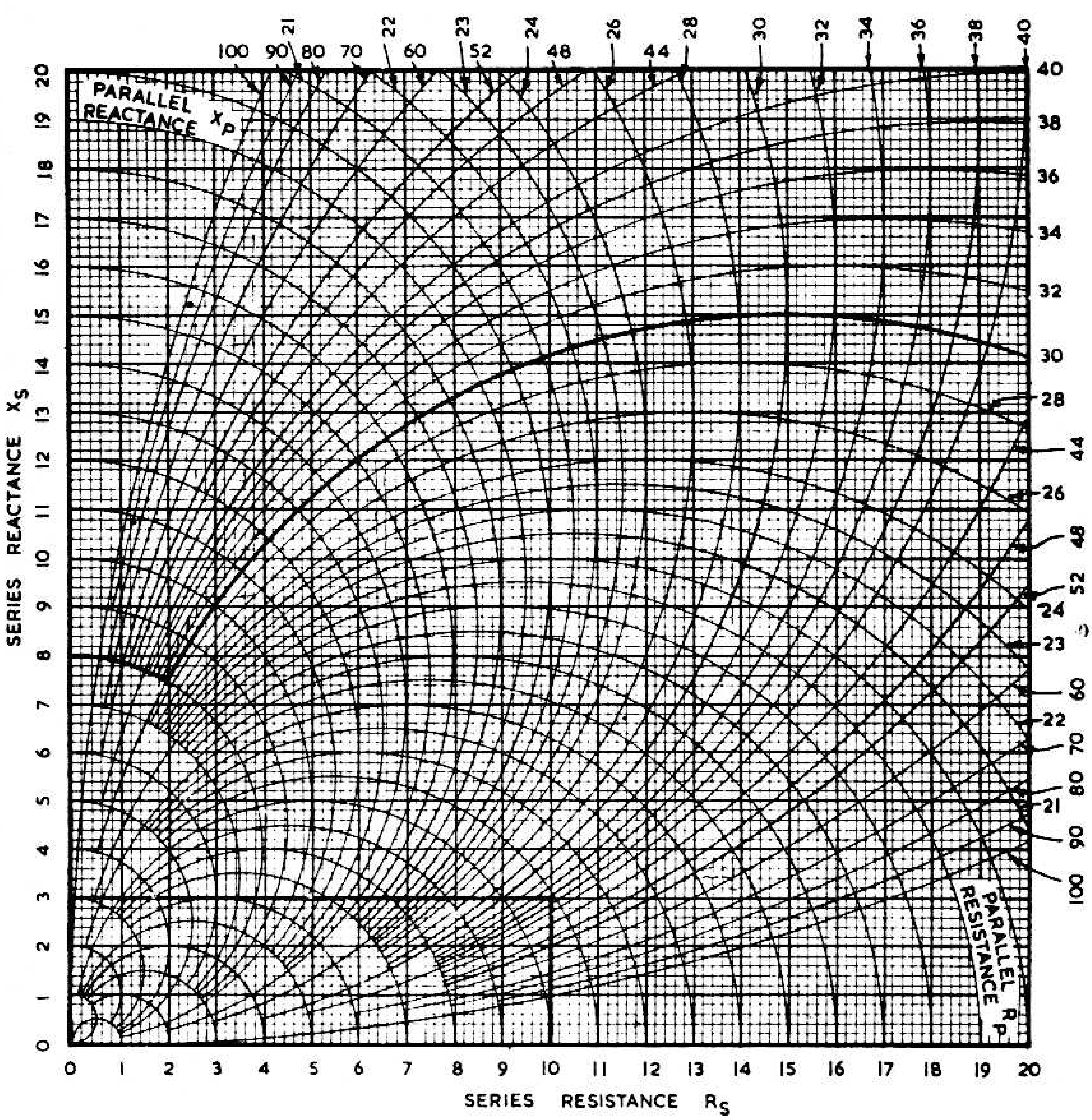


Fig. 4.22C. *Chart for conversion between series resistance and reactance and equivalent parallel resistance and reactance.*

to the equivalent parallel form (B) and vice versa.

$$R_s = \frac{R_p X_p^2}{R_p^2 + X_p^2} = \frac{X_p}{\dfrac{X_p}{R_p} + \dfrac{R_p}{X_p}} \tag{56}$$

$$X_s = \frac{X_p R_p^2}{R_p^2 + X_p^2} = \frac{R_p}{\dfrac{X_p}{R_p} + \dfrac{R_p}{X_p}} \tag{57}$$

The following approximations hold with an error within 1%:

(1) If $R_p > 10X_p$, or if $R_s < 0.1\ X_s$,

then $R_s \approx X_p^2/R_p$; $X_s \approx X_p$ (58)

and $R_p \approx X_s^2/R_s$; $X_p \approx X_s$. (59)

(2) If $R_p < 0.1\ X_p$, or if $R_s > 10X_s$

then $R_s \approx R_p$; $X_s \approx R_p^2/X_p$ (60)

and $R_p \approx R_s$; $X_p \approx R_s^2/X_s$ (61)

Fig. 4.22C can be used to make this conversion.

Example 1. Find the parallel circuit equivalent to a series connection of a 10 ohm resistor (R_s) and an inductor with a reactance of 3 ohms (X_s).

Method: Enter the chart vertically from $R_s = 10$ and horizontally from $X_s = 3$. These lines intersect at $R_p = 10.9$ and $X_p = 36$, so the equivalent parallel connection requires a 10.9 ohm resistor and an inductor with a reactance of 36 ohms.

Example 2. Find the series circuit equivalent to a parallel connection of a 30 000 ohm resistor and a capacitor with a reactance of −8000 ohms.

Method: Enter the chart on the arcs $R_p = 30$ and $X_p = 8$. The intersection is at $R_s = 2$ and $X_s = 7.5$, so the appropriate series elements are $R_s = 2000$ ohms and $X_s = -7500$ ohms.

SECTION 7: NETWORKS

(i) Introduction to networks (ii) Kirchhoff's Laws (iii) Potential Dividers (iv) Thevenin's Thereom (v) Norton's Theorem (vi) Maximum Power Transfer Theorem (vii) Reciprocity Theorem (viii) Superposition Theorem (ix) Compensation Theorem (x) Four-terminal networks (xi) Multi-mesh networks (xii) Non-linear components in networks (xiii) Phase-shift networks (xiv) Transients in networks (xv) References to networks.

(i) Introduction to networks

A network is any combination of impedances (" elements ")—whether resistances, inductances, mutual inductances or capacitances. Ohm's law may be applied either to the voltage drop in any element, or in any branch, or to the voltage applied to the whole network, involving the total network current and total network impedance, provided that the impedances of the elements are constant. Other laws and theorems which may be used for the solution of network problems are described below.

In network analysis it is assumed that the impedances of elements remain constant under all conditions; that is that the elements are **linear devices.** Some types of resistors and capacitors and all air-cored inductors are linear, but iron-cored inductors and amplifying valves are **non-linear**. Other non-linear devices include granule-type microphones, electrolytic condensers, glow lamps, barretters (ballast tubes), electric lamp filaments, temperature-controlled resistances such as thermistors*, and thyrite.† However, it is usually found that satisfactory results may be obtained by applying the average characteristics of the non-linear devices under their operating conditions. Further consideration of non-linear components is given in Sect. 7(xii).

*Thermistors are resistors with a high negative temperature coefficient. See Sect. 9(i)n.

†Thyrite is a conductor whose resistance falls in the ratio 12.6 : 1 every time the voltage is doubled, over a current ratio 10 000 000 to 1. See K. B. McEachron " Thyrite, a new material for lightning arresters " General Electric Review (U.S.A.) 33.2 (Feb. 1930) 92.

Rectifiers, whether thermionic or otherwise, are non-linear devices ; they are frequently represented by an equivalent circuit having a fixed series resistance in the conducting direction and infinitely high resistance in the other. This approximation is very inaccurate at low levels where they have effective resistance varying as a function of the applied voltage, while some rectifiers pass appreciable current in the reverse direction.

Most elements (resistors, capacitors and inductors) transmit energy equally in either direction and are referred to as " bilateral ", but thermionic valves operate only in one direction (" unilateral ") ; when the latter form part of a network it is necessary to exercise care, particularly if they are represented by equivalent circuits.

Hints on the solution of network problems

A complete solution of a network involves the determination of the current through every element (or around every mesh). With simple networks the normal procedure is to apply Kirchhoff's Laws—Sect. 7(ii)—until all the currents and their directions have been determined. The voltage drop across any element may then be derived from a knowledge of the impedance of the element and the current through it.

As a first stage it is important to simplify the circuit, and to draw an equivalent circuit diagram for analysis.

If in any arm there are two or more resistors connected in series, the equivalent circuit diagram should be drawn with

$$R = R_1 + R_2 + \ldots$$

Similarly with inductance $L = L_1 + L_2 + \ldots$

and with capacitance

$$C = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots}.$$

If in any part of the circuit there are two or more elements of the same kind in parallel (whether R, L, or C) the resultant should be determined and applied to the equivalent circuit diagram.

An exception to this rule is where it is merely required to calculate the output voltage from a passive resistive 4-terminal network. In this case it is sometimes helpful to arrange the network in the form of a potential divider, or sequence of dividers, and to use the method of Sect. 7(iii).

It is very important to mark on the equivalent circuit diagram the directions or polarities of the applied voltages (whether direct or alternating) and the assumed directions of the currents ; if any one of the latter is incorrect, this will be shown by a negative sign in the calculated value. A clockwise direction for the flow of current around any mesh is conventional.

In some cases it may be found simpler to reduce a passive 4-terminal network to an equivalent T or π section—see Sect. 7(x)—than to analyse it by means of Kirchhoff's Laws.

Definitions

An **element** is the smallest entity (i.e. a distinct unit) which may be connected in a network—e.g. L, C or R.

An **arm** is a distinct set of elements, electrically isolated from all other conductors except at two points.

A **series arm** conducts the main current in the direction of propagation.

A **shunt arm** diverts a part of the main current.

A **branch** is one of several parallel paths.

A **mesh** is a combination of elements forming a closed path.

A **two-terminal network** is one which has only two terminals for the application of a source of power or connection to another network.

A **four-terminal network** is one which has four terminals for the application of a source of power or connections to other networks. The common form of four-terminal network has two input and two output terminals ; this term is used even when one input terminal is directly connected to one output terminal, or both earthed.

A passive network is one containing no source of power.

An **active network** is one containing one or more sources of power (e.g. batteries, generators, amplifiers).

The **input circuit** of a network is that from which the network derives power.

The **output circuit** of a network is that into which the network delivers power.

Impedance matching—two impedances are said to be matched when they have the same magnitude and the same phase angle.

Reference may also be made to I.R.E. Standard 50IRE4.S1 published in Proc. I.R.E.39.1 (Jan. 1951) 27.

Examples

An amplifier is a four-terminal active network.

An attenuator is a four-terminal passive network.

A conventional tone control is a two-terminal passive network.

Differentiating and Integrating Networks

Based on the fundamental mathematical analysis of the circuit, the following terms are sometimes used in connection with 4-terminal networks.

Differentiating Networks—(1) Series resistance and shunt inductance
or (2) Series capacitance and shunt resistance

Integrating Networks —(1) Series inductance and shunt resistance
or (2) Series resistance and shunt capacitance.

(ii) Kirchhoff's Laws

(1) **The algebraic sum of all the instantaneous values of all currents flowing towards any junction point in a circuit is zero at every instant.**

This is illustrated for d.c. in Fig. 4.23. It will be seen that all junctions and corner points are lettered for reference. The polarities of the two batteries and their voltages are marked. The currents are marked in the obvious directions or, if this is not clear, then arbitrarily in either direction (clockwise around each loop is preferred). Both currents and voltages are referred to by the point lettering,

e.g. i_{ab} is the current flowing from a to b

i_{ba} is the current flowing from b to a

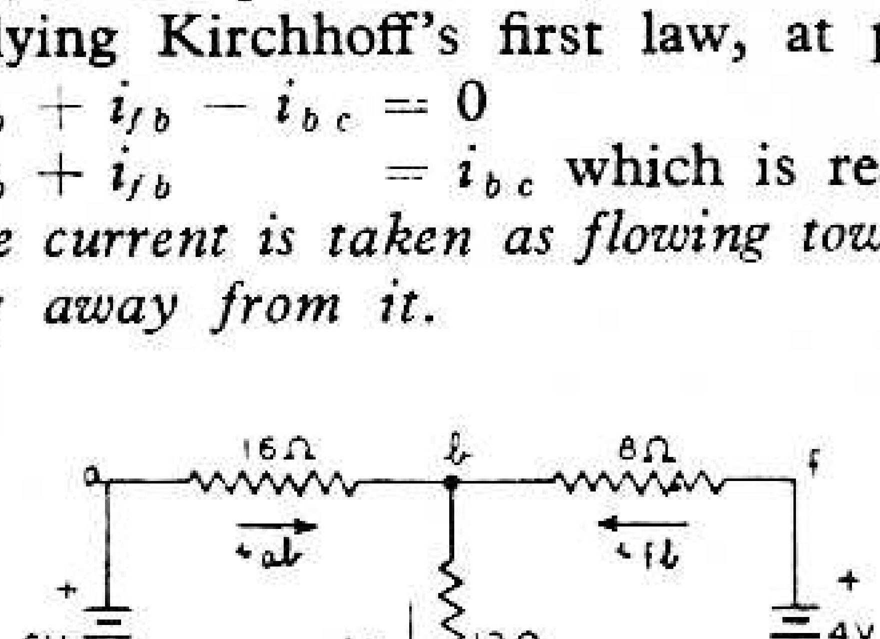
e_{bc} is the potential of point b with respect to point c.

Applying Kirchhoff's first law, at point b

$$i_{ab} + i_{fb} - i_{bc} = 0$$

or $$i_{ab} + i_{fb} = i_{bc}$$ which is really obvious.

Positive current is taken as flowing towards the junction point ; negative current as flowing away from it.



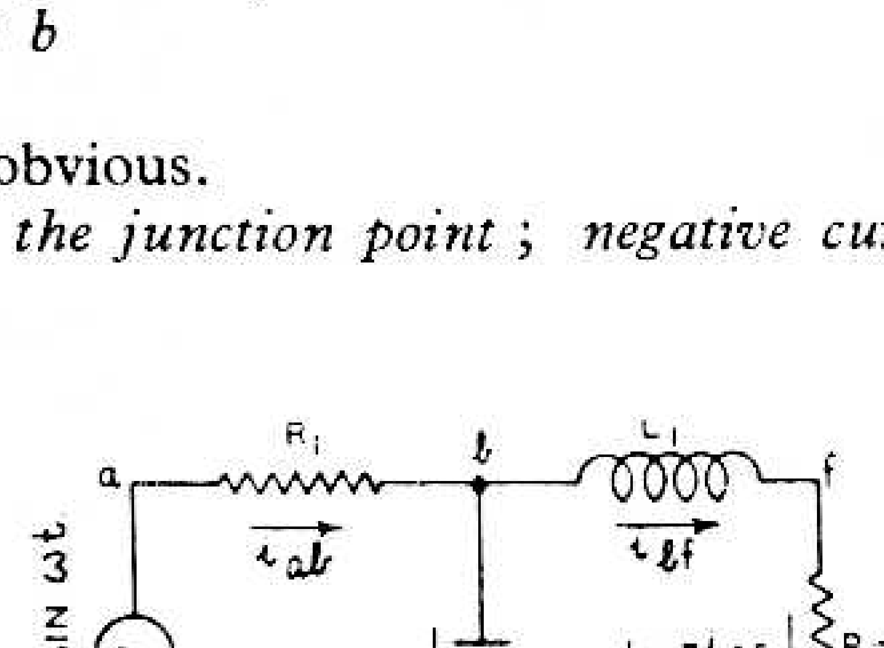


Fig. 4.23. *Network incorporating* 3 *elements and* 2 *d.c. voltage sources.*

Fig. 4.24. *Network incorporating* 4 *elements and one a.c. voltage source.*

This law is illustrated for a.c. in Fig. 4.24 which follows the same general rules as for d.c. The instantaneous generator voltage e is shown in an arbitrary direction and the directions of the currents are then determined.

At point b,

$$i_{ab} - i_{bf} - i_{bc} = 0$$

following the general procedure as in the d.c. case.

(2) **The total rise or fall of potential at any instant in going around any closed circuit is zero.**

This is illustrated for d.c. in Fig. 4.23, taking each closed circuit in turn and proceeding clockwise in each case. Any voltage source is here regarded as positive if it assists in sending current clockwise around the closed circuit (i.e. voltage rise). The voltage across any impedance is regarded as negative if the current arrow is in the same direction as the direction of travel around the closed circuit (i.e. voltage drop).

Circuit $d\ a\ b\ c\ d$: $+6 - 16\, i_{ab} - 10\, i_{bc} = 0$ (1)

Circuit $d\ a\ b\ f\ g\ c\ d$: $+6 - 16\, i_{ab} + 8\, i_{fb} - 4 = 0$ (2)

Circuit $f\ b\ c\ g\ f$: $+4 - 8\, i_{fb} - 10\, i_{bc} = 0$ (3)

Applying Kirchhoff's first law to point b,

$$i_{ab} + i_{fb} = i_{bc} \qquad (4)$$

To find the values of the three unknown currents, it is necessary to apply three suitable equations.

[*Similarly for all other cases—the total number of equations must be equal to the number of unknowns. The number of equations based on Kirchhoff's first law should be one less than the number of junction points ; those based on his second law should equal the number of* **independent** *closed paths.*]

Equations (1), (3) and (4) would be sufficient, since (2) merely duplicates parts of (1) and (3).

From (4) : $i_{ab} = i_{bc} - i_{fb}$

Applying in (1) : $+6 - 16(i_{bc} - i_{fb}) - 10\, i_{bc} = 0$

Therefore $+6 - 16\, i_{bc} + 16\, i_{fb} - 10\, i_{bc} = 0$

Therefore $+6 - 26\, i_{bc} + 16\, i_{fb} = 0$ (5)

Adding twice (3): $+8 - 20\, i_{bc} - 16\, i_{fb} = 0$

Therefore $+14 - 46\, i_{bc} = 0$

Therefore $i_{bc} = 14/46$ ampere.

The other currents may be found by applying this value in (5) and then in (4).

Kirchhoff's Second Law is illustrated for a.c. in Fig. 4.24. Here again, as in all cases, it is assumed that we move around each loop of the network in a clockwise direction.

The voltage across—

a resistance is $-Ri$

an inductance is $-j\omega Li$

a capacitance is $+j(1/\omega C)i$

} where i is in the clockwise direction around the loop.

Circuit $d\ a\ b\ c\ d$: $e - R_1 i_{ab} + j\,(1/\omega C_1) i_{bc} = 0$

Therefore $R_1 i_{ab} - j(1/\omega C_1) i_{bc} = e$ (6)

Circuit $b\ f\ g\ c\ b$: $-j\omega L_1 i_{bf} - R_2 i_{bf} - j(1/\omega C_1) i_{bc} = 0$

Therefore $(j\omega L_1 + R_2) i_{bf} + j(1/\omega C_1) i_{bc} = 0$ (7)

Applying Kirchhoff's first law to junction b :

$$i_{ab} - i_{bf} - i_{bc} = 0$$

Therefore $i_{ab} = i_{bf} + i_{bc}$ (8)

Adding (6) and (7)

$$R_1 i_{ab} + (j\omega L_1 + R_2) i_{bf} = e$$

Applying (8), $R_1 i_{bf} + R_1 i_{bc} + (j\omega L_1 + R_2) i_{bf} = e$

Therefore $R_1 i_{bc} + (R_1 + R_2 + j\omega L_1) i_{bf} = e$ (9)

Multiplying (7) by $(-jR_1\omega C_1)$, remembering that $j^2 = -1$,

$$R_1 i_{bc} + (R_1 \omega^2 L_1 C_1 - jR_1 R_2 \omega C_1) i_{bf} = 0 \qquad (10)$$

Subtracting (10) from (9),

$$[(R_1 + R_2 - R_1 \omega^2 L_1 C_1) + j(\omega L_1 + R_1 R_2 \omega C_1)]\, i_{bf} = e \qquad (11)$$

which gives the value of i_{bf} when e is known.

The value of i_{bc} may be found by substituting this value of i_{bf} in eqn. (7) ; i_{ab} may then be determined by eqn. (8).

(iii) Potential Dividers

The fundamental form of potential divider (also known as voltage divider or potentiometer) is shown in Fig. 4.25. Here a direct line voltage E is " divided " into two voltages E_1 and E_2, where $E = E_1 + E_2$. If no current is drawn from the junction (or tap) B, the voltage across BC is given by

$$E_2 = \left(\frac{R_2}{R_1 + R_2}\right)E, \tag{1}$$

and $I_1 = I_2 = E/(R_1 + R_2)$ (2)

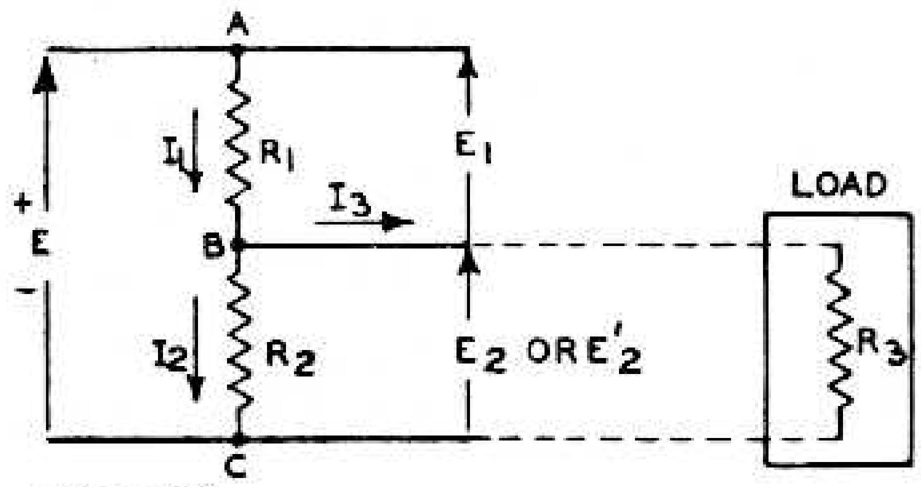


FIG. 4·25

Fig. 4.25. *Potential divider across a d.c. line.*

On load

When a current I_3 is drawn from B, the simplest analysis is to consider the effective load resistance R_3 which will draw a current I_3 at a voltage E_2', i.e. $R_3 = E_2'/I_3$. We now have resistances R_2 and R_3 in parallel, and their total effective resistance is therefore

$$R' = R_2R_3/(R_2 + R_3).$$

In this case the voltage divider is composed of R_1 and R' in series, and the voltage at the point B is given by

$$E_2' = \left(\frac{R'}{R_1 + R'}\right)E$$

$$= \left(\frac{R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}\right)E$$

$$E_2' = \left(\frac{R_2}{R_1 + R_2}\right)E - \left(\frac{R_1R_2}{R_1 + R_2}\right)I_3 \tag{3}$$

The first term on the right hand side is the no-load voltage E_2; the second term is the further reduction in voltage due to I_3—this being a linear equation.

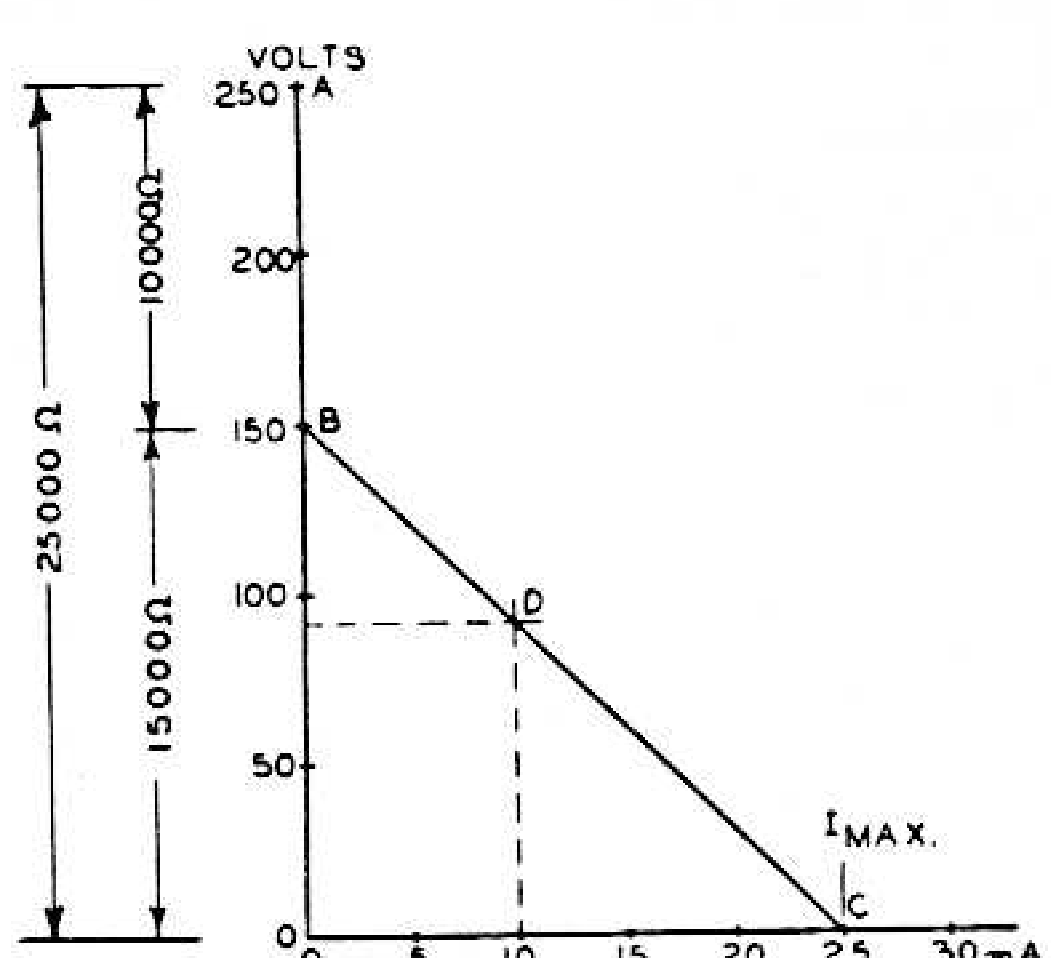


FIG.4·26

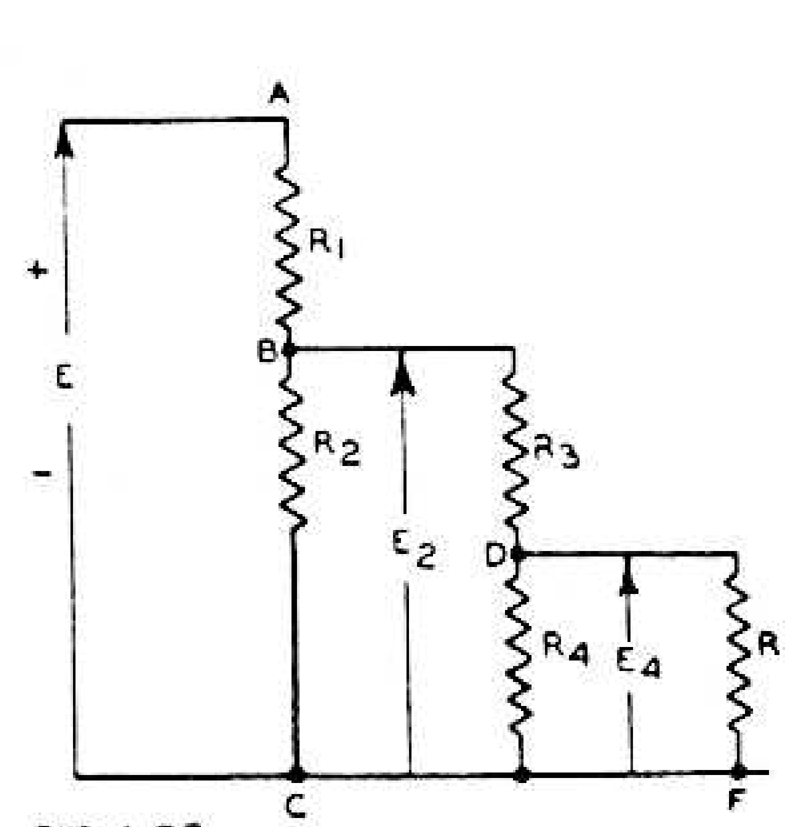


FIG 4·27

Fig. 4.26. *Graphical method for determining the output voltage from a potential divider.*

Fig. 4.27. *A potential divider with a load in the form of another potential divider.*

Eqn. (3) may be plotted* as in Fig. 4.26 where, as a typical example, $E = 250$ V, $R_1 = 10\,000$ and $R_2 = 15\,000$ ohms, so that $(R_1 + R_2) = 25\,000$ ohms. E_2, for

*Cundy, P. F. "Potential Divider Design," W.W. 50.5 (May 1944) 154.

no load, is obviously 150 volts (point B) while point C is for the condition of maximum current (I_{max}) and zero voltage. The latter may be determined by putting $E_2' = 0$ in eqn. (3) which gives

$$I_{max} = E/R_1 \tag{4}$$

In this condition we have, effectively, a resistance R_1 only, in series with a load of zero resistance.

If points B and C in Fig. 4.26 are joined by a straight line, we may then determine the voltage for any value of load current (I_3) between zero and maximum. For example, with a load current of 10 mA, the voltage will be 90 V (point D).

It is sometimes useful to calculate on the basis of a drop in voltage (based on the no-load voltage E_2) of so many volts per milliamp of load current; in this case the drop is 150 volts for 25 mA, or 6 volts per milliamp. This rate of voltage drop is actually the negative slope of the line BC.

The equivalent series source resistance (R_s) is given by

$$R_s = \frac{E_2}{I_{max}} = \frac{150}{25 \times 10^{-3}} = 6000 \text{ ohms.}$$

This may be put into the alternative form

$$R_s = \frac{R_1 R_2}{R_1 + R_2}. \tag{5}$$

"**Regulation**" is defined differently in American and British practice.

American definition: The percentage voltage regulation is the difference between the full-load and no-load voltages, divided by the *full-load* voltage and multiplied by 100.

British definition: The percentage voltage regulation is the difference between the full-load and no-load voltages, divided by the *no-load* voltage and multiplied by 100.

In the example above for a current of 10 mA,

$$\text{Regulation} = \frac{150 - 90}{150} \times 100 = 40\% \text{ by British definition.}$$

The line BD in Fig. 4.26 may be called the "**regulation characteristic**" for the conditions specified above.

To find the load current corresponding to a specified value of E_2', eqn. (3) may be re-arranged in the form

$$I_3 = \frac{E}{R_1} - E_2'\left(\frac{R_1 + R_2}{R_1 R_2}\right) \tag{6}$$

$$\text{or } I_3 = I_{max} - E_2'\left(\frac{R_1 + R_2}{R_1 R_2}\right) \tag{7}$$

Special Case 1

If it is known that the voltage drops from E_2 at no load to E_2' for a load current I_3, then the voltage E_x corresponding to a load current I_x is given by

$$E_x = E_2 - (E_2 - E_2')(I_x/I_3) \tag{8}$$

Special case 2

If the voltages across the load (E_x, E_y) for two different values of load current (I_x, I_y) are known, the voltage at zero load current is given by

$$E_2 = \frac{E_x I_y - E_y I_x}{I_y - I_x} \tag{9}$$

This is often useful for determining the no load voltage when the voltmeter draws appreciable current. A method of applying this with a two range voltmeter has been described*, and the true voltage is given by

*Lafferty, R. E. "A correction formula for voltmeter loading" (letter) Proc. I.R.E. 34.6 (June 1946) 358.

$$E_2 = \frac{(S - 1) E_x}{S - (E_x/E_y)} \tag{10}$$

where S = ratio of the two voltmeter scales used for the two readings E_x and E_y,
E_x = voltmeter reading on the higher voltage scale,
and E_y = voltmeter reading on the lower voltage scale.
If $S = 2$, then

$$E_2 = \frac{E_x}{2 - (E_x/E_y)}. \tag{11}$$

Complicated divider network

When a voltage divider has another voltage divider as its load (Fig. 4.27) the best procedure is to work throughout in resistances and voltages, and to leave currents until after the voltages have been determined. The final equivalent load resistance (R_5) must be determined before commencing calculations, then proceed—

R_4 in parallel with R_5: $R' = R_4R_5/(R_4 + R_5)$
R_3 in series with R': $R'' = R_3 + R'$
R'' in parallel with R_2: $R''' = R_2R''/(R_2 + R'')$

Then $$E_2 = \left(\frac{R'''}{R_1 + R'''}\right)E \tag{12}$$

and $$E_4 = \left(\frac{R'}{R_3 + R'}\right)E_2 \tag{13}$$

A somewhat similar procedure may be adopted in any divider network.

(iv) Thévenin's Theorem (pronounced " tay-venin's ")

This theorem* may be expressed in various ways, of which one is :

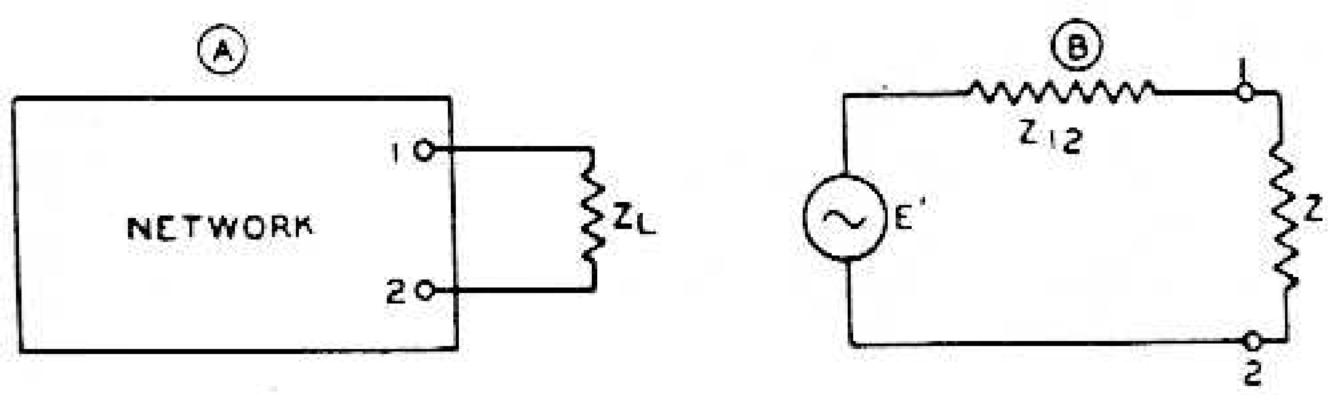


Fig. 4.28. *(A) An impedance connected to two terminals of a network (B) Thevenin's equivalent circuit.*

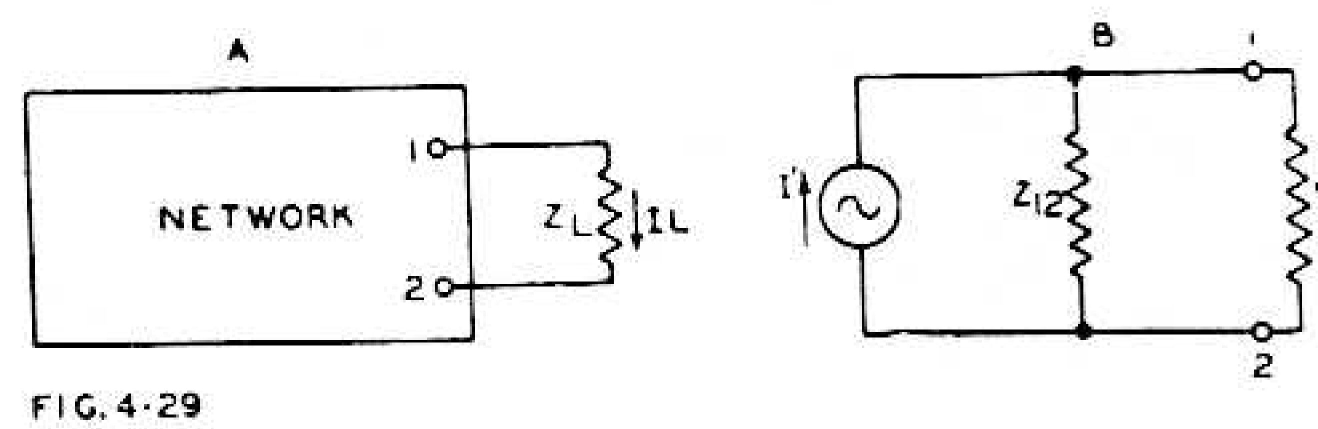


Fig. 4.29. *(A) an impedance, carrying a current I_L, connected to two terminals of a network ; (B) Norton's equivalent circuit.*

The current in any impedance, Z_L, connected to two terminals of a network consisting of any number of impedances and generators (or voltage sources) is the same as though Z_L were connected to a simple generator, whose generated voltage is the open-circuited voltage at the terminals in question, and whose impedance is the impedance of the network looking back from the terminals, with all generators replaced by impedances equal to the internal impedances of these generators.

In Fig. 4.28, (A) shows an impedance Z_L whose two ends are connected to the terminals 1, 2 of any network. Diagram (B) shows Thevenin's equivalent circuit, with a generator E' and series impedance Z_{12} where :

E' is the voltage measured at the terminals 1, 2, with Z_L removed,
and Z_{12} is the impedance of the network measured across the terminals 1, 2, when looking backwards into the network, with all generators out of operation and each replaced by an impedance equal to its internal impedance.

*For the proofs of this and subsequent theorems, see W. L. Everitt, Ref. 1 pp. 47-57.

(v) Norton's Theorem

This is similar in many ways to Thevenin's Theorem, but provides a constant current generator and a shunt impedance.

The current in any impedance Z_R, connected to two terminals of a network, is the same as though Z_R were connected to a constant current generator whose generated current is equal to the current which flows through the two terminals when these terminals are short-circuited, the constant-current generator being in shunt with an impedance equal to the impedance of the network looking back from the terminals in question.

In Fig. 4.29, (A) shows an impedance Z_L through which flows a current I_L, connected to a network. Diagram (B) shows Norton's equivalent circuit, with a constant current generator delivering a current I' to an impedance Z_{12} in shunt with Z_L. As in diagram A, the current through Z_L is I_L.

Here $I' = E'/Z_{12}$

where E' and Z_{12} are the same as in Fig. 4.28B (Thevenin's Theorem).

(vi) Maximum Power Transfer Theorem

The maximum power will be absorbed by one network from another joined to it at two terminals, when the impedance of the receiving network is varied, if the impedances (looking into the two networks at the junction) are conjugates* of each other.

This is illustrated in Fig. 4.30 where E is the generated voltage, Z_g the generator internal impedance and Z_L the load impedance. In the special case where Z_g and Z_L are pure resistances,

Z_g will become R_g

Z_L will become R_L

and maximum power transfer will occur when $R_L = R_g$.

In the general case, $Z_g = R_g + jX_g$ and $Z_L = R_L + jX_L$ while for maximum power transfer $R_L = R_g$ and $X_L = -X_g$.

In other words, if Z_g is inductive, Z_L should be capacitive, and vice versa.

If the magnitude of the load impedance may be varied, but not the phase angle, then the maximum power will be absorbed from a generator when the absolute value of the load impedance is equal to the absolute value of the impedance of the supply network.

See Ref 3 (References to networks). Sect. 7(xv).

(vii) Reciprocity Theorem

In any system composed of linear bilateral impedances, if an electromotive force E is applied between any two terminals and the current I is measured in any branch, their ratio (called the "transfer impedance") will be equal to the ratio obtained if the positions of E and I are interchanged.

In Fig. 4.31 a generator supplies a voltage E to a network, and an ammeter A reads the current I_2. The transfer impedance is E/I_2. If now E and A are reversed, the new transfer impedance will have the same value as previously. In other words, E being unchanged, the ammeter reading in the new position will be the same as previously.

This theorem proves that a network of bilateral impedances transmits with equal effectiveness in both directions, when generator and load have the same impedance.

(viii) Superposition Theorem

In any network consisting of generators and linear impedances, the current flowing at any point is the sum of the currents which would flow if each generator were considered separately, all other generators being replaced at the time by impedances equal to their internal impedances.

This theorem considerably simplifies the analysis of any network containing more than one generator. It is important to note the linearity requirements, as the theorem

*Two impedances are conjugates of each other when their resistive components are equal, and their reactive components are equal in magnitude but opposite in sign.

breaks down under other conditions. It is therefore only applicable to valves when these are being operated to give negligible distortion.

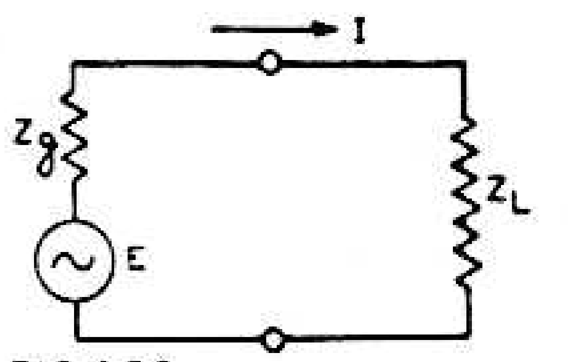


FIG. 4·30

Fig. 4.30. *A generator, with internal impedance Z_g, connected to a load Z_L, to illustrate the maximum power transfer.*

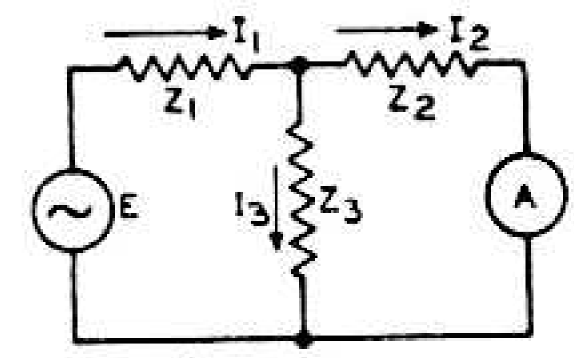


FIG. 4·31

Fig. 4.31. *A generator E supplying voltage to a network, with an ammeter A to read the current I_2. The reciprocity theorem states that, when A and E are reversed, the transfer impedance E/I_2 will be unchanged.*

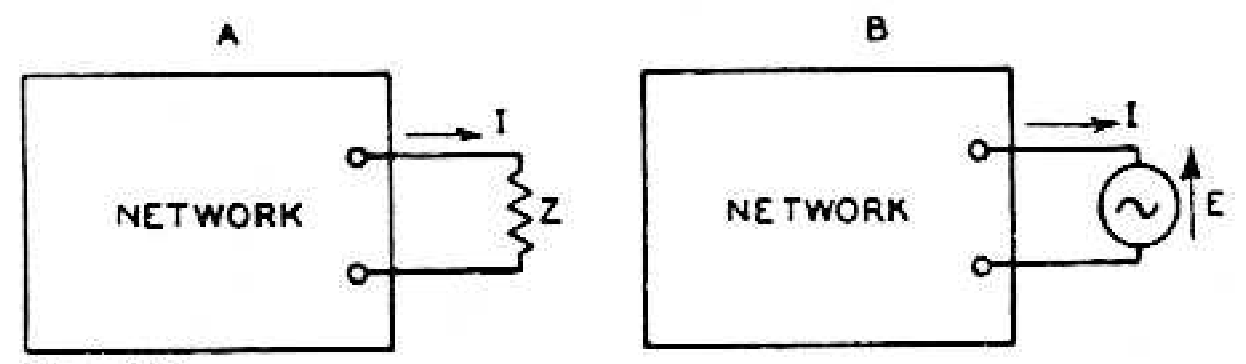


FIG. 4·32

Fig. 4.32. *(A) An impedance Z in a network with current I and voltage drop ZI (B) Equivalent circuit having identical results so far as current and voltage drop are concerned, with a generator developing a voltage E = ZI. This illustrates the Compensation Theorem.*

(ix) Compensation Theorem

An impedance in a network may be replaced by a generator of zero internal impedance, whose generated voltage at any instant is equal to the instantaneous potential difference produced across the replaced impedance by the current flowing through it.

This is illustrated in Fig. 4.32 where in (A) a current I is flowing through an impedance Z with a voltage drop IZ. This is equivalent to an identical network, as in (B), where Z has been replaced by a generator of zero internal impedance, whose generated voltage (E) is equal in magnitude to IZ, and is in a direction opposing the flow of current.

(x) Four-terminal networks

The most common fundamental types of four terminal networks are illustrated in Fig. 4.33, where (A) is a T section, (B) is a Π section and (C) a Lattice section. Both A and B are called 3 element networks, and C a 4 element network, from the number of arms containing impedances. In conventional operation, the left-hand terminals 1, 2, are regarded as the input terminals, to which is connected some generator, or other network containing a generator. Terminals 3, 4, are normally regarded as the output terminals, across which is connected a load impedance Z_L.

If we are concerned only with the observable impedances between terminals, it is possible—by selecting suitable values—to convert a T section to a Π section, and vice versa, but only for one particular frequency. This equivalence is independent of the character of the generator or load.

Equivalent T section — *Equivalent Π section*

$$Z_1 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C} \qquad Z_A = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_2}$$

$$Z_2 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C} \qquad Z_B = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_3} \qquad (14)$$

$$Z_3 = \frac{Z_A Z_C}{Z_A + Z_B + Z_C} \qquad Z_C = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_1}$$

The bridged T section of Fig. 4.33E may be reduced to the equivalent T section of Fig. 4.33F, or vice versa, by using the same equivalent values as for equivalent T and Π sections (above).

Any complex four-terminal network can be reduced, at a single frequency, to a single T or Π section. The procedure is to measure (or calculate) the three constants :

Z_{o1} = the impedance at the input terminals when the output terminals are open-circuited.

Z_{o2} = the impedance at the output end looking back into the network with the input terminals open-circuited.

Z_{s1} = the impedance at the input end with the output terminals short-circuited.

When these are known, the constants for the equivalent T and Π sections are :

Equivalent T section

$$Z_1 = Z_{o1} - \sqrt{Z_{o2}(Z_{o1} - Z_{s1})}$$

$$Z_2 = Z_{o2} - \sqrt{Z_{o2}(Z_{o1} - Z_{s1})}$$

$$Z_3 = \sqrt{Z_{o2}(Z_{o1} - Z_{s1})}$$

Equivalent Π section

$$Z_A = \frac{Z_{s1}Z_{o2}}{Z_{o2} - \sqrt{Z_{o2}(Z_{o1} - Z_{s1})}}$$

$$Z_B = \frac{Z_{s1}Z_{o2}}{\sqrt{Z_{o2}(Z_{o1} - Z_{s1})}} \qquad (15)$$

$$Z_C = \frac{Z_{s1}Z_{o2}}{Z_{o1} - \sqrt{(Z_{o1} - Z_{s1})Z_{o2}}}$$

It would also be possible to derive expressions for the equivalent sections, using Z_{s2} in place of Z_{s1}, where Z_{s2} = impedance at the output end with the input terminals short-circuited.

A lattice network may be reduced to an equivalent T or Π network (see any suitable text-book) but it is interesting to note that it is essentially a " bridge " circuit. Fig. 4.33 (C) may be re-drawn as in (D) without any change being made.

Four-terminal networks are considered further in Sect. 8 ; they may also be treated as multi-mesh networks as in Sect. 7(xi) below.

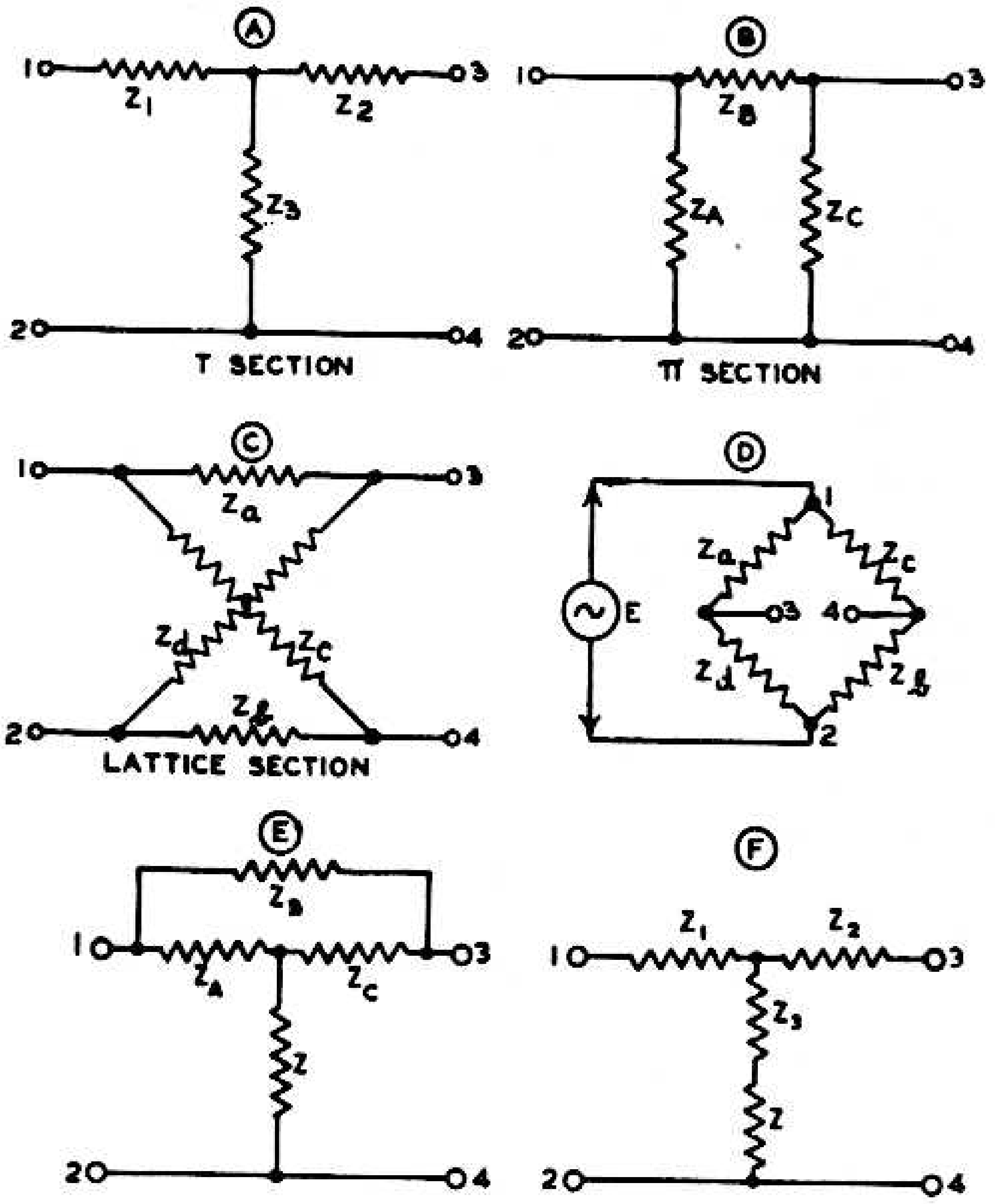


FIG. 4.33

Fig. 4.33. Four Terminal Networks (A) T Section (B) Π Section (C) Lattice Section (D) Lattice Section redrawn in the form of a bridge (E) Bridged T Section (F) T Section equivalent to Bridged T.

(xi) Multi-Mesh Networks

A typical flat multi-mesh network is shown in Fig. 4.33G. A " flat " network is defined as one which can be flattened out without having any lines crossing over each other ; the treatment in this handbook is limited to flat networks. Junctions at which the current can divide are called branch points (A, B, C, D, E). In Fig. 4.33G there are 5 meshes, and the circulating mesh current of each is marked (I_1, I_2, etc.) in the conventional clockwise direction. The simplest form of solution is by means of the **Mesh Equations.** The basis for the use of these equations is given in Ref. 4, Sect. 7(xv). Impedances in the network are numbered Z_{10}, Z_{20}, etc., when they form part of one mesh only (mesh 1, mesh 2, etc.). Impedances which are common to two meshes are numbered Z_{12}, Z_{13}, etc., and are called **mutual impedances,** the suffixes indicating the meshes to which they are common. The applied alternating voltages are also numbered E_1, E_2, etc. where the suffix indicates the mesh number. If there is more than one voltage source in any mesh, E_1 etc. will indicate the vector sum of these voltages.

There must be the same number of mesh equations as there are meshes in the network. The mesh equations are written in the general form for n meshes as

$$\left.\begin{array}{l} Z_{11}I_1 + Z_{12}I_2 + Z_{13}I_3 + \quad \ldots Z_{1n}I_n = E_1 \\ Z_{21}I_1 + Z_{22}I_2 + Z_{23}I_3 + \quad \ldots Z_{2n}I_n = E_2 \\ \ldots \\ Z_{n1}I_1 + Z_{n2}I_2 + Z_{n3}I_3 + \quad \ldots Z_{nn}I_n = E_n \end{array}\right\} \quad (16)$$

where each Z is of the form $R + j(\omega L - 1/\omega C)$ and Z_{11}, Z_{22}, etc. is called the **self-impedance** of the individual mesh, i.e., the impedance round the mesh if all other branches of the network other than those included in the mesh in question were open-circuited.

For example, in Fig. 4.33G—

$$\begin{aligned} Z_{11} &= Z_{10} + Z_{13} + Z_{12} \\ &= R_{10} + jX_{10} + R_{13} + jX_{13} + R_{12} + jX_{12} \\ &= R_{10} + j(\omega L_{10} - 1/\omega C_{10}) + R_{13} + j(\omega L_{13} - 1/\omega C_{13}) \\ &\quad + R_{12} + j(\omega L_{12} - 1/\omega C_{12}) \end{aligned}$$

i.e.

$$\begin{aligned} Z_{11} = (R_{10} + R_{13} + R_{12}) + j[(\omega L_{10} + \omega L_{13} + \omega L_{12}) \\ - (1/\omega C_{10} + 1/\omega C_{13} + 1/\omega C_{12})] \end{aligned} \quad (17)$$

Note that Z_{21} is the same as Z_{12} etc. and that the signs of the mutual impedances may be positive or negative (see below).

An impedance that is common to two branches is considered to be a positive **mutual impedance** when the arrows representing the corresponding mesh currents pass through the impedance in the same direction ; or a negative mutual impedance if the arrows pass through the impedance in opposite directions.

Thus in Fig. 4.33G the arrows representing the corresponding mesh currents pass through the impedances Z_{12}, Z_{13}, Z_{24} and Z_{34} in opposite directions, so that these constitute negative mutual impedances, and have negative signs in eqn. (16).

A **mutual inductance** may be defined as positive or negative according to whether it acts with a polarity the same as, or opposite to, that of a corresponding common inductance*.

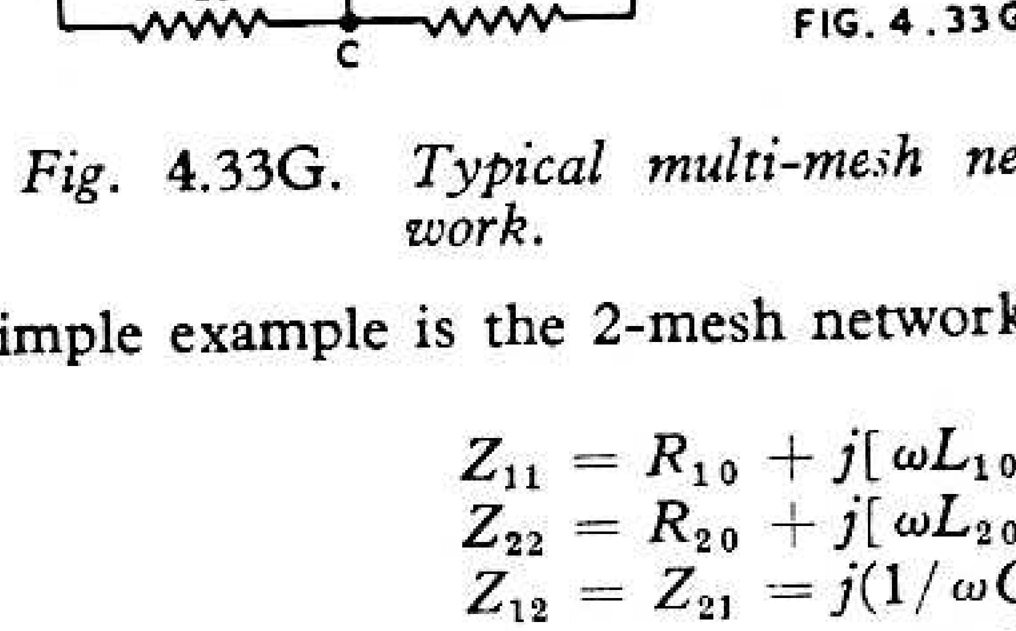


FIG. 4.33G

Fig. 4.33G. *Typical multi-mesh network.*

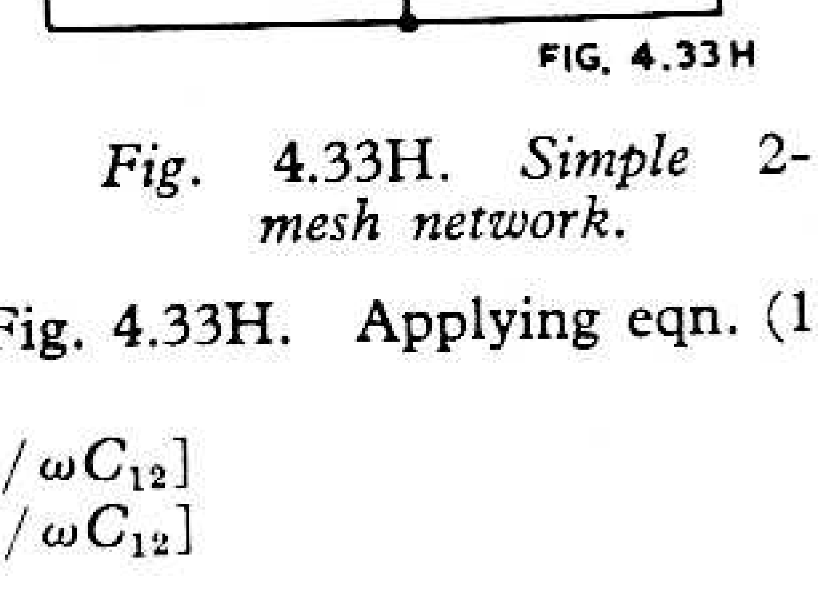


FIG. 4.33H

Fig. 4.33H. *Simple 2-mesh network.*

A simple example is the 2-mesh network of Fig. 4.33H. Applying eqn. (16) we have

$$Z_{11} = R_{10} + j[\omega L_{10} - 1/\omega C_{12}] \quad (18)$$

$$Z_{22} = R_{20} + j[\omega L_{20} - 1/\omega C_{12}] \quad (19)$$

$$Z_{12} = Z_{21} = j(1/\omega C_{12}) \quad (20)$$

Since there are two meshes, there will be two mesh equations—

$$\left.\begin{array}{l} Z_{11}I_1 + Z_{12}I_2 = E_1 \\ Z_{21}I_1 + Z_{22}I_2 = 0 \end{array}\right\} \quad (21)$$

*The opposite definition is also used.

These linear simultaneous equations may be solved by elimination, but any more complicated network would have to be solved by the use of Determinants, for which see any suitable mathematical textbook.

The total current in any common branch may be determined by the difference between the two mesh currents.

The method of handling **mutual inductance** is illustrated by the 2-mesh network of Fig. 4.33I. Here

$$Z_{11} = (R_{10} + R_{12}) + j(\omega L_{10} + \omega L_{12} - 1/\omega C_{10}) \quad (22)$$
$$Z_{22} = (R_{12} + R_{20}) + j(\omega L_{12} + \omega L_{20} - 1/\omega C_{20}) \quad (23)$$
$$Z_{12} = Z_{21} = R_{12} + j(\omega L_{12} \pm \omega M_{12}) \quad (24)$$

and the two mesh equations will be as eqn. (21).

The polarity of M_{12} in eqn. (24) must be determined in accordance with the accepted convention, as described in connection with eqn. (16).

A solution of eqn. (16) by means of Determinants shows that in the general case the Determinant D is given by

$$D = \begin{vmatrix} Z_{11}Z_{12} & . & . & Z_{1n} \\ Z_{21}Z_{22} & . & . & . & Z_{2n} \\ . & . & . & . \\ Z_{n1}Z_{n2} & . & . & . & Z_{nn} \end{vmatrix} \quad (25)$$

and the current I_k in the k^{th} mesh that flows as the result of the voltage E_j acting in the j^{th} mesh is

$$I_k = E_j \frac{B_{jk}}{D} \quad (26)$$

where B_{jk} is the principal minor of D multiplied by $(-1)^{j+k}$. This minor is formed by cancelling the j^{th} row and the k^{th} column and then moving the remainder together to form a new determinant with one less row and column than D, where D is the determinant defined by eqn. (25). The row cancelled corresponds to the mesh containing the input voltage, the column cancelled to the mesh containing the required current.

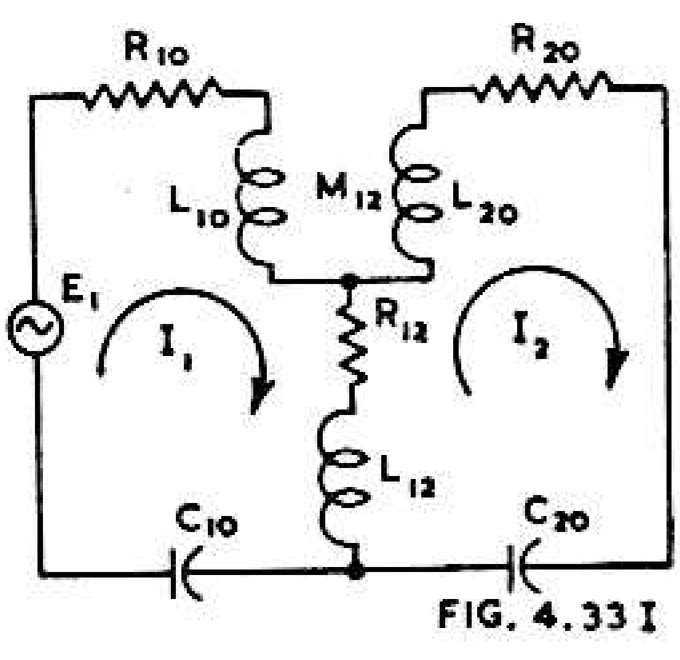


FIG. 4.33 I

Fig. 4.33 I. Two-mesh network incorporating mutual inductance.

The **input impedance** of a passive network with a single applied voltage (Fig. 4.33J) is given by

$$\text{Input impedance} = E_1/I_1 = D/B_{11} \quad (27)$$

where E_1 = voltage applied to input terminals
I_1 = input current
D = determinant defined by eqn. (25)
and B_{11} is the minor of D obtained by cancelling the first row and column.

The **transfer impedance** of a 4-terminal network (Fig. 4.33K) is defined as the ratio of the voltage applied to the input terminals to the resulting current through the load impedance connected to the output terminals (i.e., the n^{th} mesh).

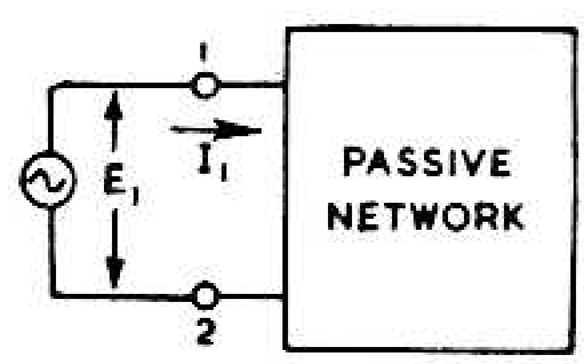


FIG. 4.33 J

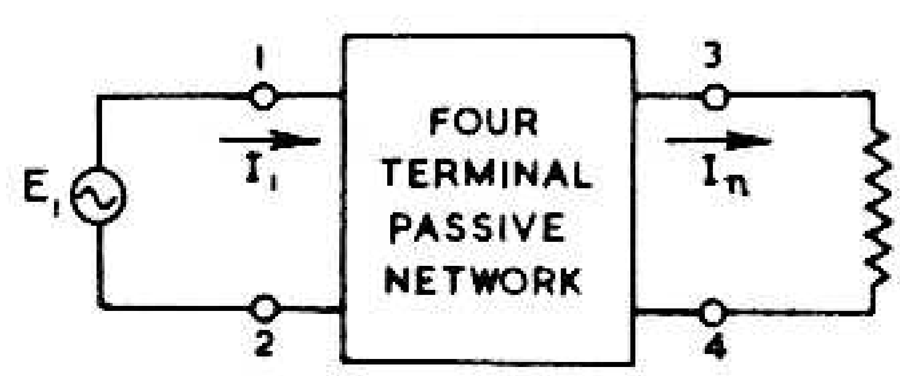


FIG. 4.33 K

Fig. 4.33J. Illustrating input impedance of network.
Fig. 4.33K. Four terminal network illustrating input and transfer impedances.

$$\text{Transfer impedance} = E_1/I_L = D/B_{1n} \quad (28)$$

where D = determinant defined by eqn. (25)

and B_{1n} is the minor of D obtained by cancelling the first row and n^{th} column, and applying the factor $(-1)^{n+1}$.

References to multi-mesh networks: Refs. 1, 2, 4, 5, 6, 7. Sect. 7(xv).

(xii) Non-linear components in networks

In radio engineering the principal non-linear components in networks are valves, although certain resistors and iron-cored inductances are also non-linear. The non-linearity can only be neglected when the voltage swing across any non-linear component is so limited that the characteristic is substantially constant over the range of operation.

In a non-linear component the impedance is not constant, but varies with the applied voltage*. Each such impedance must be treated as having an impedance which is a function of voltage,

i.e. $$Z = F(e).$$

The mathematical theory of circuits containing non-linear components is complicated and, in the general form, outside the scope of this handbook. Those who are interested are referred to the list at the end of this subsection.

The treatment of non-linearity in valve characteristics is covered in Chapter 27 for detection and Chapter 25 for frequency conversion. Distortion through curvature of valve characteristics is covered in Chapter 2 Sect. 9 and Chapter 13.

References to non-linearity
(other than those covered elsewhere)

Chaffee, E. L. "Theory of Thermionic Vacuum Tubes" (McGraw-Hill Book Company, New York and London, 1st edit. 1933) Chapter 21.

Llewellyn, F. B., and L. C. Peterson. "Vacuum tube networks" Proc. I.R.E. 32.3 (March 1944) 144.

(xiii) Phase-shift networks

The bridge type phase-shift network of Fig. 4.34 has the advantage of providing full range phase shifting from 0 to 180°, with constant attenuation (6db) for all degrees of phase shift†.

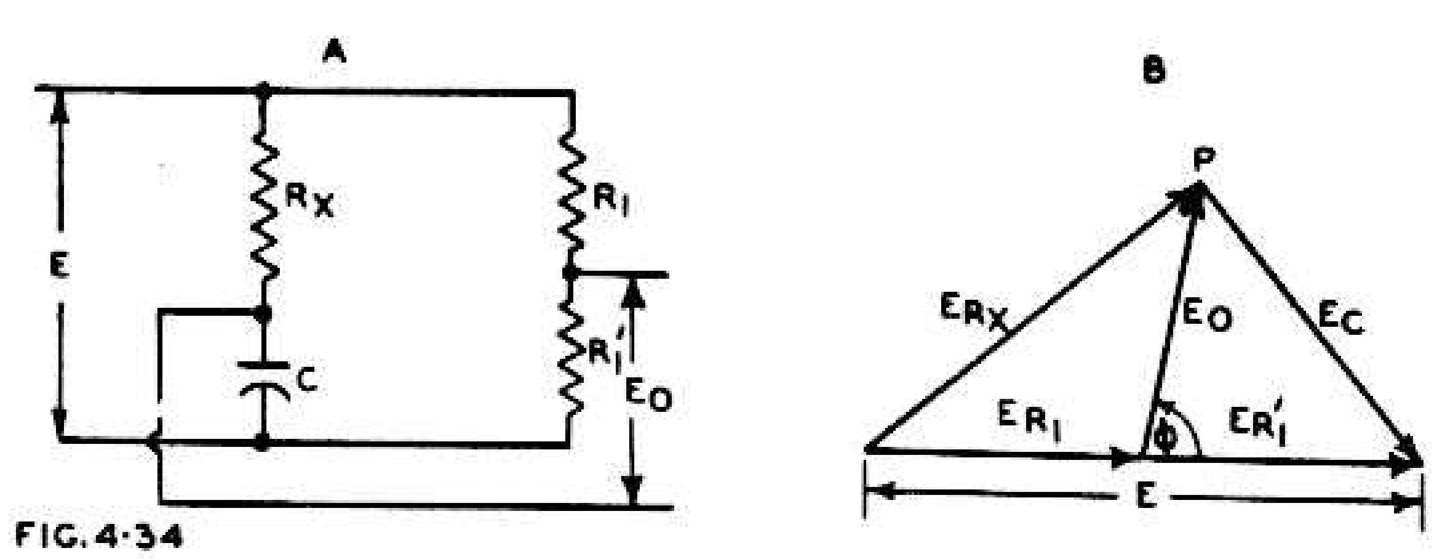


Fig. 4.34 (*A*) *A phase-shift network providing full range phase shifting from* 0 *to* 180° *with* 6 *db attentuation for all degrees of phase shift* (*B*) *vector diagram of voltage relationships.*

It may be shown, when $R_1 = R_1' = 1/\omega C$, that

$$\left|\frac{E_o}{E}\right| = \tfrac{1}{2} \qquad \text{and } \phi = \tan^{-1}\left(\frac{2R_1R_x}{R_x^2 - R_1^2}\right)$$

which is illustrated by the vector diagram Fig. 4.34B, where E_{R1} and E_{R1}' are equal, and both equal to E_o; E_{Rx} and E_c are at right angles, but their vector sum is E. Point P is therefore on the circumference of a semi-circle.

*It is assumed here that the impedance is not a function of time; this occurs in the case of barretters and lamp filaments.

†Lafferty, R. E. "Phase-shifter nomograph," Elect. 19.5 (May 1946) 158.

(xiv) Transients in Networks

The treatment of networks earlier in this Section has been on the basis of a steady applied direct voltage, or a steady alternating voltage, or a combination of the two. It is also important to know the transient currents which may flow during the period from the application of a voltage until the steady state has been reached or during the period from the disconnection of the voltage until a steady state has been reached, or due to some sudden change in operating conditions after the steady state has been reached.

The simple cases of a capacitance and inductance each in series with a resistance, on direct charge and discharge, have been covered in Sects. 4 and 5. These charge and discharge characteristics are of a logarithmic form, and non-oscillatory. However, in any network including L, C and R, the transient characteristics tend to be oscillatory. These oscillatory transients may be more or less heavily damped, and normally become of negligible value after a short period of time. On the other hand, they may continue as sustained oscillations.

Amplifiers, particularly those with feedback, should have a sufficiently damped oscillatory transient in the output circuit when the input is increased instantaneously from zero to some predetermined steady voltage (this is called "unit step" input). In practice it is more convenient to use a periodic rectangular wave input, with the time period of the " flat top " sufficiently long to allow for the decay of damped oscillatory transients. A C.R.O. is commonly used for observation of such a waveform.

Alternatively, an impulse having very short time duration may be applied to the network or amplifier. This differs from unit step input in that the input voltage returns to zero almost instantaneously.

The complete mathematical analysis of all but very simple networks is very complicated and specialized, and outside the scope of this Handbook.

For further information see Refs. 4, 7 and 8.

(xv) References to Networks

1. Everitt, W. L. (book) " Communication Engineering." (McGraw-Hill Book Co. Inc., New York and London, 1937).

2. Shea, T. E. (book) " Transmission Networks and Wave Filters " (D. Van Nostrand Co. Inc., New York, 1943).

3. Ellithorn, H. E. " Conditions for transfer of maximum power." Comm. 26.10 (Oct. 1946) 26.

4. Guillemin, E. A. (book) " Communication Networks " Vol. 1. (John Wiley and Sons Inc. New York ; Chapman and Hall Ltd., London, 1931).

5. Terman, F. E. (book) " Radio Engineers' Handbook " (McGraw-Hill Book Co., New York and London, 1943).

6. Johnson, K. S. (book) " Transmission Circuits for Telephonic Communication " (D. Van Nostrand Co. Inc. New York, 1931).

7. Valley, G. E., and H. Wallman (book) " Vacuum Tube Amplifiers " (McGraw-Hill Book Co. New York and London, 1948).

8. Gardner, M. F., and J. L. Barnes (book) " Transients in Linear Systems " (John Wiley and Sons, Inc. New York, 1942).

See Supplement for additional references.

SECTION 8 : FILTERS

(i) Introduction to filters (ii) Resistance-capacitance filters, high-pass and low-pass (iii) Special types of resistance-capacitance filters (iv) Iterative impedances of four-terminal networks (v) Image impedances and image transfer constant of four-terminal networks (vi) Symmetrical networks (vii) " Constant k " filters (viii) M Derived filters (ix) Practical filters (x) Frequency dividing networks (xi) References to filters.

(i) Introduction to filters

A filter is any passive* network which discriminates between different frequencies, that is to say it provides substantially constant " transmission " over any desired range of frequencies and a high degree of attenuation for all other frequencies.

Filters are conveniently grouped as under :

Low pass filters—transmission band from zero (or some very low) frequency to a specified frequency. Attenuation for all higher frequencies.

High pass filters—transmission band from some specified frequency to very high frequencies. Attenuation for all lower frequencies.

Band pass filters—transmission band from one to another specified frequency. Attenuation for all lower and higher frequencies.

Band elimination filters—" traps."

Practical filters, particularly of the simple variety, have only a gradual change in attenuation. The sharper the required change in attenuation, the more complicated becomes the filter.

Some very simple filters are :

1. The grid coupling condenser and grid resistor of an amplifier (high pass resistance-capacitance filter).
2. The series condenser and variable resistor of a conventional tone control (low pass resistance-capacitance, with adjustable attenuation).
3. The smoothing filter of a power supply, including one or two inductances and two or three capacitances (low pass filter).
4. An overcoupled i-f transformer (tuned band pass filter).
5. A tuned aerial coil or r-f transformer (tuned narrow band pass filter).

See Chapter 6 for mathematics.

(ii) Resistance—capacitance filters

Fig. 4.35A shows a r.c. high pass filter as for grid coupling in an amplifier. This is essentially a voltage divider in which C forms a reactive, and R a resistive, arm.

If the generator has zero resistance and if there is no loading on the output, the ratio of output to input voltages is given by

$$\frac{E_o}{E_i} = \frac{R}{R + jX_c} \quad \text{where } X_c = -1/2\pi fC.$$

$$\text{Therefore } \left|\frac{E_o}{E_i}\right| = \frac{R}{\sqrt{R^2 + X^2}} = \frac{1}{\sqrt{1 + (X/R)^2}} \tag{1}$$

If we select a frequency (f_1) at which $|X_c| = R$, then $|E_o/E_i| = 0.707$ which is practically equivalent to an attenuation of 3 db. This frequency is the reference point used for design ; it is called the theoretical cut-off frequency. Its value is given by†

$$f_1 = 1/(2\pi RC) \text{ c/s} \tag{2}$$

where R and C are in ohms and farads (or in megohms and microfarads). The value RC is called the **time constant** and is measured in seconds (see Sect. 4(iv)) so that

*i.e. not including a valve or generator.
†A nomogram to determine the value of f_1 is given by E. Frank " Resistance Capacitance Filter Chart " Elect. 18.11 (Nov. 1945) 164.

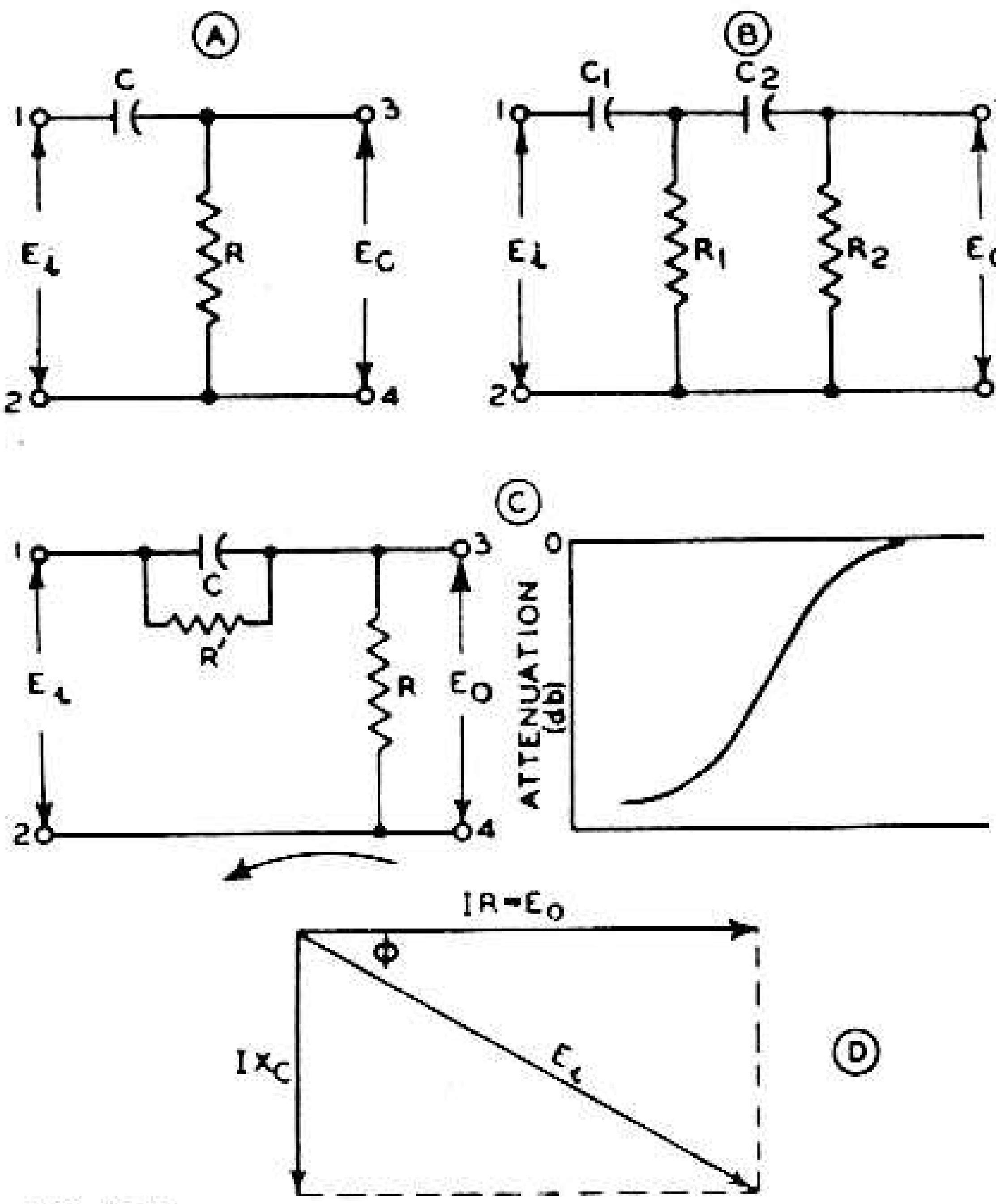


Fig. 4.35. *(A) is a resistance capacitance high-pass filter ; (B) is a two section filter ; (C) is a modified form of A, providing attenuation within top and bottom limits ; (D) is the vector diagram of A.*

the low frequency response of an amplifier is sometimes defined by specifying a time constant of so many microseconds. If the time constant is, say 3000 microseconds,

$$RC = 3000 \times 10^{-6} \text{ seconds,}$$

so that

$$f_1 = 1/(2\pi \times 3000 \times 10^{-6}) = 53 \text{ c/s.}$$

Fig. 4.36 shows the attenuation in db plotted against frequency for $R = 1$ megohm and selected values of C. It will be seen that the slope of the attenuation characteristics approaches 6 db per octave (i.e. the attenuation increases by 6 db every time the frequency is halved), and is very close indeed to this value for attenuations beyond 10 db. Each of these characteristics is exactly the same shape as the other, only moved bodily sideways.

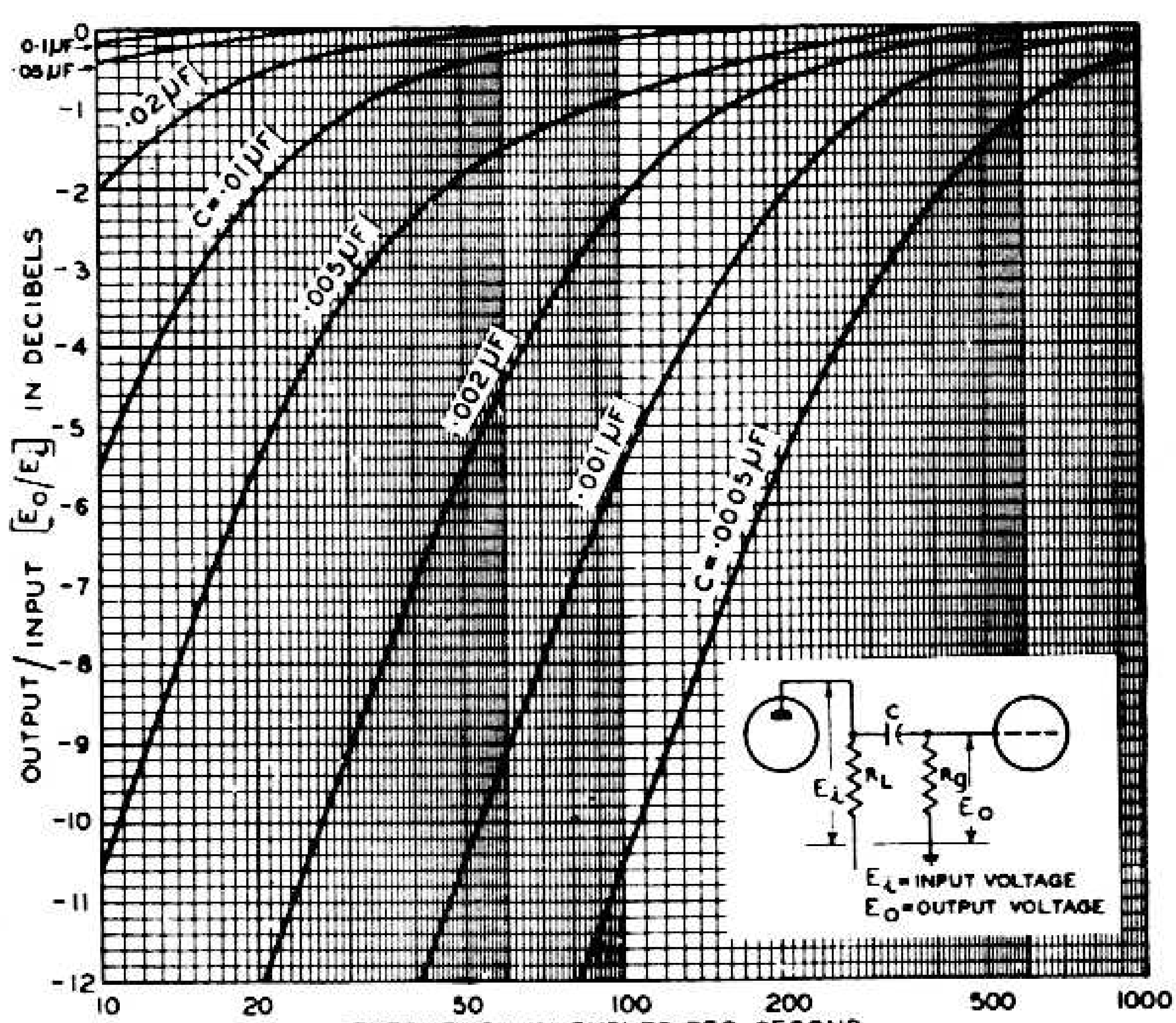


Fig. 4.36. *Attenuation in decibels versus frequency for a resistance-capacitance coupling or filter (Fig. 4.35A) in which the total resistance (R) = 1 megohm. If applied to a resistance-coupled valve amplifier,* $R = R_g + r_p R_L/(r_p + R_L)$. *If* r_p *is less than* 10 000 *ohms, the error in neglecting the second term is less than* 1%. *If the valve is a pentode, R may be taken as* $(R_g + R_L)$ *with a sufficient accuracy for most purposes.*

If R = 0.5 *megohm, multiply values of C by* 2 *and similarly in proportion for other resistances.*

One of these attenuation characteristics is shown in Fig. 4.36A together with the straight line AB which is the tangent to the curve and has a constant slope of 6 db/octave (or 20 db/decade). The point of intersection between AB and the zero db line is point A which corresponds to the theoretical cut-off frequency f_1. For ease in calculation, the "straight-line" approximate characteristic CAB is sometimes used in calculations in place of the actual attenuation characteristic, the maximum error being 3 db.

In Fig. 4.35(B) there are two such filters in cascade and further sections may also be added. A two-section filter in which $R_1 = R_2$ and $C_1 = C_2$ will have somewhat more than twice the attenuation in decibels of a single section filter, and the ultimate slope of the attenuation characteristic will approach 12 db/octave.

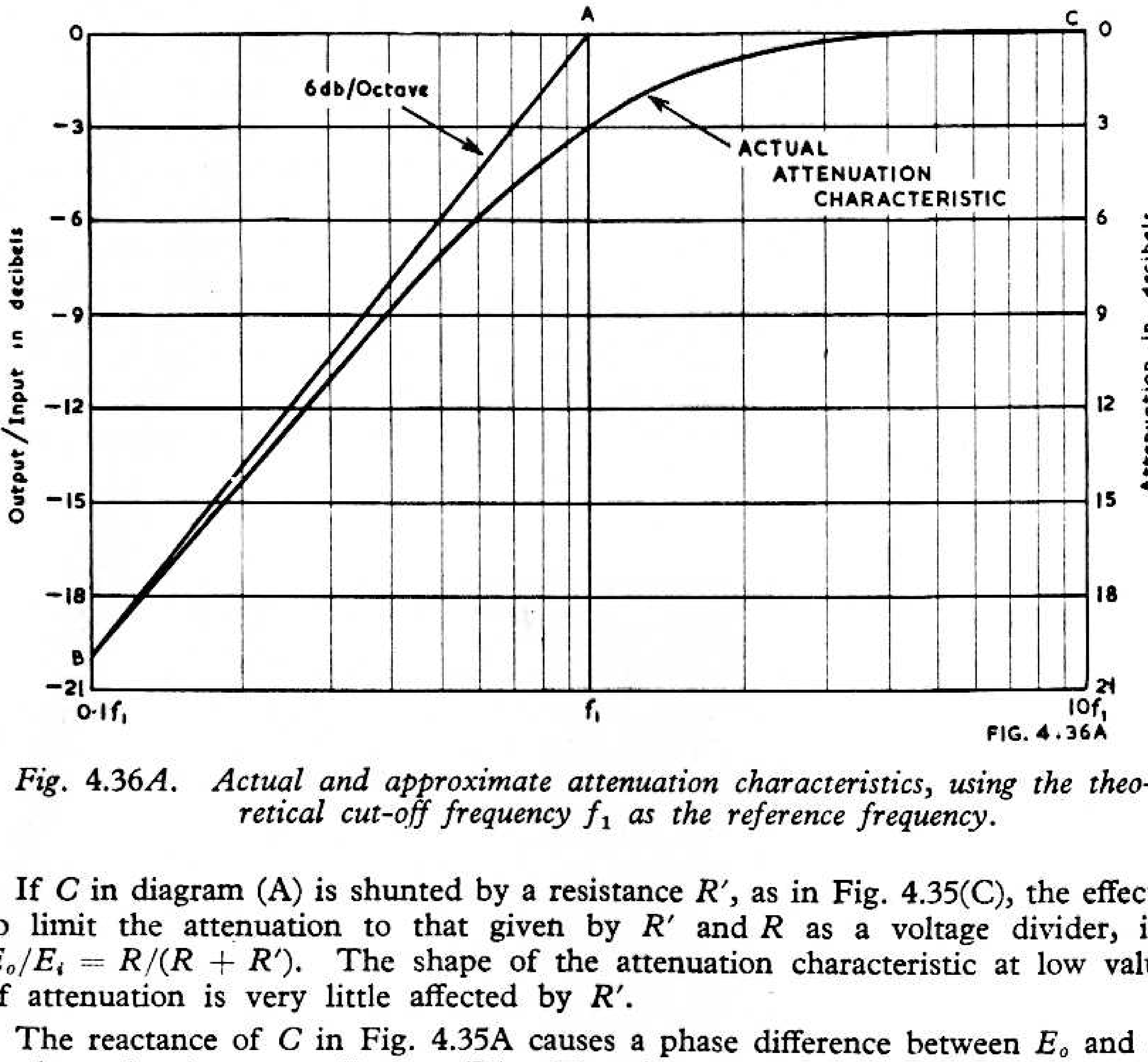


Fig. 4.36*A*. *Actual and approximate attenuation characteristics, using the theoretical cut-off frequency f_1 as the reference frequency.*

If C in diagram (A) is shunted by a resistance R', as in Fig. 4.35(C), the effect is to limit the attenuation to that given by R' and R as a voltage divider, i.e., $E_o/E_i = R/(R + R')$. The shape of the attenuation characteristic at low values of attenuation is very little affected by R'.

The reactance of C in Fig. 4.35A causes a phase difference between E_o and E_i, as shown by the vector diagram (D). The phase angle is given by

$$\cos \phi = E_o/E_i \quad (3)$$

For an attenuation of 3 db, $E_o/E_i = 0.707$ and $\phi = 45°$; thus E_o leads E_i by 45°.

Fig. 4.37 shows a typical **r.c. low-pass filter,** as used for tone control, decoupling in multistage amplifiers, or smoothing filters for power supplies when a large voltage drop in the filter is permissible. It is readily seen that this is the same as Fig. 4.35A except that R and C have been interchanged. The theoretical cut-off frequency f_1, at which $X_c = R$, is therefore unchanged and equal to $1/(2\pi RC)$. Provided that the generator impedance is zero, and that there is no load across the output terminals, the ratio of output to input voltages is given by

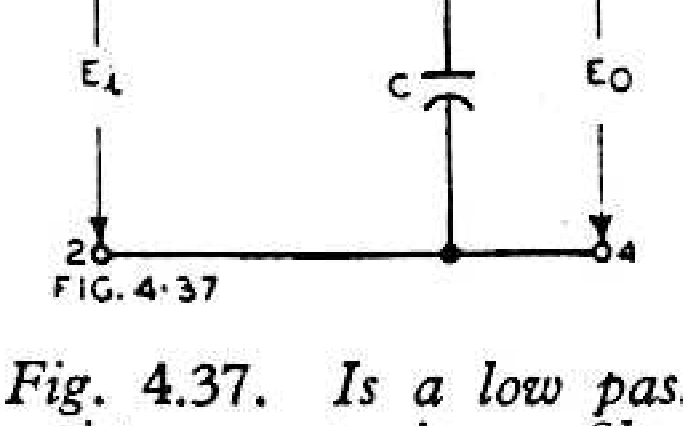


Fig. 4.37. *Is a low pass resistance-capacitance filter*

$$\frac{E_o}{E_i} = \frac{jX_c}{R + jX_c} \quad \text{where } X_c = -1/2\pi fC$$

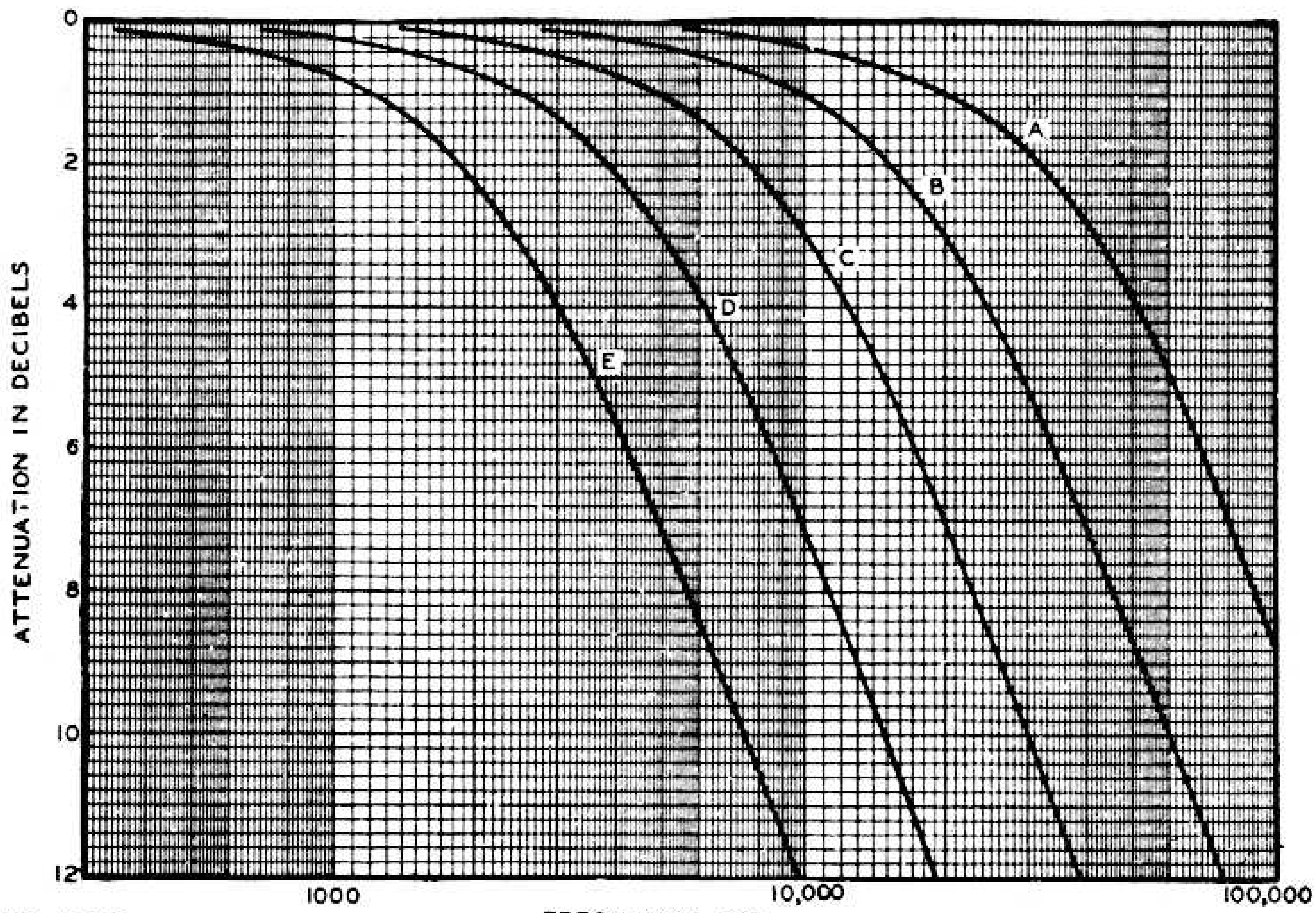


FIG. 4·38

Fig. 4.38. *Shows the attenuation characteristics for the low-pass filter of Fig.* 4.37 *for the values of R and C shown below*:

R (ohms)	*C* ($\mu\mu F$)				
	Curve A	*B*	*C*	*D*	*E*
400,000	10	20	40	80	160
200,000	20	40	80	160	320
100,000	40	80	160	320	640
50,000	80	160	320	640	1280
20,000	200	400	800	1600	3200
10,000	400	800	1600	3200	6400

Therefore $$\left|\frac{E_o}{E_i}\right| = \frac{X_c}{\sqrt{R^2 + X_c^2}} = \frac{1}{\sqrt{1 + (R/X_c)^2}} \qquad (4)$$

If the generator has a resistance R_g, then R in the eqn. (4) should include R_g.

Attenuation characteristics derived from this equation are given in Fig. 4.38 in a form capable of adaptation to most problems involving limited attenuation.

The rate of attenuation approaches 6 db/octave, and the shape of the curve is a mirror-image of that for the high-pass filter (Figs. 4.35A and 4.36).

If the filter is used as a smoothing filter, the ratio E_o/E_i is frequently 0.1 or less and under these conditions (with a maximum error of 1%)—

$$\left|\frac{E_o}{E_i}\right| \approx \frac{1}{2\pi fRC} \qquad (5)$$

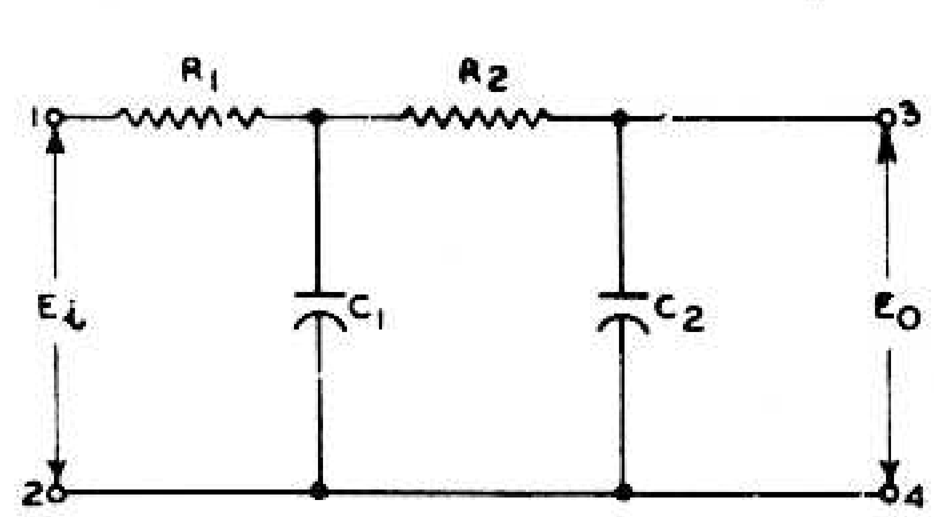


FIG. 4·39

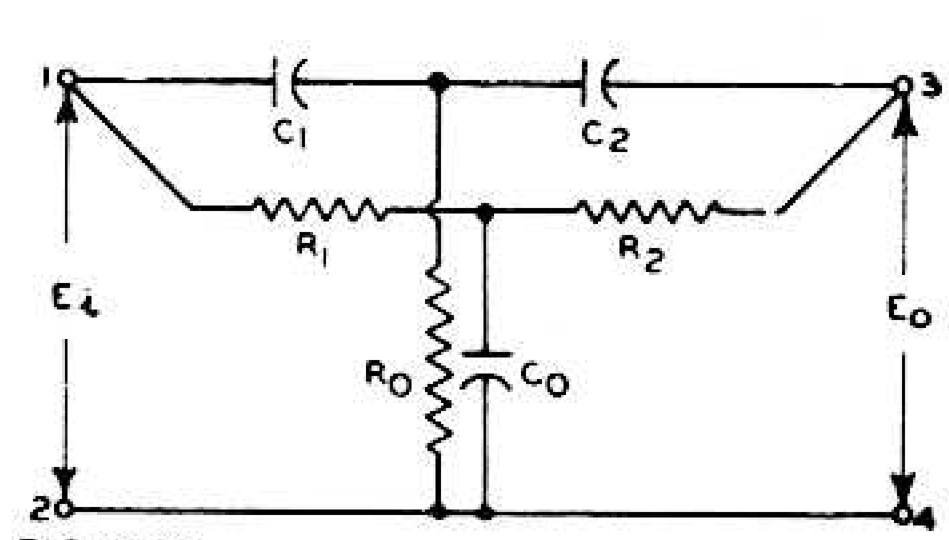


FIG. 4·40

Fig. 4.39 *is a two section low pass resistance-capacitance filter.*
Fig. 4.40 *is a Parallel T Network for the complete elimination of a particular input frequency from the output voltage.*

If additional filtering is required, it is possible to use two or more filter sections (Fig. 4.39). Provided that $R_1 > 10X_{c1}$; $R_2 > 10X_{c1}$ and $R_2 > 10X_{c2}$, the voltage ratio is given (with a maximum error of 3%) by

$$\left|\frac{E_o}{E_i}\right| \approx \frac{1}{40f^2 \times R_1R_2C_1C_2} \tag{6}$$

(iii) Special types of resistance-capacitance filters

An important method of eliminating a particular input frequency from the output voltage is the **Parallel T Network** as in Fig. 4.40. In the usual symmetrical form*,

$R_1 = R_2 = 2R_o$; $C_1 = C_2 = \frac{1}{2}C_o$; and $R_1 = 1/(2\pi f C_1)$;

infinite attenuation is obtained at a frequency f where

$$f = 1/(2\pi R_1 C_1) \tag{7}$$

provided that the output is unloaded.

The mathematical analysis of the general case has been published†, showing that the unsymmetrical case provides an improvement in the discrimination.

The case with finite generator resistance and terminated into a resistive load has also been analysed‡.

(iv) Iterative impedances of four-terminal networks

A four-terminal network or filter is shown in Fig. 4.41 with a generator E_g, having a series generator impedance Z_g, applied to the input terminals, 1, 2, and a load impedance Z_L across the output terminals 3, 4. The input impedance under these conditions, looking into the network from terminals 1, 2 is shown as Z_{k1}. The output impedance, looking into the network from terminals 3, 4 is shown as Z_{k2}. When Z_L is adjusted to be equal to Z_{k1} we have a special case which is of interest when similar filter sections are to be connected together in a chain so that each section is a load on the one preceding it. Under those conditions we call Z_{k1} an **iterative impedance** of the network

We must also adjust Z_g to be equal to Z_{k2}, this being the second iterative impedance.

If the network is a single T section, as in Fig. 4.42, and if $Z_1 = Z_2$, then

$$Z_{k1} = Z_{k2} = \sqrt{Z_1(Z_1 + 2Z_3)} \tag{8}$$

and the two iterative impedances are equal.

If the network is a single L section (as in Fig. 4.42 but with $Z_2 = 0$) then the iterative impedances are given by

$$Z_{k1} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_3} + \frac{Z_1}{2} \tag{8a}$$

$$Z_{k2} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_3} - \frac{Z_1}{2} \tag{8b}$$

If we now define a quantity P, called the iterative transfer constant, such that

$$P = \log_\epsilon (I_1/I_2) \tag{9a}$$

where I_1 = input current
and I_2 = output current
the formulae for the L section may be written in the alternative forms

$$Z_{k1} = \frac{Z_1\epsilon^P}{\epsilon^P - 1} = Z_3(\epsilon^P - 1) \tag{9b}$$

$$Z_{k2} = Z_{k1}\epsilon^{-P}$$

where $\cosh P = 1 + \frac{Z_1}{2Z_3} = \frac{Z_1 + 2Z_3}{2Z_3}$.

(See Chapter 38 Sect. 21 Table 73 for hyperbolic sines, cosines and tangents.)

*Scott, H. H., "A new type of selective circuit and some applications." Proc. I.R.E. 26.2 (Feb. 1938) 226.
†Wolf, A. "Note on a parallel-T resistance-capacitance network," Proc. I.R.E. 34.9 (Sept. 1946) 659. See also Hastings, A. E. "Analysis of a resistance-capacitance parallel-T network and applications," Proc. I.R.E. 34.3 (March 1946) 126P; McGaughan, H. S. "Variation of an RC parallel-T null network," Tele-Tech. 6.8 (Aug. 1947) 48.
‡Cowles, L. C. "The parallel-T resistance-capacitance network" Proc. I.R.E. 40.12 (Dec. 1952) 1712. Corr. 42.10 (Oct. 1954) 1547.

For a T network with iterative impedances,

$$P = \cosh^{-1}[(Z_1 + Z_2 + 2Z_3)/2Z_3] \qquad (10)$$

If a number of networks are connected in a chain, with each section having the same two iterative impedances Z_{k1} and Z_{k2} as Fig. 4.43, we may regard the combined networks as equivalent to a single network having iterative impedances Z_{k1} and Z_{k2} respectively. The iterative transfer constant (P) is then given by

$$P = (A_1 + A_2 + A_3) + j(B_1 + B_2 + B_3) \qquad (11)$$

where P_1 = propagation constant of the first section of the network
$= A_1 + jB_1$
$P_2 = A_2 + jB_2$ etc.
A_1 = attenuation constant (nepers)
and B_1 = phase constant (radians).

(v) Image impedances and image transfer constant of four-terminal networks

An alternative way of expressing the constants of a network is by the use of image impedances. A T network is shown in Fig. 4.44 with generator and load impedances Z_{I1} and Z_{I2} respectively. It is possible to adjust Z_A, Z_B and Z_C to give an input impedance (looking into the network from terminals 1, 2 with load connected) equal

Fig. 4.41 *shows a four terminal network with generator and load.*

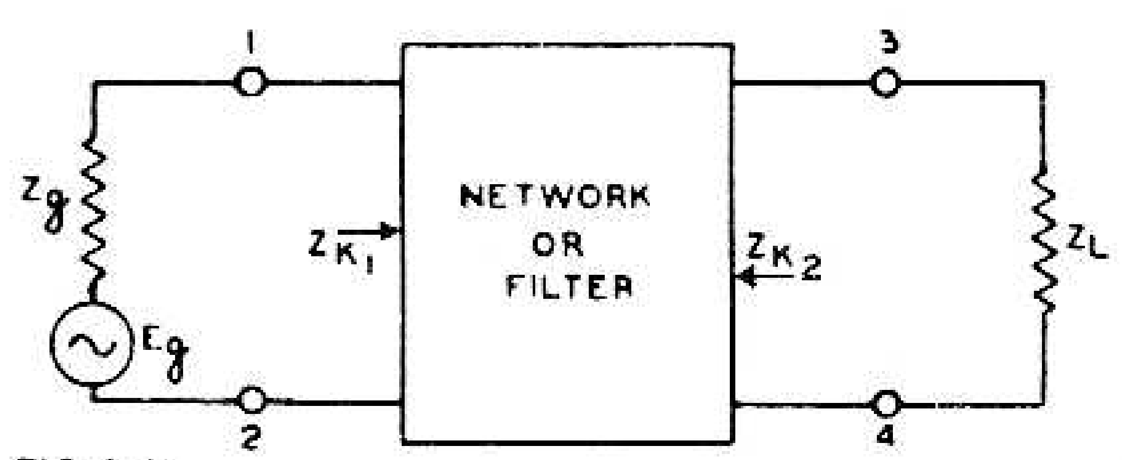


FIG. 4·41

Fig. 4.42 *is a single T section network applicable to Fig.* 4.41.

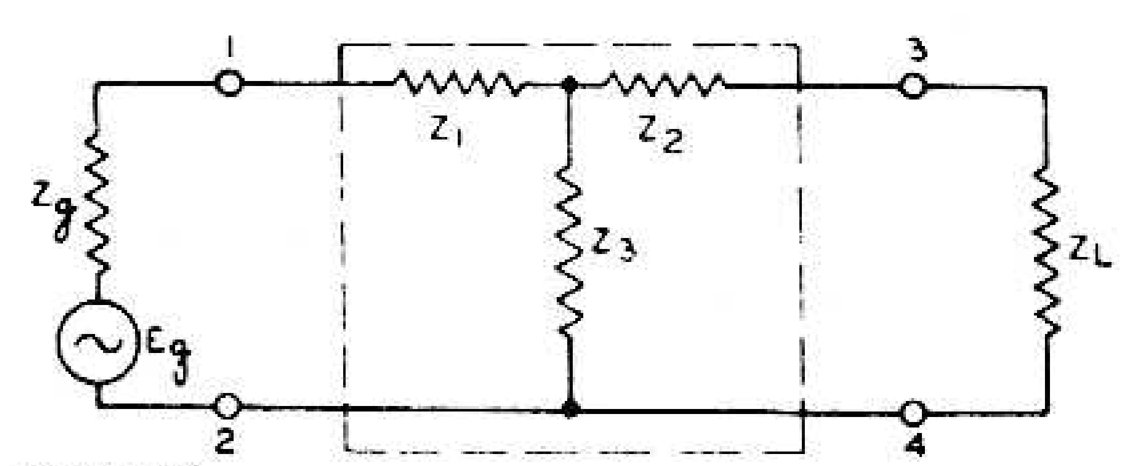


FIG. 4·42

Fig. 4.43 *is a combination of three networks in cascade, with iterative impedance relationships.*

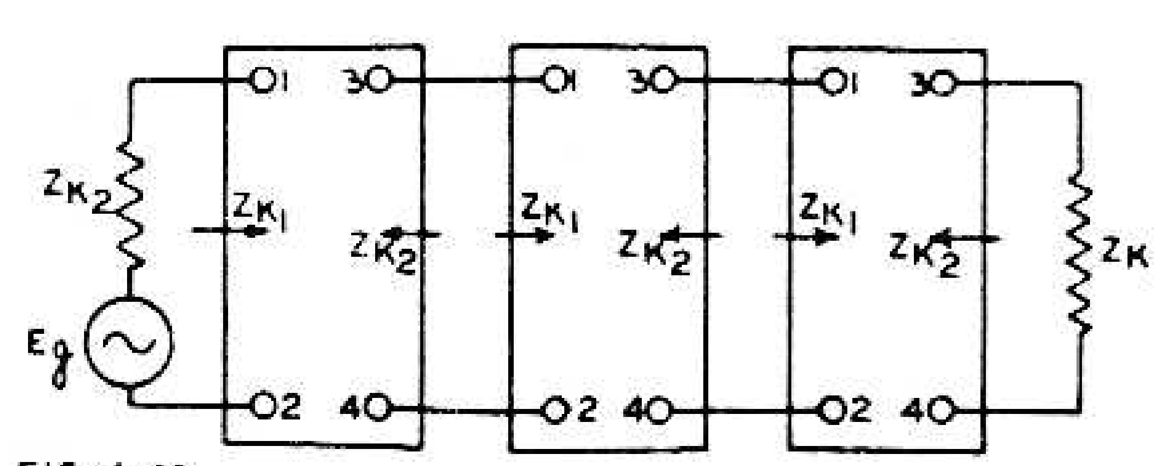


FIG. 4·43

Fig. 4.44 *is a single T section network terminated in its image impedances.*

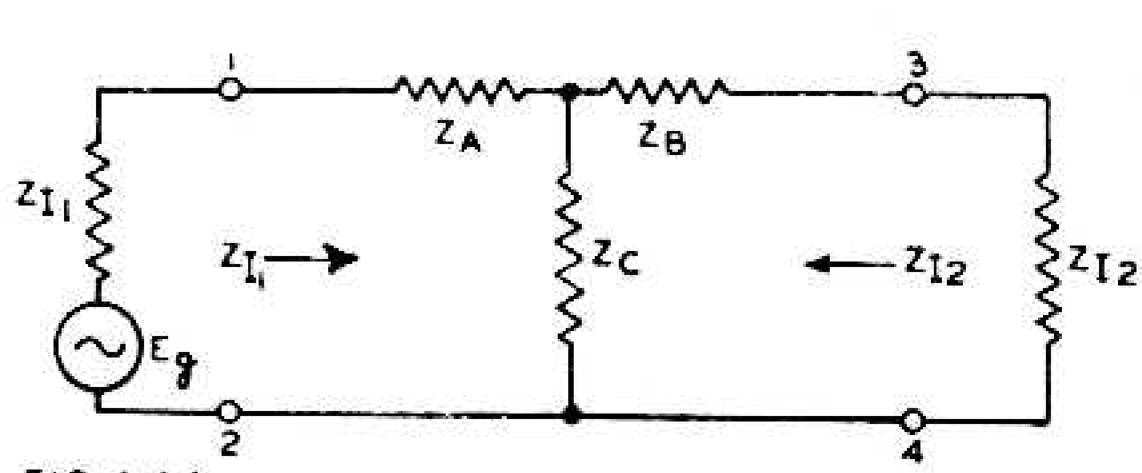


FIG. 4·44

to Z_{I1}, and at the same time to give an output impedance (looking from terminals 3, 4 with generator connected) equal to Z_{I2}. Under these conditions the impedance on each side of terminals 1, 2 is an " image " of the other (since they are both identical) and similarly with the impedances on each side of terminals 3, 4. The two image impedances are given by

$$Z_{I1} = \sqrt{\frac{(Z_A + Z_C)(Z_A Z_B + Z_A Z_C + Z_B Z_C)}{(Z_B + Z_C)}} \qquad (12a)$$

$$Z_{I2} = \sqrt{\frac{(Z_B + Z_C)(Z_A Z_B + Z_A Z_C + Z_B Z_C)}{(Z_A + Z_C)}} \qquad (12b)$$

Similarly a Π network as shown in Fig. 4.44A may have the values of the elements adjusted so that the network is terminated in its image impedances. The two image impedances are then given by

$$Z_{I1} = Z_1\sqrt{\frac{(Z_2 + Z_3)Z_3}{(Z_1 + Z_3)(Z_1 + Z_2 + Z_3)}} \qquad (13a)$$

$$Z_{I2} = Z_2\sqrt{\frac{(Z_1 + Z_3)Z_3}{(Z_2 + Z_3)(Z_1 + Z_2 + Z_3)}} \qquad (13b)$$

The two image impedances may also be expressed in terms of the open-circuit and short-circuit impedances (Sect.7(ix))

$$Z_{I1} = \sqrt{Z_{01}Z_{s1}} \qquad (14)$$

$$Z_{I2} = \sqrt{Z_{02}Z_{s2}} \qquad (15)$$

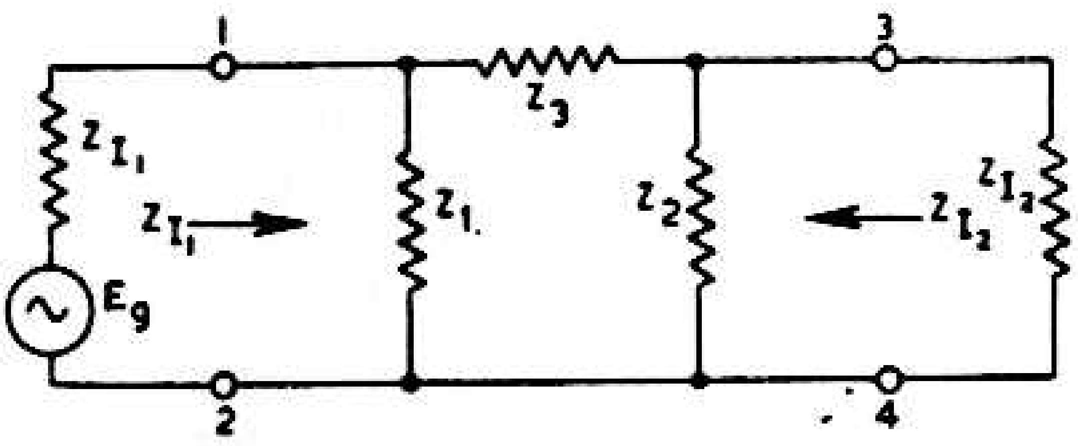


FIG. 4.44A

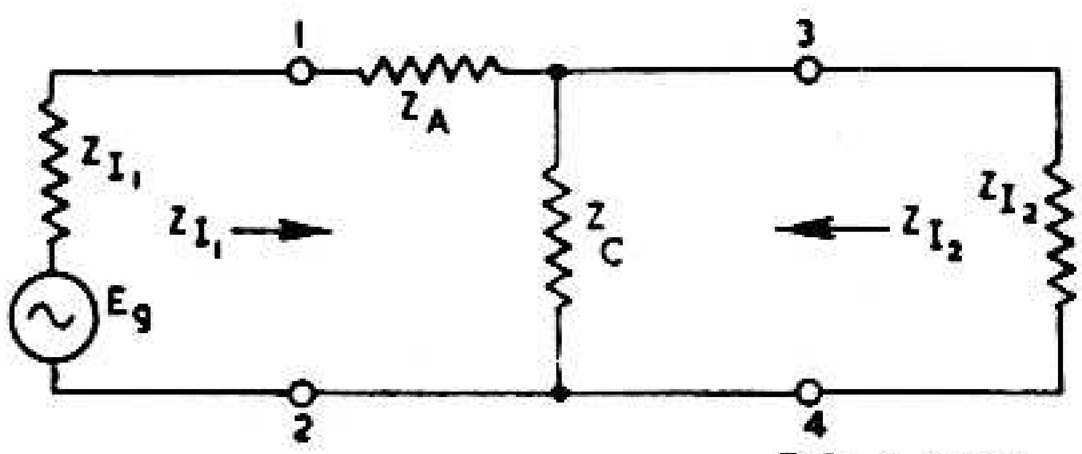


FIG. 4.44B

Fig. 4.44A is a single Π section network terminated in its image impedances.
Fig. 4.44B is a single L section network terminated in its image impedances.

The transfer of power is indicated by the **image transfer constant** θ whose value is given by

$$\theta = \tfrac{1}{2}\log_\epsilon \frac{E_1 I_1}{E_2 I_2} = \log_\epsilon \frac{I_1}{I_2}\sqrt{\frac{Z_{I1}}{Z_{I2}}} = \log_\epsilon \sqrt{\frac{I_1}{I_2}\cdot\frac{I_2'}{I_1'}} \qquad (16)$$

provided that the network is terminated in its image impedances,
where E_1 and I_1 are voltage and current at terminals 1, 2
E_2 and I_2 are voltage and current at terminals 3, 4
I_1' and I_2' are currents at terminals 1, 2 and 3, 4
respectively with transmission in the reversed direction.

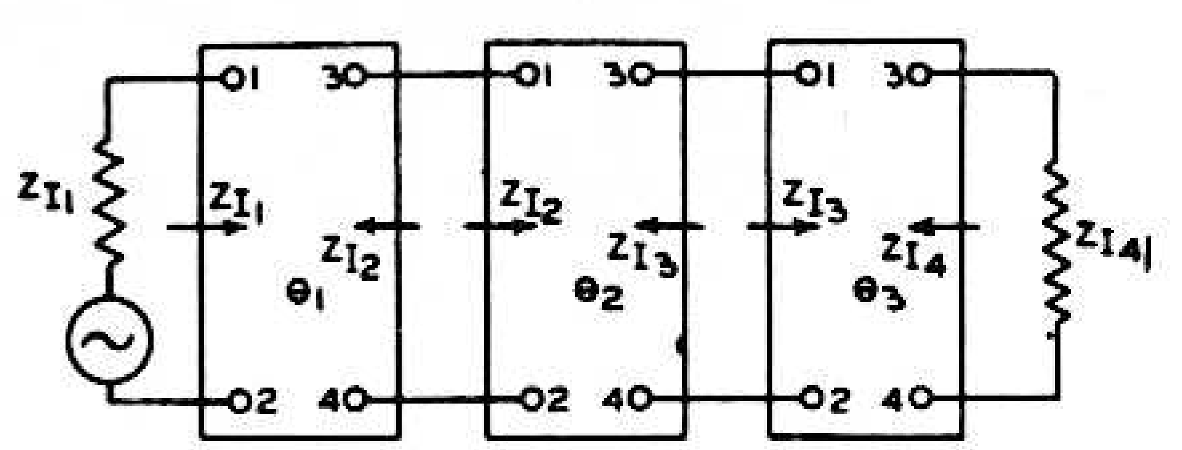


FIG. 4-45

Fig. 4.45 is a combination of three networks in cascade, with image impedance relationships.

When every section of a filter is working between its image impedances, there are no reflection effects. Fig. 4.45 shows a three section group connected on an image impedance basis. This is equivalent to a single network having image impedances Z_{I1} and Z_{I4} respectively,

and $\theta = (\alpha_1 + \alpha_2 + \alpha_3) + j(\beta_1 + \beta_2 + \beta_3) = \alpha + j\beta$ (16a)

where $\theta_1 = \alpha_1 + j\beta_1$, etc.

The real part $(\alpha_1 + \alpha_2 + \alpha_3 = \alpha)$ of the image transfer constant θ is called the **image attenuation constant,** and the imaginary part $(\beta_1 + \beta_2 + \beta_3 = \beta)$ is called the **image phase constant.**

Where the values of the elements of the T and Π networks are known, the value of the image transfer constant is given by
T Network (Fig. 4.44):

$$\tanh \theta = \sqrt{\frac{(Z_A Z_B + Z_A Z_C + Z_B Z_C)}{(Z_A + Z_C)(Z_B + Z_C)}} \quad (16b)$$

Π Network (Fig. 4.44A):

$$\tanh \theta = \sqrt{\frac{Z_3(Z_1 + Z_2 + Z_3)}{(Z_1 + Z_3)(Z_2 + Z_3)}} \quad (16c)$$

If the image impedances and transfer constant are known, the impedances of T, Π and L networks are given by
For T Section (Fig. 4.44):

$$Z_C = \frac{\sqrt{Z_{I1}Z_{I2}}}{\sinh \theta} \quad (17a)$$

$$Z_B = \frac{Z_{I2}}{\tanh \theta} - Z_C \quad (17b)$$

$$Z_A = \frac{Z_{I1}}{\tanh \theta} - Z_C \quad (17c)$$

For a Π Section (Fig. 4.44A):

$$Z_3 = \sqrt{Z_{I1}Z_{I2}} \sinh \theta \quad (17d)$$

$$Z_2 = \frac{1}{\dfrac{1}{Z_{I2} \tanh \theta} - \dfrac{1}{Z_3}} \quad (17e)$$

$$Z_1 = \frac{1}{\dfrac{1}{Z_{I1} \tanh \theta} - \dfrac{1}{Z_3}} \quad (17f)$$

For an L Section (Fig. 4.44B):

$$Z_A = \sqrt{Z_{I1}(Z_{I1} - Z_{I2})} \quad (17g)$$

$$Z_C = Z_{I2}\sqrt{\frac{Z_{I1}}{Z_{I1} - Z_{I2}}} \quad (17h)$$

$$\cosh \theta = \sqrt{(Z_{I1}/Z_{I2})} \quad (17i)$$

(vi) Symmetrical networks

When a network is symmetrical, that is when it may be reversed in the circuit with respect to the direction of propagation without alterations in the voltages and currents external to the network, the two iterative impedances become equal to each other and to the two image impedances:

$$Z_K = Z_{K1} = Z_{K2} = Z_I = Z_{I1} = Z_{I2} \quad (18)$$

(this is sometimes called the characteristic impedance)
Also $P = \theta = \alpha + j\beta$ (19)
where α = image attenuation constant
and β = image phase constant.

(vii) "Constant k" filters

A "constant k" filter is one in which

$$Z_1 Z_2 = k^2 \quad (20)$$

where Z_1 and Z_2 are the two arms of a filter section, and k is a constant, independent of frequency. Fig. 4.46 shows a symmetrical T type section terminated in its image impedances, which is a "constant k" filter provided that $Z_1 Z_2 = k^2$. The constant k^2 has the dimensions of a resistance squared, so that we replace k^2 by R^2 in the following analysis. This requirement is fulfilled when Z_1 and Z_2 are reciprocal reactances; the simplest case is when Z_1 is a capacitance with zero resistance and Z_2 an inductance with zero resistance or vice versa. Some popular combinations are given below :

Original (Z_1)	Reciprocal (Z_2)
L	$C = L/R^2$
$L + r$ in series	$C = L/R^2$ in shunt with R^2/r
$C + r$ in series	$L = R^2C$ in shunt with R^2/r
$L + C$ in series	$C' = L/R^2$ in shunt with $L' = R^2C$
$r + L + C$ in series	R^2/r in shunt with $C' = L/R^2$ and with $L' = R^2C$

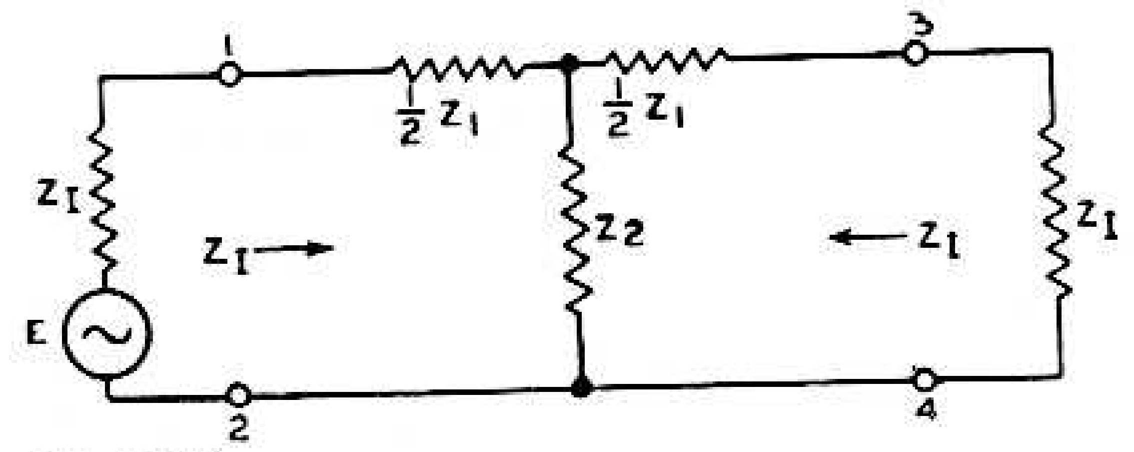


FIG. 4·46

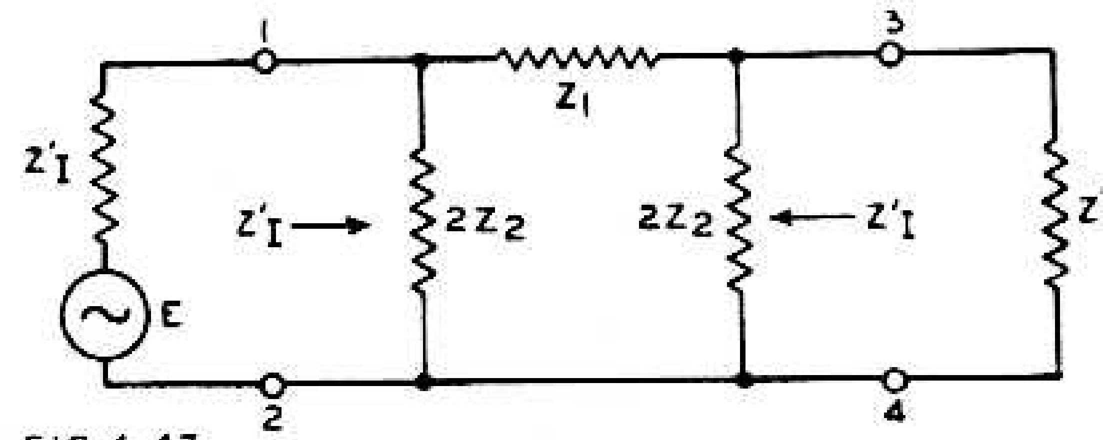


FIG. 4·47

Fig. 4.46 *is a symmetrical T section terminated in its image impedances.*
Fig. 4.47 *is a symmetrical Π section terminated in its image impedances.*

Z_I is called the **mid-series image impedance** in a symmetrical T section (Fig. 4.46) while Z_I' is called the **mid-shunt image impedance** in a symmetrical *Π* section (Fig. 4.47) :

T section : mid-series image impedance

$$Z_I = \sqrt{Z_1Z_2 + (Z_1{}^2/4)} = R\sqrt{1 + (Z_1/2R)^2} \tag{21}$$

Π section : mid-shunt image impedance

$$Z_I' = \sqrt{\frac{Z_1Z_2}{1 + Z_1/4Z_2}} = \frac{R}{\sqrt{1 + (Z_1/2R)^2}} \tag{22}$$

Therefore $Z_IZ_I' = R^2 = Z_1Z_2$ (23)

If Z_1 is a pure reactance (X_1),

then $Z_I = R\sqrt{1 - (X_1/2R)^2}$ (24)

and $Z_I' = \dfrac{R}{\sqrt{1 - (X_1/2R)^2}}$ (25)

The image transfer constant θ for either T or *Π* sections is given by

$$\cosh \theta = 1 + (Z_1/2Z_2) \tag{25a}$$

Half sections have exactly half the image transfer constant of a full section.

In the pass band : $\alpha = 0$ (25b)

$\cos \beta = 1 + (Z_1/2Z_2)$ (25c)

In the stop band : $\cosh \alpha = |\ 1 + (Z_1/2Z_2)\ |$ (25d)

Phase shift $= 0$ or $\pm\ 180°$ (25e)

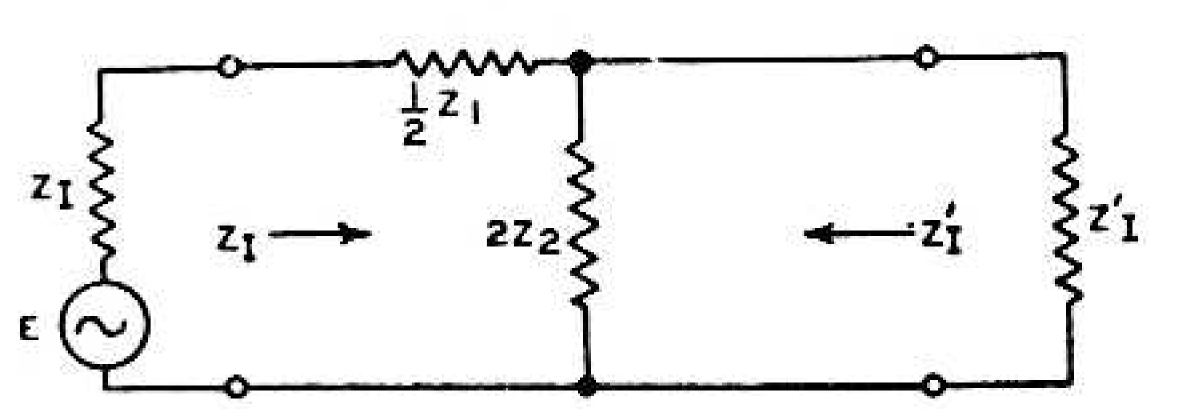


FIG. 4·48

Fig. 4.48 *is a half section terminated in its image impedances.*

Fig. 4.48 is a half-section terminated in its image impedances, which in this case are unequal. Two such half-sections, with the second one reversed left-to-right, are equivalent to a single T section.

$$Z_I = \sqrt{Z_1Z_2 + (Z_1{}^2/4)} \tag{25f}$$

$$Z_I' = \sqrt{\frac{Z_1Z_2}{1 + (Z_1/4Z_2)}} \tag{25g}$$

An ideal filter in which the reactances have zero loss has zero attenuation for all frequencies that make $(Z_1/4Z_2)$ between 0 and -1 ; this range of frequencies is called the **pass band.** All other frequencies are attenuated and are said to lie in the **stop band** (or attenuation band) of the filter.

Low pass filter—constant *k* type

Fig. 4.49 shows three forms of simple low-pass filters of the constant *k* type. In each case

$Z_1 = j\omega L$

and $Z_2 = 1/j\omega C$.

Now $Z_1 Z_2 = R^2$, therefore $L/C = R^2$, where R may have any convenient value. The mid-series image impedance is

$$Z_I = R\sqrt{1 - (\omega L/2R)^2} = R\sqrt{1 - (f/f_0)^2} \quad (26)$$

where $f_0 = 1/\pi\sqrt{LC}$, f_0 = cut-off frequency.

When $f = f_0$, $Z_I = 0$ and there is infinite attenuation if L and C have no resistance. The mid-shunt image impedance is

$$Z_I' = \frac{R}{\sqrt{(1 - (\omega L/2R)^2}} = \frac{R}{\sqrt{1 - (f/f_0)^2}} \quad (27)$$

When $f = f_0$, Z_I' becomes infinite.

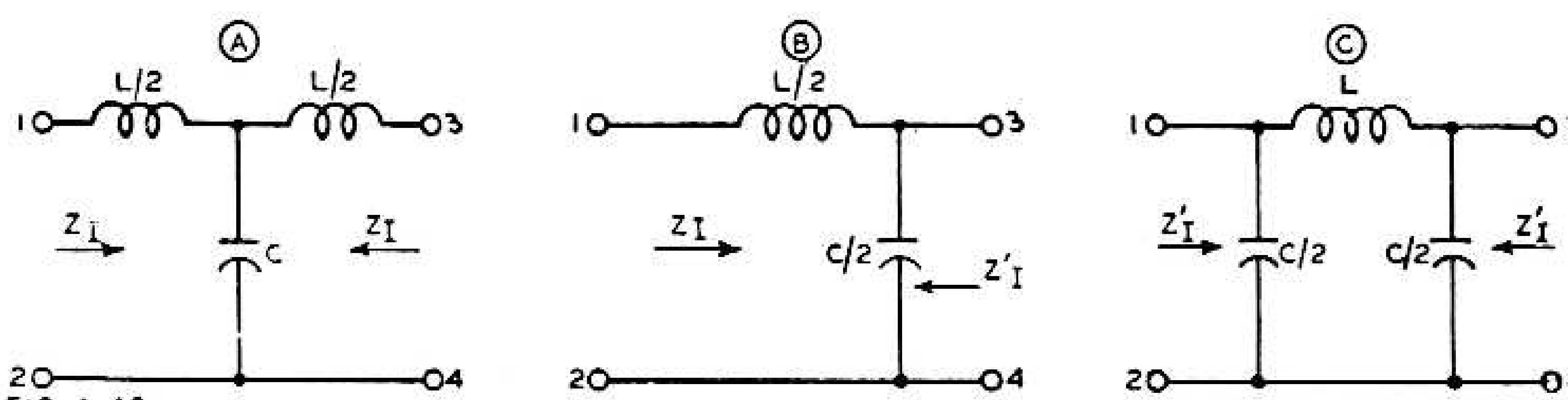


FIG. 4·49

Fig. 4.49. *Three varieties of low-pass constant k filters*: (*A*) *T section* (*B*) *Half section* (*C*) *Π section.*

With both T and Π arrangements, the ideal filter has zero attenuation for frequencies less than f_0, a sharp cut-off at f_0, and a very rapid attenuation immediately above f_0. However the rate of attenuation gradually falls as the frequency is increased, and approaches 12 db/octave for the single section at frequencies much greater than f_0.

Both image impedance characteristics are purely resistive below f_0 and purely reactive at higher frequencies.

The **phase shift** varies from zero at zero frequency to 180° at f_0, but is constant at 180° for all frequencies higher than f_0.

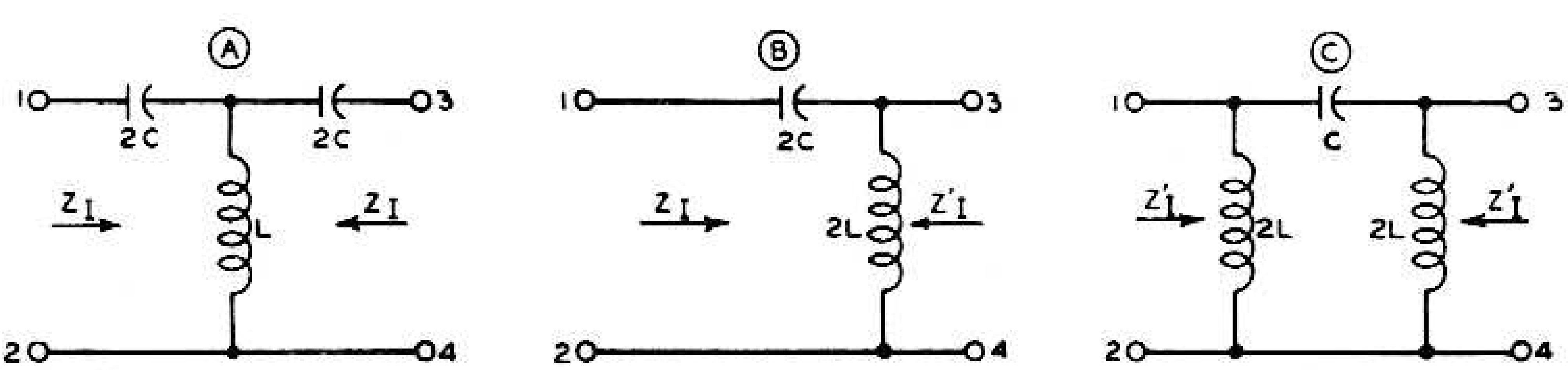


FIG. 4·50

Fig. 4.50. *Three varieties of high-pass constant k filters* (*A*) *T section* (*B*) *Half section* (*C*) *Π section.*

High-pass filter—constant *k* type

Fig. 4.50 shows three forms of simple high-pass filters of the constant *k* type In each case

$Z_1 = 1/j\omega C$

$Z_2 = j\omega L.$

Now $Z_1 Z_2 = R^2$, therefore $L/C = R^2$, where R may have any convenient value. The mid-series image impedance is

$$Z_I = R\sqrt{1 - (1/2R\omega C)^2} = R\sqrt{1 - (f_0/f)^2} \quad (28)$$

where $f_0 = 1/(4\pi\sqrt{LC})$, f_0 = cut-off frequency.

The mid-shunt image impedance is

$$Z_I' = \frac{R}{\sqrt{1-(1/2R\omega C)^2}} = \frac{R}{\sqrt{1-(f_0/f)^2}} \qquad (29)$$

With both T and Π arrangements, the ideal filter has zero attenuation for frequencies greater than f_0, a sharp cut-off at f_0, and a very rapid attenuation immediately below f_0. However, the rate of attenuation gradually falls as the frequency is decreased, and approaches 12 db/octave for the single section for frequencies much less than f_0.

The **phase shift** varies from nearly zero at very high frequencies to 180° at f_0, but is constant at 180° for all frequencies lower than f_0.

Both image impedance characteristics are purely resistive above f_0, and purely reactive at lower frequencies.

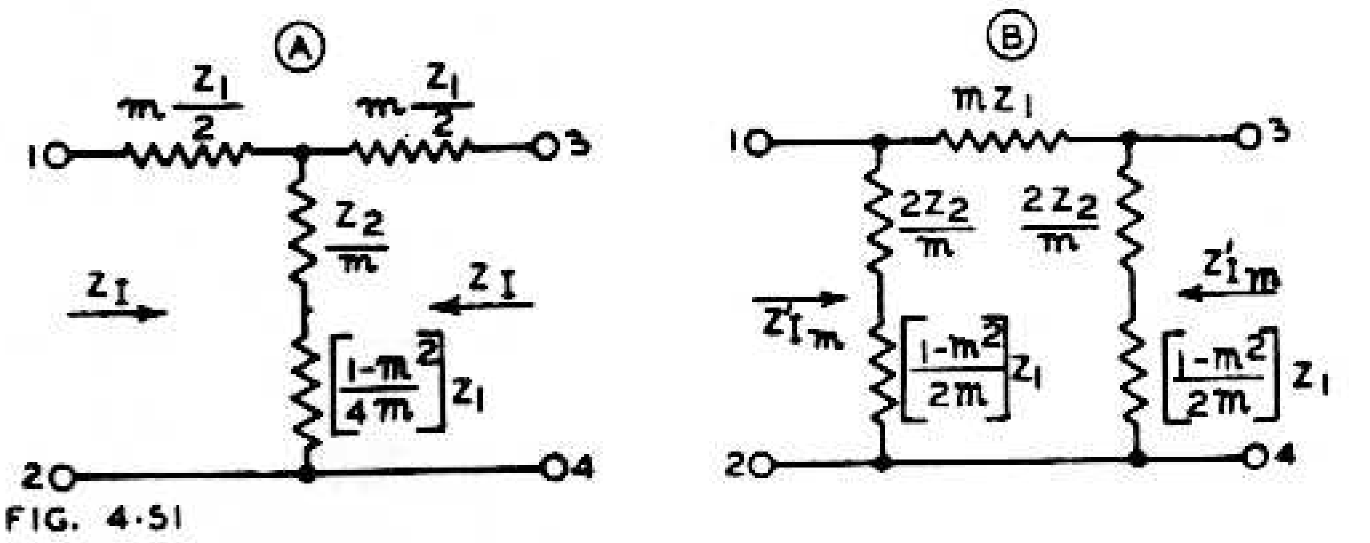


Fig. 4.51. *(A) T section (B) Π section, series m-derived filters.*

(viii) M-derived filters

This is a modified form of the constant k filter. Fig. 4.51A shows a T section series m-derived filter, which has the same value of Z_I as its prototype (Fig. 4.46 and eqn. 21). In fact, when $m = 1$, it becomes the prototype. In general, m may have any value between 0 and 1. It may be joined, at either end, to a constant k or m-derived section, or half-section, having an image impedance equal to Z_I. This derived filter has the same pass band and cut-off frequency f_0 as the prototype, but different attenuation characteristics with sharper cut-off and infinite attenuation at the resonant frequency of the shunt arm, provided that the elements have zero loss.

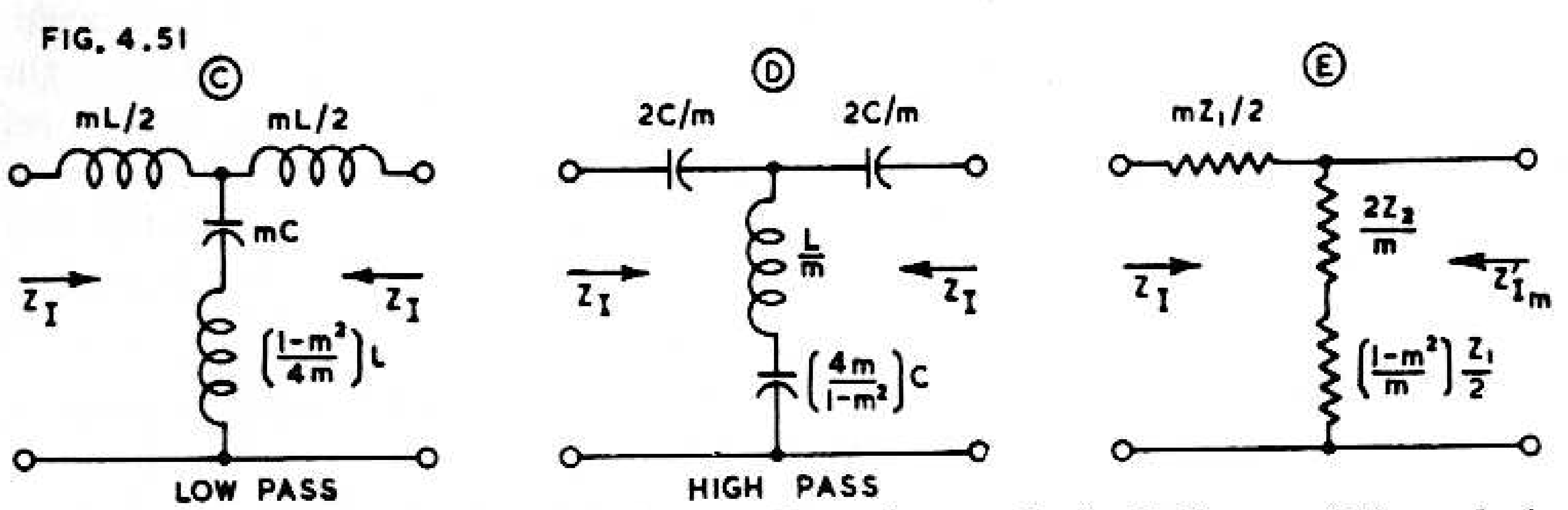


Fig. 4.51. *(C) Low pass (D) High pass T section m-derived filters (E) m-derived half-section for matching purposes.*

Values of Z_1 and Z_2 are as for the prototype (constant k) filter. A low-pass T section series m-derived filter is shown in Fig. 4.51C, and an equivalent high-pass section in Fig. 4.51D. In both cases the shunt arm becomes resonant at a frequency $f\infty$ given by

$$\text{Low pass} \quad f\infty = \frac{1}{\pi\sqrt{(1-m^2)LC}} \qquad (30)$$

$$\text{High pass} \quad f\infty = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}} \qquad (31)$$

In the theoretical case when the reactances have zero loss, the shunt arm will have zero impedance and therefore infinite attenuation at frequency $f\infty$.

The cut-off frequency f_0 is given by

$$f_0 = 1/(4\pi\sqrt{LC}) \qquad (32)$$

and the following relationships hold.

Low pass	**High pass**	
$f\infty = f_0/(\sqrt{1 - m^2})$	$f\infty = f_0\sqrt{1 - m^2}$	(33)
$m = \sqrt{1 - (f_0/f\infty)^2}$	$m = \sqrt{1 - (f\infty/f_0)^2}$	(34)

The frequency of "infinite" attenuation (sometimes called the peak attenuation frequency) may be controlled by varying the value of m, which variation does not affect the image impedances.

It is generally desirable, for good attenuation characteristics, that the ratio of the cut-off frequency to the frequency of peak attenuation in a high-pass filter should be as high as possible, and not less than 1.25, and in a low pass filter it should be as low as possible and not greater than 0.8. The value of m as given by eqn. (34) is 0.6 when the ratio is 1.25 or 0.8 respectively.

The equivalent Π section series m-derived filter is shown in Fig. 4.51B but here the mid-shunt image impedance is different, and this section cannot be connected at either end to a constant k or m-derived T section except through a half-section to match the respective image impedances.

$$Z_{Im}' = Z_I' \left[1 + (1 - m^2)(Z_1/2R)^2\right] \tag{35}$$

In other respects (B) has the same characteristics as (A). An m-type half-section for matching sections having different image impedances is Fig. 4.51E, the values shown being for matching a constant k section on the left and a series m-derived Π section on the right. The value of m should be approximately 0.6 to provide the most nearly constant value of image impedance in the pass band.

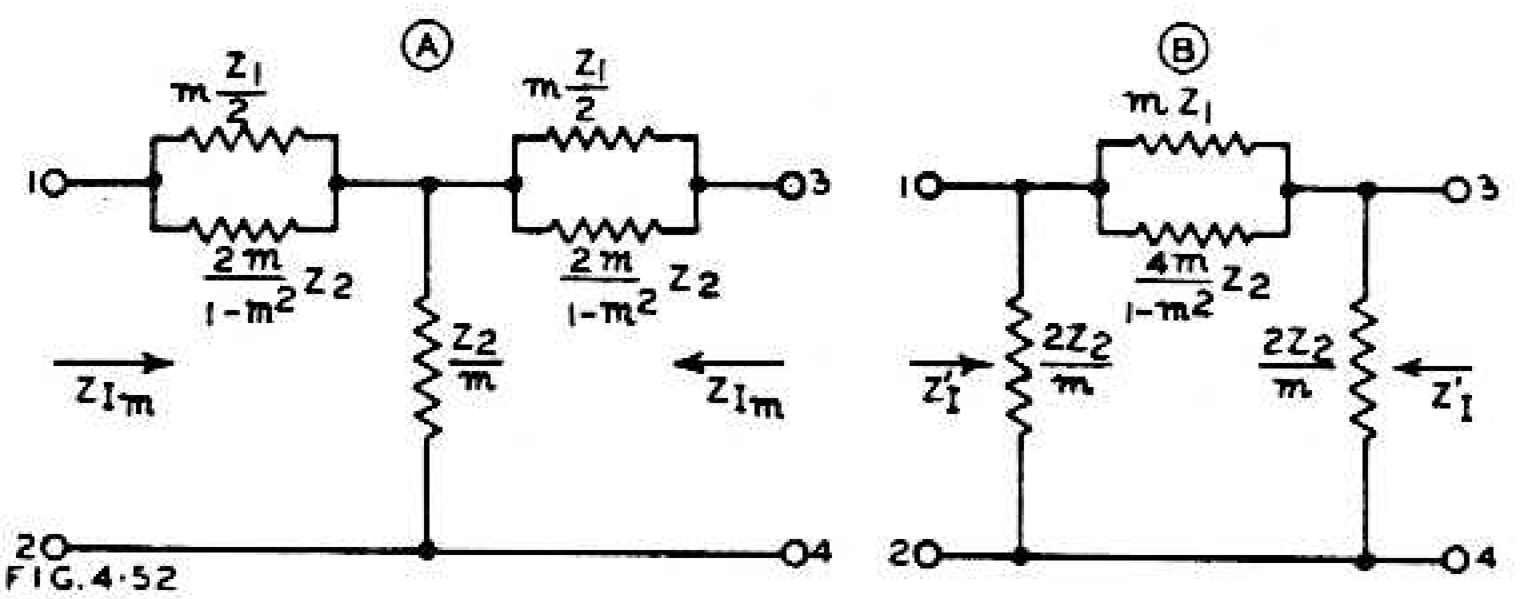


Fig. 4.52. (A) T section (B) Π section shunt m-derived filters.

Two forms of shunt m-derived filters are shown in Fig. 4.52. The Π section (B) may be joined at either end to a constant k section or half-section of mid-shunt image impedance Z_I'. The T section requires the medium of a half-section to match the impedances before being so connected.

$$\text{(A)}\ Z_{Im} = \frac{Z_I}{1 + (1 - m^2)(Z_1/4Z_2)} \tag{36}$$

(B) Z_I' is as for Fig. 4.47 (Eqn. 22)

$$\text{Therefore } Z_{Im}Z_{Im}' = Z_IZ_I' = R^2 \tag{37}$$

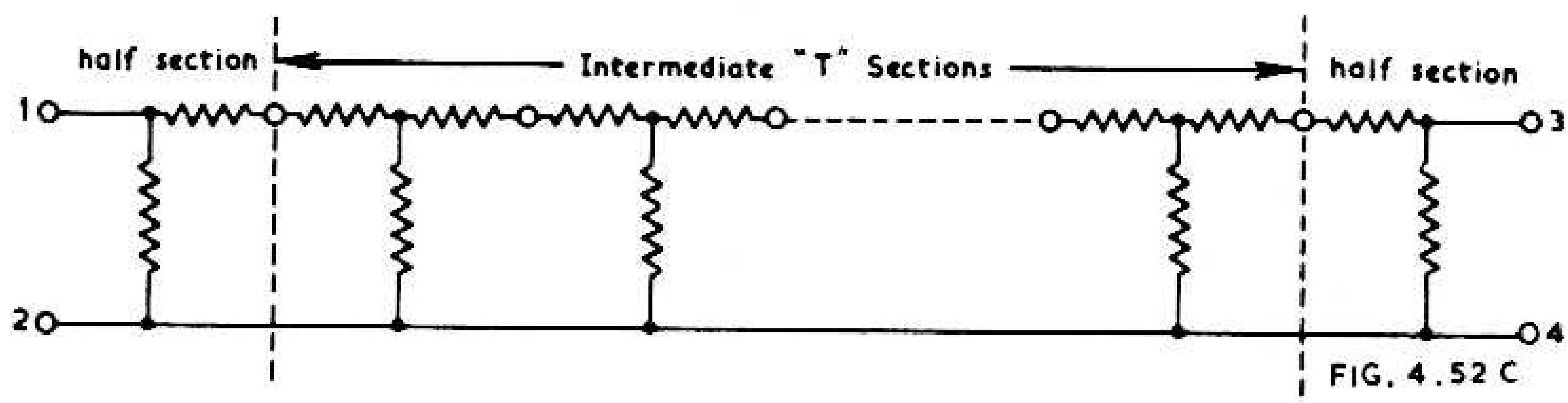
The design of multiple-section filters

A multiple-section filter is made up from any desired number of intermediate sections (either T or Π), and usually of the m-derived type, together with a terminal half-section at each end. All sections in the filter are matched at each junction on an image-impedance basis. The intermediate sections usually have different values of m such that frequencies which are only slightly attenuated by one section are strongly attenuated by another. The image impedances of these sections are far from constant over the pass band of the filter ; hence the necessity for using suitable terminal half-sections.

The terminal half-sections should be designed with a value of m approximately equal to 0.6 to provide the most nearly constant image impedance characteristics in the pass band ; with this value of m the image impedance is held constant within 4% over 90% of the pass band. Each of the terminal end sections should, however, be so designed that its frequency of peak attenuation is staggered with respect to the other, and the intermediate sections should then be staggered for the best overall attenuation characteristic.

The total attenuation or phase shift of the combined multi-section filter would be given by the sum of the attenuations or phase shifts respectively of the individual sections or half-sections.

When the design has been completed for all the sections in the filter, the practical form will be obtained by neglecting the junction points between sections and by adding together the values of the inductors in series in each arm, and those of the capacitors in parallel in each arm. If capacitors are in series, or inductors in parallel, the total effective value should be calculated and used in the practical form of the filter.



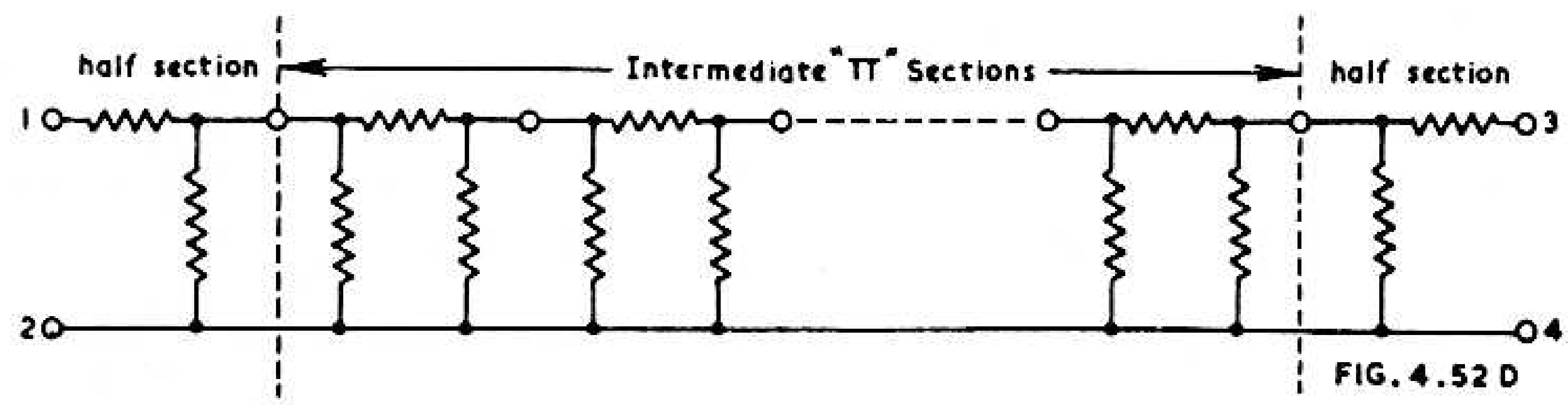


Fig. 4.52. *(C and D) Method of building up a multi-section filter using T or Π intermediate sections.*

For further information on the design of m-derived filters, reference may be made to Shea (Ref. 1) pp 244-285 ; Guillemin (Ref. 4) Vol. 2, pp. 324-338 ; Johnson (Ref. 6) pp. 192-195, 204-228, 293-303 ; Terman (Ref. 2) pp. 226-238 ; Everitt (Ref. 3) pp. 194-214 ; (Ref 6) pp. 130-152 ; (Ref. 7) pp. 6-33 to 6-62. References are listed in Sect. 8(xi).

(ix) Practical filters

In practice a filter network is usually terminated, not by its image impedance which is a function of frequency, but by a resistance of fixed value R where

$$R = \sqrt{Z_1 Z_2} \qquad (38)$$

As a result, the impedance mismatching causes some attenuation in the pass-band, which is increased further by the unavoidable losses in the inductors. Moreover, the attenuation in the attenuation-band is less than for the ideal case, and of course never reaches infinity except in the case of a null network.

(x) Frequency dividing networks

Frequency dividing networks are of two types, the filter type (which has only approximately constant input impedance) and the constant-resistance type.

With either type, the **cross-over frequency** (f_c) is the frequency at which the power delivered to the two loads is equal. This occurs with an attenuation of 3 db for each load, with an ideal dividing network having no loss.

The nominal **attenuation per octave** beyond the crossover frequency may be :

6db : available with constant-resistance, but attenuation not sufficiently rapid for general use with loudspeaker dividing networks (Fig. 4.53A)

$L_0 = R_0/(2\pi f_c)$ $\qquad C_0 = 1/(2\pi f_c R_0)$

12db : available with either type, and very suitable for general use with loudspeaker dividing networks. Fig. 4.53B shows the constant-resistance type. This is a very popular ar-angement.

$L_1 = R_0/(2\sqrt{2}\pi f_c)$ $\qquad L_2 = R_0/(\sqrt{2}\pi f_c)$

$C_1 = 1/(\sqrt{2}\pi f_c R_0)$ $\qquad C_2 = 1/(2\sqrt{2}\pi f_c R_0)$

The filter type has identical circuit connections but the condensers and inductors are unequal (see Chapter 21 Sect. 3).

18db : available with filter type. This is the maximum rate of attenuation normally used with loudspeaker dividing networks (Fig. 4.53C).

$L_4 = R_0/(2\pi f_c)$	$L_3 = (1 + m)R_0/(2\pi f_c)$
$L_6 = 2R_0/(2\pi f_c)$	$L_4 = R_0/(2\pi f_c)$
$L_7 = R_0/(1 + m)(2\pi f_c)$	$L_5 = R_0/(4\pi f_c)$
$C_5 = 1/(2\pi f_c R_0)$	$C_3 = 1/(\pi f_c R_0)$
$C_6 = 1/(4\pi f_c R_0)$	$C_4 = 1/(1 + m)(2\pi f_c R_0)$
$C_7 = (1 + m)/(2\pi f_c R_0)$	$C_5 = 1/(2\pi f_c R_0)$

(L in henrys ; C in farads)

See also Chapter 21 Sect. 4.

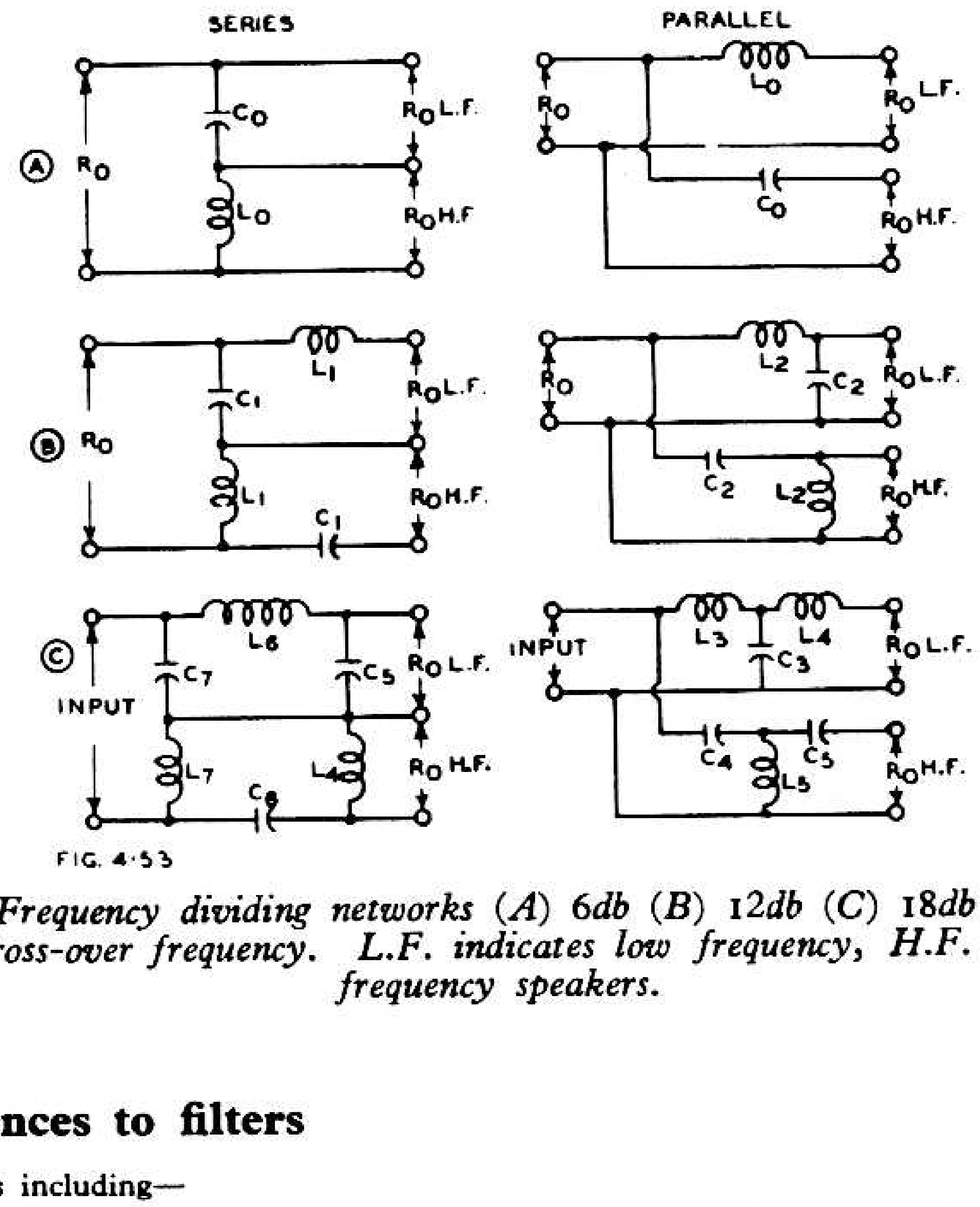


Fig. 4.53. Frequency dividing networks (A) 6db (B) 12db (C) 18db for the octave beyond the cross-over frequency. L.F. indicates low frequency, H.F. indicates high frequency speakers.

(xi) References to filters

Many textbooks including—

1. Shea, T. E. " Transmission Networks and Wave Filters " (D. van Nostrand Company, New York, 1943).

2. Terman, F. E. " Radio Engineers' Handbook " (McGraw-Hill Book Company, New York and London, 1943).

3. Everitt, W. L. " Communication Engineering " (McGraw-Hill Book Co. Inc. New York and London, 1937).

4. Guillemin, E. A. " Communication Networks " Vols. 1 and 2 (John Wiley and Sons Inc. New York ; Chapman and Hall Ltd. London, 1931).

5. Johnson, K. S. " Transmission Circuits for Telephonic Communication " (D. van Nostrand Co. Inc. New York, 1931).

6. " Reference Data for Radio Engineers " (Federal Telephone and Radio Corporation, 3rd ed. 1949).

7. Pender, H., and K. McIlwain " Electrical Engineers' Handbook : Electric Communication and Electronics " (John Wiley and Sons, Inc. New York ; Chapman and Hall Ltd., London, 1950).

Charts for the prediction of audio-frequency response :

Crowhurst, N. H. " The prediction of audio-frequency response " Electronic Eng. No. 1—Circuits with single reactance element, 23.285 (Nov. 1951) 440. No. 2—Circuits with two reactive elements, 23.286 (Dec. 1951) 483 ; 24.287 (Jan. 1952) 33 ; 24.288 (Feb. 1952) 82. No. 3—Single complex impedance in resistive network, 24.291 (May 1952) 241. No. 4—Step circuits 24.293 (July 1952) 337.

See Supplement for additional references.

SECTION 9: PRACTICAL RESISTORS, CONDENSERS AND INDUCTORS

(i) Practical resistors (ii) Practical condensers (iii) Combination units (iv) Practical inductors (v) References to practical resistors and condensers.

(i) Practical resistors

Resistors are in two main groups, wire-wound and carbon, although each group is subdivided. Wire-wound resistors are available with ordinary and non-inductive windings. Nichrome wire is suitable for operation at high temperatures but has a large temperature coefficient; it is suitable for use with ± 5% or 10% tolerances. Advance (or constantan, or eureka) is limited in its operating temperature, but is used for tolerances of about ± 1%. Wire-wound resistors are also graded by the type of coating material.

Carbon resistors are divided into insulated and non-insulated types, while the resistance material may be composition or cracked carbon (high stability). The resistance material may be either solid (e.g. rod) or in the form of a film.

For American and English standard specifications on resistors see Chapter 38 Sect. 3(i) and (ii), including standard resistance values. For Colour Codes see Chapter 38 Sect. 2.

(a) Tolerances

Every resistor has tolerances in resistance, and the price increases as the percentage tolerance is made smaller. Composition resistors are usually obtainable with the following tolerances:

± 5% For critical positions only
± 10% Desirable for semi-critical use in radio receivers and amplifiers (e.g., plate, screen and bias resistors)
± 20% For non-critical positions only (e.g. grid resistors).

"High stability" carbon resistors are available with resistance tolerances of ± 5%, ± 2% and ± 1% (Ref. A26).

Wire-wound resistors are available with almost any desired tolerances in resistance (± 5%, 10% are usual values in radio receivers).

Comment on tolerances in components

When a manufacturer of resistors or capacitors selects simultaneously for large quantities of each of three tolerances, ± 5%, ± 10% and ± 20%, there is a distinct possibility that the ± 10% tolerance group may be nearly all outside the ± 5% tolerances, and therefore in two "channels" differing by more than 10%. Similarly with ± 10% and ± 20% tolerances. It is therefore good engineering practice to design on the expectation of a large percentage of components lying close to the two limits.

(b) Stability

The resistance of carbon resistors tends to drift with time. Ordinary composition resistors may drift as much as ± 2% during storage for 3 months at 70°C and normal humidity (Ref. A27). See also Ref. A35.

Some high stability carbon resistors are limited to a maximum change in resistance of ± 0.5% after 3 months' storage at 70°C (Ref. A26).

(c) Dissipation

Composition resistors are usually available with nominal dissipation ratings of ¼, ½, 1, 2, 4 and 5 watts (JAN-R-11). The English RIC/113 standard (Ref. A27) includes ratings of 1/10, ¼, ½, ¾, 1 and 2.5 watts. Other manufacturers produce ⅓, ⅕, 1½ and 3 watt ratings.

English **high stability resistors** are available with 1/10, ⅛, ¼, ½, ¾, 1, 1½ and 2 watt ratings (Ref. A26).

In accordance with the American R.M.A. Standard and JAN-R-11 characteristics *A*, *B*, *C* and *D* [see Chapter 38 Sect. 3(i)], composition resistors may be used at their maximum ratings with **ambient temperatures** up to 40°C.

The JAN-R-11 characteristic *G* and some English resistors (Refs. A3, A26, A27) may be used with maximum ratings up to an ambient temperature of 70°C. In one case (Ref. A27) higher ratings than the nominal value are permitted below 70°C ambient temperature (see below).

At ambient temperatures greater than the maximum rating, resistors may only be used with reduced dissipation and maximum voltage, in accordance with **derating curves**.

The following is a typical power dissipation derating curve for the temperature limits 40°C and 110°C (JAN-R-11 types, *A*, *B*, *C*, *D*) :

Temperature	40°	50°	60°	70°	80°	90°	100°	110°
Dissipation	100%	83%	66%	50%	33%	17%	5%	0

The following is the English derating curve for high stability Grade 1, RIC/112 (Ref. A26) :

Temperature	70°	80°	90°	100°	110°	120°	130°	140°	150°
Dissipation	100%	87.5%	75%	62.5%	50%	37.5%	25%	12.5%	0

The following is the English rating and derating curve for Grade 2, RIC/113 (Ref. A27) :

Temperature	40°	50°	60°	70°	80°	90°	100°	110°
Dissipation	175%	150%	125%	100%	75%	50%	25%	0

A typical **voltage de-rating curve** is the American R.M.A., see Chapter 38 Sect. 3(i).

It is good engineering practice to select a composition resistor, for any particular application, such that the actual dissipation is about 60% of the resistor rated dissipation, after making allowance for any de-rating due to high ambient temperature.

Wirewound resistors are usually available with nominal dissipation ratings of ¼, ½, 1, 2, 3, 5, 8, 10, 12, 16, 20 watts and upwards. These are also normally available with tappings.

Wirewound resistors, vitreous enamelled, are usually available in dissipation ratings of 5 or 10 watts and upwards. These are also normally available with tappings.

Adjustable voltage dividers are manufactured in 8 or 10 watt dissipation ratings, together with larger sizes.

" **Radio** " **voltage dividers** usually have resistances of 25,000 and 15,000 ohms suitable for connection across 250 volts. The maximum dissipation is something like 5 watts.

(d) Voltage ratings

With low resistances, the applied voltage is always limited by the permissible dissipation. With high resistances there is an additional condition to be met in the maximum voltage rating. In general the maximum voltage is between 250 and 500 volts for ¼ to 1 watt ratings, although there are a few below 250 volts, some (½ watt and over) with maximum voltages of 700 volts or more, and some (¾ watt and over) with maximum voltages of 1000 volts or more.

(e) Temperature rise

A temperature rise of from 40°C to 62°C (with 40°C ambient temperature) is to be expected with ordinary composition resistors at maximum ratings. English resistors which are rated at an ambient temperature of 70°C have a maximum surface temperature not greater than 120°C (Grade 2) or 150°C (Grade 1, high stability) (Refs. A26, A27).

A typical small wire-wound resistor may have a maximum temperature of 110°C for 1 watt or 135°C for 2 watts dissipation.

A typical large bare or organic-coating wire-wound resistor has a maximum surface temperature from 170°C to 220°C.

A typical lacquered wire-wound resistor has a maximum surface temperature of about 130°C.

A typical vitreous-enamelled resistor normally has a maximum surface temperature from 200°C to 270°C, but even higher temperatures are sometimes used.

High dissipation resistors should be mounted vertically to allow good air circulation, and should be spaced well away from other resistors or components. When mounted in a confined space they should be used at about half the rated maximum dissipation.

(f) Effect of temperature on resistance

The resistance of composition resistors always tends to rise as the temperature is reduced below 20° or 25°C, but at higher temperatures the resistance may either rise or fall, or may fall and then rise (for curves see Ref. A14).

For a temperature rise from 25° to 80°C, the change of resistance is usually under ± 7% for low values of resistance.

For the same rise of temperature, high stability resistors have a change of resistance not greater than + 0 − 2.2% for resistances up to 2 megohms (1 and 2 watts, Ref. A26). See also Ref. A36.

The resistance of composition resistors is also affected by a temporary severe change in ambient temperature, followed by return to normal (Ref. A35).

The resistance of a wire-wound resistor generally increases as the temperature is increased, the change being usually not more than 0.025% per °C in the case of low temperature units.

See also Chapter 38 Sect. 3(i) and (ii) for standard specifications.

The effect of soldering

Small (¼ watt) composition resistors are subject to as much as 3% change in resistance, approximately half of which may be permanent, due to soldering; larger dissipation resistors are usually not affected more than 1% (Ref. A24). The maximum permissible change in resistance due to soldering is ± 2% (Class 2) or ± 0.3% (Class 1, high stability) for resistors coming under the English RIC Specifications (Refs. A26, A27).

(g) Effect of voltage on resistance

The resistance of a composition resistor decreases when the voltage applied across it increases. The percentage change increases as the resistance increases. For a 1 megohm resistor, a typical percentage fall in resistance is given by (Erie Resistor Co.):

Size	¼	½	1	2	watts
Voltage from zero to	200	350	500	500	volts
Fall in resistance	2.1	2.5	1.3	1.5	%

The voltage coefficient is defined as (Ref. A27):

$$\text{Voltage coefficient} = \frac{100\,(R_1 - R_2)}{R_2\,(E_1 - E_2)}$$

where R_1 and E_1 are the resistance and voltage respectively at the normal maximum rating, and R_2 and E_2 are the values at one-tenth of E_1. Limiting values of the voltage coefficient are (Ref. A27):

0.025% per volt for values below 1 megohm
0.05% per volt for values above 1 megohm.

This effect is much reduced by the use of " high stability " resistors, a typical value being 0.4% fall in resistance for 1 megohm, with voltage change from zero to 500 volts (Dubilier). The limiting value of the voltage coefficient for Specification RIC/112 (Ref. A26) is 0.002% per volt.

Some applications require a resistor whose resistance falls as the applied voltage is increased; a wide range of characteristics is available (e.g. Carborundum " Globar" ceramic resistors).

See Chapter 38 Sect. 3(i) for standard specifications.

(h) Effect of humidity on resistance

The effect of humidity is to increase the resistance by up to about 3% under normal conditions. Extreme tropical humidity may cause an increase in resistance generally less than 10%. Some insulated resistors have less than 1% change in humidity due to humidity tests (e.g., I.R.C. type BTA). See Chapter 38 Sect. 3(i) for humidity tests and limits.

(i) Capacitance of resistors

Every resistor has a capacitance which, at the lower radio frequencies, may be considered as a capacitance between the two ends of the resistor (usually between 0.1 and 1.0 $\mu\mu$F for composition resistors). This capacitance may usually be neglected in normal applications in radio receivers. At higher frequencies it is necessary to consider the capacitance as being distributed along the resistance element. This leads to a reduction in resistance which, unlike the end-to-end capacitance, is not removable by tuning [see (k) below]. References A5, A6, A7, A8, A9, A14.

(j) Inductance of resistors

Every resistor has an inductance, partly due to the inductance of the resistor itself and partly due to the leads. However, experience shows that at high frequencies the effect of the inductance is negligible compared with the effects of capacitance.

Typical values of inductance of composition resistors (Dubilier) are given below :

Resistance		100 ohms	1 megohm	
Inductance	¼ watt	0.0007	0.06	μH
	1 watt	0.017	2.0	μH

Wire-wound resistors have much greater inductance than composition types, but where they must be used it is possible to adopt a " non-inductive " winding which reduces the inductance to a low value.

(k) Effect of frequency on resistance

Largely as a result of the distributed capacitance effects, the effective resistance of a rod type carbon or composition resistor falls as the frequency is increased. With resistances up to approximately 1 megohm, the theoretical curve of effective resistance plotted against frequency is given in Ref. A29 (Fig. 5). The effective resistance drops to 90% of the d.c. value

when $f = \dfrac{0.3}{C_d R_{dc}}$ cycles per second where C_d = total distributed capacitance of rod ;

or as an approximation, assuming $C' \approx C_d/3$, when $f \approx 1/10C'R_{dc}$ cycles per second where C' = equivalent shunt capacitance at low frequencies (this is usually the published capacitance of the resistor).

If C' is measured with the resistor in its operating position in relation to other components and metal parts, the proximity effects will be included.

C' is constant below 4 Mc/s (Ref. A29).

References A4, A5, A6, A7, A8, A9, A14, A29.

We give below some experimental values :

Ratio of effective resistance to d.c. resistance

R (megohms d.c.) × f (Mc/s) =	0.1	0.5	1.0	5	10	20
I.R.C. type BTR (½W)	0.98	0.93	0.89	0.62	0.46	0.30
I.R.C. type BTA (1W)	0.95	.80	.71	.48	.37	.24
I.R.C. type BT-2 (2W)	0.80	.53	.40	.19	.14	.11
I.R.C. type BTS	1.00	.89	.79	.61	.57	—
I.R.C. type F (lower limit)	—	—	—	.84	.80	.75
Allen Bradley GB-1	0.85	.60	.48	.24	.17	.12
Allen Bradley EB½	0.90	.68	.57	.46	.23	.15
Speer SCT ½	0.92	.70	.60	.35	.27	.20

Reference A14.

(l) Noise of resistors

All resistors have an inherent minimum noise voltage due to **thermal agitation** (" Johnson noise ") which is given at 30°C (80°F) by

$$e = 1.29 \times 10^{-10}\sqrt{\Delta f \times R}$$

where e = r.m.s. noise voltage
$\triangle f$ = bandwidth in c/s of the noise measuring instrument
and R = resistance in ohms.

The thermal agitation noise of ideal resistors at 30°C is tabulated below for a bandwidth of 5 000 c/s.

Resistance	1000	10 000	100 000	1 Meg	8 Meg	ohms
Noise	0.29	0.91	2.9	9.1	25.7	μV

When a current flows through the resistor, there is a small increase in the magnitude of the thermal agitation noise.

In addition to the thermal agitation noise, carbon and metallized resistors also have a noise voltage which is approximately proportional to the direct voltage applied across the resistance. This has been called **current noise** or **resistance fluctuation noise** (Ref. A25). The frequency distribution of this additional noise component, unlike thermal agitation noise, is not uniform but its value decreases with increasing frequency from 30 c/s upwards. The amount of noise varies according to the material and construction of the resistor, and even varies considerably from one resistor to another equivalent type. The " current noise " voltages of some typical English composition resistors (Ref. A3) are given below :

Resistance	1000	10 000	100 000	1 Meg	8 Meg	ohms
Average noise	0.03	0.18	0.35	0.5	0.6	μV/V
Max. noise	0.13	0.62	1.3	1.8	2.2	μV/V

Noise values up to 20 μV/V have been measured in commercial radio resistors (Ref. A25). The English Specification RIC/113 (Grade 2) issued June 1950, gives the limit of noise as $\log_{10} R \, \mu$V/V ; this is equivalent to 6 μV/V for 1 megohm.

Lower " current noise " voltages are produced by high stability cracked carbon resistors (see below) and also by palladium film resistors (Ref. A21).

In addition to the steady " current noise " fluctuations, all carbon composition resistors show abnormal **fluctuations** which do not appear to bear any simple relationship to the steady " current noise " (Ref. A21).

References A2, A3, A4, A13, A14, A15, A16, A17, A18, A19, A20, A21, A25, A32.

(m) High stability cracked carbon resistors (Refs. A10, A26, A28)

These not only have high short and long period stability and close tolerances (up to ± 1%) but also have low noise, low voltage coefficient and low temperature coefficient. They have practically no non-linearity of resistance and the inductance is very low except for the higher resistance values. They are manufactured in England with resistances from 10 ohms to 10 megohms and dissipations from ⅛ to 2 watts. The extremely high resistance values are only obtainable in the higher wattage ratings. See also (b), (e), (f) and (g) above, and Chapter 38 Sect. 3(i) for standards.

The historical development, constructions and special features of cracked carbon resistors, with a very extensive bibliography, are given in Ref. A28.

These resistors are particularly suited for use in low-level high-gain a-f amplifiers, and in r-f applications up to 100 Mc/s. The inductance varies from 0.001 μH for a small 100 ohm resistor to 2 μH for a large (spiral element) 1 megohm resistor.

The I.R.C. deposited carbon resistors have tolerances of ± 1%, ± 2% and ± 5%. The voltage coefficient is approx. 10 parts per million per volt. The temperature coefficient varies linearly from − 0.025 to − 0.05 (10 MΩ type DCH) or − 0.065 (10 MΩ type DCF). Maximum dissipations for high stability are ¼ and ½ watt ; when high stability is not essential the values are ½ and 1 watt.

(n) Negative temperature coefficient resistors (Thermistors)

Negative temperature coefficient resistors are sometimes used in a.c./d.c. receivers to safeguard the dial lamps when the set is first switched on. For example, one such NTC resistor has a resistance of 3 000 ohms cold and only 200 ohms when the heater current of 0.1 ampere is passing through it. By its use the initial surge on a 230 volt supply may be limited to 0.12 ampere. See also page 1267.

They are also known as " Varistors." Refs. A22, A28, A31, A34.

(o) Variable composition resistors (" potentiometers ")

Standard variable composition resistors are described in Chapter 38 Sect. 3(viii). In radio receivers and amplifiers these are commonly used as diode load resistances and as volume controls (attenuators). In general it is desirable to reduce to a minimum or eliminate entirely any direct voltage across them, and any current drawn by the moving contact. Whether or not a current is passed through the terminations, noise voltages appear across any two of the three terminations and, so long as the rotor is stationary, the noise does not differ from that of a fixed resistor of equal resistance. However, when the rotor is turned, additional noise is produced which is of the order of 1 or 2 millivolts per volt applied across the extreme terminations for a speed of rotation of one full rotation per second. The noise produced at any point of the track is approximately proportional to the voltage gradient at this point ; consequently the rotation-noise is greater over that portion of a logarithmic resistance characteristic where the most rapid change in resistance occurs, than for the other end of the same characteristic or for a linear characteristic. When a logarithmic characteristic is used for volume control, the rotation noise will therefore be much lower at settings for low volume where noise would be most noticeable (Ref. A30).

Some receiver manufacturers avoid the increased rotation noise caused by direct diode current in a volume control by using a fixed diode load resistor with capacitance coupling to a separate volume control having at least four times the resistance of the diode load resistor.

Variable composition resistors are available with a choice of up to 6 tapers ; tappings may be provided at 38% and 62% effective rotation (Mallory). They are also available, if desired, with a switch.

(ii) Practical condensers

(a) Summary of characteristics

A condenser has tolerances in the value of its capacitance. For most radio receiver applications, tolerances of $\pm$ 20% or even higher may be used. The closest tolerances available with paper dielectric condensers are $\pm$ 10% [see Chapter 38 Sect. 3(iii)]. Where closer tolerances are required it is necessary to adopt mica dielectric condensers in which the tolerance may be any standard value between $\pm$ 20% and $\pm$ 2% [Chapter 38 Sect. 3(v)].

The capacitance changes with frequency, with temperature, and with age.

A condenser has inductance, so that at some high frequency it becomes series-resonant.

A condenser has a.c. resistance, and dissipates energy in the form of heat ; the loss is approximately proportional to the square of the frequency and is also affected by the temperature. This energy loss is partly dielectric loss (which predominates at low frequencies) and partly electrode and lead losses (which predominate at higher frequencies). It may be replaced for the purpose of calculation by the " equivalent series resistance."

A condenser thus has a complex impedance, with resistive, capacitive and inductive components. The impedance may be capacitive over one range of frequencies, inductive over another, and resistive at one or more frequencies.

A condenser with a solid or liquid dielectric takes a longer time to charge than an ideal condenser having the same capacitance; this effect is due to " dielectric absorption." When such a condenser is short-circuited it fails to discharge instantaneously —a second discharge may be obtained a few seconds later. As a consequence the capacitance of such a condenser is a function of the duration of the applied direct voltage ; when it is used in an a.c. circuit, the capacitance decreases as the frequency rises. This effect is pronounced with paper dielectric, but is very small with mica dielectric condensers.

A condenser has d.c. leakage, and behaves as though it were an ideal condenser with a high resistance shunted across it.

A condenser with a solid dielectric tends to deteriorate during service, and may break down even when it is being operated within its maximum limits.

(b) The service life of a condenser

Condensers under voltage may be subject to gradual deterioration and possible breakdown, due to the solid dielectric (if any) and other insulation. This deterioration is much more rapid with some dielectrics than with others, and also varies considerably between different batches from the same factory.

Except for electrolytic condensers, there should be no deterioration with age except while under voltage ; the service life may therefore be taken as the time of operation under voltage.

The maximum temperature of a condenser has a pronounced effect on the service life. In some types, every 10°C rise in temperature causes 50% decrease in life ; other types are less sensitive to temperature.

The safe working voltage of a condenser at a given temperature is much less than the " ultimate dielectric strength "—as low as one tenth of this value in some cases. The " factory test " voltage is intermediate between these two values, but it is no guide to the safe working voltage. The only satisfactory procedure, if long life is essential, is to obtain from the manufacturer the safe working voltage at the proposed temperature of operation.

Condensers for operation on a.c. should have maximum a.c. ratings for safe working voltage, frequency and temperature. The peak operating voltage, whether a.c. or pulse, should be within the maximum voltage rating. The maximum surge voltage (usually during " warming up ") should be within the surge rating, if quoted, or alternatively should not exceed the maximum working voltage by more than 15%.

(c) Electrolytic condensers*

An electrolytic condenser provides more capacitance in a given space and at a lower cost per microfarad than any other. It is usually manufactured with a capacitance of 4 μF or more. The capacitance **tolerances** may be − 20%, + 100%, or − 20%, + 50%, while JAN-C-62 permits + 250%. Unlike other types, electrolytic condensers may only be used on substantially direct voltage, and they must be correctly connected with regard to polarity. Electrolytic condensers are generally used in radio receivers and amplifiers with a steady direct voltage plus an a.c. ripple. The highest rated voltage rarely exceeds 500 V peak, even for use under the most favourable conditions.

The capacitance increases somewhat with **increase of temperature**, and decreases rapidly with temperatures below − 5°C. The capacitance also decreases with **age**—one dry type shows a 5% decrease in capacitance after 7000 hours operation at 20°C ambient temperature, and a 20% decrease at 40°C.

The **capacitance** at 10 000 **c/s** is usually less than that at low frequencies. Typical wet types at 10 000 c/s have only from 30% to 50% of the capacitance at 50 c/s. Typical dry types are better in this regard, having capacitances at 10 000 c/s from 42% to 85% of that at 50 c/s, at 20°C. However, the **temperature** has a marked effect on the capacitance versus frequency characteristic. One etched-foil type (Ref. B13) has 42% of its nominal capacitance at 20°C, 95% at 33°C, and 107% at 50°C.

The **series-resistance** of a new condenser at ordinary working temperature is fairly low (not more than 25 ohms for 8 μF at 20°C, 450 V working) but it rises rapidly at higher temperatures and temperatures below 10°C. The series resistance rises considerably during life and eventually may be the cause of unsatisfactory operation of a receiver.

Among dry electrolytics, those with etched foil anodes are much inferior to those with plain foil electrodes when used for a-f by-passing, owing to their high impedance particularly at the higher frequencies. For example, at 10 000 c/s an ideal 8 μF capacitor has an impedance of approximately 2 ohms, whereas typical plain-foil electrolytics have impedances from 3.5 to 6.5 ohms and those with etched-foil electrodes have impedances from 8 to 22 ohms (Ref. B13).

*The following remarks apply to aluminium electrodes. However tantalum electrodes are also used (Ref. B17).

The **dissipation factor** is a function of both frequency and temperature. One 10 μF etched-foil capacitor has a dissipation factor of 10% at 100 c/s, 68% at 1000 c/s and 92% at 10 000 c/s, at 20°C. The dissipation factor decreases rapidly with increase of temperature, and at 1000 c/s is 30% at 33°C and 13% at 50°C for the same capacitor (Ref. B13).

When electrolytic condensers are required to be operated across **voltages of more than** 450 **volts**, two or more condensers may be connected in series but the effective total capacitance will then be half (or less) that of the single unit. In such a case it is advisable to connect a resistor, say 0.25 megohm, across each capacitor.

Electrolytic condensers have **self-healing properties**—after a momentary surge of over-voltage, resulting in break-down of the dielectric, the electrolytic condenser is more likely to recover than a non-electrolytic type. Wet electrolytic condensers are very good in this respect.

Electrolytic condensers have an appreciable **leakage current** ; this may be from 0.002 to 0.25 mA per microfarad and varies considerably with the type of condenser and the " working voltage," being higher for higher values of working voltage. The maximum leakage current in milliamperes permitted by JAN-C-62 is (0.04 × capacitance in microfarads) + 0.3. This is equivalent to a shunt resistor of 0.7 megohm for an 8 μF condenser, 450 V rating. At voltages lower than the working voltage, the leakage current falls, but at higher voltages it increases very rapidly. The leakage current also increases rapidly at higher temperatures.

If an electrolytic condenser is **left idle** for some days, the initial leakage may be quite substantial, but it tends to become normal after a few minutes. If the condenser has been left idle for several months, the time of recovery is longer.

The **power factor** of an electrolytic condenser may be between 2% and 3% for the best condensers and is usually between 5% and 10%. Some of the older wet types had power factors of 25% or greater.

Electrolytic condensers should not be used in positions where the **ambient temperature** is high or the alternating current component is excessive, otherwise the service life will be short. Ambient temperatures up to 50°C are always satisfactory while 60°, 65°C or 70°C is permissible for many types and some may be used at higher temperatures (e.g. 85°C).

Special types are available for very low temperatures (Ref. B18).

Electrolytic condensers are in two major groups.

Wet electrolytic condensers have vents, and must be mounted vertically with the vent unobstructed. They are valuable as first filter condensers in a rectifier system. Some wet types are used as voltage regulating devices, to limit the peak voltage during the warming-up period. All wet types have rather greater leakage currents than dry types. **Dry electrolytic condensers** are very widely used as filter and by-pass condensers. They are inferior to the wet type as regards frequent and severe voltage surges and short period overloads, when they are liable to fail permanently, but are preferable to the wet type in most other respects. They are manufactured in several forms—plain foil, etched, sprayed or fabricated foil. Most modern compact units have etched or fabricated foil, but the plain foil type has a lower impedance at radio frequencies. " Surgeproof " types are available with a safe operating voltage of 450 volts but which have heavy leakage current when the voltage exceeds 500 volts. This type is able to handle very heavy ripple currents without deterioration.

Reversible dry electrolytic condensers are manufactured, but they have higher leakage currents than standard types.

Electrolytic condensers when used as **first filter condensers** in condenser-input filters require careful consideration. The d.c. voltage plus the peak value of the ripple voltage must not exceed the rated voltage of the capacitor while the ripple current must not exceed the ripple current rating. The ripple current may be measured by a low-resistance moving iron, or thermal, meter ; a moving-coil rectifier type of instrument is not suitable. Alternatively the ripple current may be calculated —see Chapter 30 Sect. 2.

Some typical **ripple current ratings** are given below (T.C.C.). Plain foil types have a higher ripple current rating than equivalent etched foil types.

Ambient temperature	20°C	40°C	60°C	70°C
8 μF 350 V " micropack " plain foil	148	125	85	32 mA
16 μF 350 V " micropack " plain foil	250	200	110	50 mA
8 μF 450 V etched foil	88	67	33	10 mA
16 μF 450 V etched foil	162	122	62	20 mA
16 μF 450 V plain foil	300	260	160	85 mA
32 μF 450 V plain foil	500	405	230	100 mA

With multiple capacitor units, only one of the units is normally intended for use as the first filter condenser ; see catalogues for identification.

See Chapter 38 Sect. 3(x) for standard ratings.

(d) Paper dielectric condensers

Impregnated paper forms a very useful dielectric, being intermediate between electrolytic and mica condensers as regards cost, size and leakage for a given capacitance. It is usually manufactured in units from 0.001 to 0.5 μF, larger values being built up from several smaller units in parallel in one container. The impregnating material may be resin, wax, oil or a synthetic compound. Some impregnating materials enable condensers to withstand extremely wide temperature ranges (e.g. − 50° to + 125°C—Sprague " Prokar " with plastic impregnant). Waxes may be used for moderate voltages and temperatures, as in radio receivers (from − 30°C to + 65°C for R.M.A. Class W). Other impregnants are used for higher temperatures (e.g., 85°C, as R.M.A. Class M ; 100°C as T.C.C. " metalpack " and " metalmite "). The permissible insulation resistance at 25°C is not less than 5000 megohms for capacitances up to 0.15 μF, falling to 1000 megohms for 1 μF, but this falls rapidly at higher temperatures, being 35% of these values at 40°C.

A typical 1 microfarad wax paper condenser designed for audio frequency applications has the following characteristics :

Frequency	1000 c/s	10 000 c/s	100	360	Kc/s
Inductance	0.2	0.2	0.2	0.2	μH
Resistance (effective)	1.1	0.43	0.3	0.25	ohms
Reactance	− 159*	− 15.9*	− 1.50	0**	ohms
Q	145	37	5	0	
Power factor	0.007	0.027	0.19	1	
Percentage power factor	0.7	2.7	19	100	%

* equal to ideal ** resonance

Some paper dielectric capacitors are impregnated with a high permittivity wax to reduce the dimensions of the capacitor (Ref. B13). These capacitors have less desirable electrical characteristics than those with normal waxes. At 10 000 c/s the capacitance falls to 89% of its value at 100 c/s ; the capacitance varies from − 19% to + 6% as the temperature is varied from − 30° to + 70°C.

Wax-impregnated paper dielectric condensers are sometimes used for grid coupling purposes from a preceding plate at high potential, but plastic-impregnated or mica dielectric is to be preferred on account of leakage. As an example, take a paper condenser with capacitance = 0.01 $\mu\mu$F at 40°C. The minimum insulation resistance will be 5000 × 0.35 = 1750 megohms. If the grid resistor has a resistance of 1 megohm and the preceding plate voltage is 175 volts, there will be a voltage of 0.1 volt on the grid as the direct result of leakage.

In most other applications the leakage may be neglected entirely.

Plastic (polystyrene) impregnated paper dielectric condensers have a very high insulation resistance, of the order of 500 000 megohms per microfarad, and are much more suitable for use as grid coupling condensers (e.g. T.C.C. Plastapacks, 50 to 5 000 $\mu\mu$F). A test after 9 months' handling under bad climatic conditions showed insulation resistances of 24 000 to 100 000 megohms (Aerovox Duranite, 0.01 to 0.22 μF). These condensers have power factors as low as those of mica condensers, while the temperature coefficient of capacitance is from − 100 to − 160 parts in a million per °C. For maximum stability they should not be operated above 60°C, but the insulation resistance remains very high even up to 75°C. These are available in capacitances from 100 to 10 000 $\mu\mu$F in tubular form and from 0.02 to 4 μF in rectangular metal boxes (Ref. B11).

Mineral oil is used as an impregnant for working voltages from 1000 to 25 000 volts and operating temperatures from $-30°$ to $+71°$C (T.C.C. "Cathodray"). The insulation resistance of a mineral oil impregnated capacitor is greater than that with petroleum jelly impregnation, in the ratio of 2.5 to 1 at 0°C, rising to 12.5 to 1 at 70°C (Ref. B13).

Paper dielectric condensers are made in two forms—inductive, and non-inductive. The former is limited to a-f applications, while the latter may be used at radio frequencies.

Ordinary paper dielectric capacitors should not be subjected to high a.c. potentials. Special types are produced by some manufacturers for use under these conditions, for example with vibrator power packs and line filters.

For Standard Specifications see Chapter 38 Sect. 3(iii) and (iv).

Metallized paper dielectric condensers

This type utilizes a metal-sprayed or metal-evaporated paper dielectric instead of the more conventional metal foil and paper construction. This construction results in considerable reduction in size, while it also has a partial self-healing property in the case of breakdown. The insulation resistance of unlacquered condensers is quite low—of the order of 100 megohm microfarads—but some of those with a lacquered film have an average insulation resistance as high as 8000 megohm microfarads at 25°C (Refs. B14, B15).

There is a gradual reduction in the insulation resistance due to the self-healing property, the degree depending on the number of punctures. It is desirable for the total circuit resistance to be not less than 500 or 1000 ohms, to reduce the carbonising effect of the arc. However if the circuit resistance is high, there may be insufficient current to clear completely any breakdown, and the insulation resistance may fall. Consequently, this type of condenser should not be used in high impedance circuits without seeking the advice of the manufacturer.

A metallized paper dielectric unit should be used with discretion as the first filter condenser following a thermonic rectifier, since the high peak breakdown current may damage the rectifier unless the circuit resistance is sufficiently high.

The ratio of reactance to resistance (Q) of one lacquered unit with a capacitance of 2 μF is 200 at 0.5 Kc/s, 140 at 2 Kc/s and 60 at 10 Mc/s ; a 0.1 μF unit has $Q = 98$ at 10 Mc/s (Ref. B15).

The inductance may be made very low, and these condensers are very useful for a-f and r-f by-passing. The usual (English) temperature limit is 71°C for d.c. operation, 60°C for a.c. In tubular form (wax-coated) these are available from 0.0001 to 2 μF with voltage ratings 150, 250, 350 and 500 V d.c. (RIC/136). Larger sizes are available with capacitances up to 20 μF (400 V d.c. or 250 V a.c.) and voltages up to 550 V d.c. (4 μF).

Voltage-temperature derating curves for Astron (U.S.A.) are 100% up to 86°C, linearly down to 38% at 120°C for units up to 1 μF ; larger units are 100% up to 76°C, down to 22% at 120°C (Ref. B16).

The effect of 5000 hours' operating life on capacitance is negligible up to 65°C and 8% at 100°C. The effect of the same operation is to increase the power factor from an initial value of 0.5% to 0.6% at 65°C, or 0.8% at 100°C (Ref. B16).

The paper is usually impregnated with wax, although mineral oil has also been used. Mineral wax impregnated units are generally preferred because of their higher breakdown voltage, although their capacitance falls about 10% as the temperature is increased from 50°C to 85°C. Mineral oil impregnated units have more constant capacitance with temperature change.

These condensers are damaged by moisture and the unit is therefore well dried initially and hermetically sealed to prevent the ingress of moisture.

References B10, B12, B14, B15, B16.

Standard Specifications—Chapter 38 Sect. 3(ix).

(e) Mica dielectric condensers

Mica has very high electrical stability and very low a.c. loss. It also permits the manufacture of condensers with close tolerances in capacitance, and low leakage. It is used in the manufacture of condensers with capacitances from 5 $\mu\mu$F to 0.047 μF for radio receiver applications, with voltage ratings from 300 to 2500 V. The insulation resistance is in excess of 3000 for the cheaper grade (Class A) and 6000 megohms for other classes (American R.M.A. REC-115). The value of Q is over 1000 for a typical capacitance of 200 $\mu\mu$F at a frequency of 1 Mc/s ; the maximum value of Q occurs at frequencies about 100 Kc/s.

Mica condensers are available in metal, moulded and ceramic casings [see Chapter 38 Sect. 3(v)].

"**Silvered mica**" condensers are used when very high precision is required. Such a condenser with a capacitance of 1000 $\mu\mu$F exhibits a capacitance change of less than 0.1% over a frequency range from low frequencies to 2 Mc/s. The effect of temperature on capacitance is a change of less than 60 parts in a million for 1°C temperature change (RIC/137).

A typical 0.001 microfarad silvered mica condenser has the following characteristics :

Frequency	1000 c/s	10 000 c/s	100	500	Kc/s
Resistance (effective)	0.024	0.024	0.024	0.024	ohm
Q	3400	5500	7000	5800	
Power factor	0.00029	0.00018	0.00014	0.00017	
Percentage power factor	0.029	0.018	0.014	0.017	%

Silvered mica condensers are normally available with capacitances from 5 to 20 000 $\mu\mu$F with tolerances $\pm$ 1%, $\pm$ 2%, $\pm$ 5%, $\pm$ 10% and $\pm$ 20% (subject to minimum tolerance $\pm$ 1 $\mu\mu$F) (RIC/137). The average temperature coefficient is $+ 25 \times 10^{-6}$ per °C, with limits from $+ 5 \times 10^{-6}$ to $+ 50 \times 10^{-6}$ per °C(U.I.C.).

Standard Specifications—Chapter 38 Sect. 3(v).

(f) Ceramic dielectric condensers

Ceramic dielectric condensers may be grouped under five heads :

1. Types intended primarily for temperature compensation, having a series of negative and positive temperature coefficients with close tolerances on the coefficients.

2. Types having temperature coefficients nearly zero.

3. General purpose types with a broad spread of temperature coefficients. This may be further divided into two groups, those having positive and negative temperature coefficients.

4. Types with temperature coefficients not specified. These are available with capacitances from 0.5 $\mu\mu$F upwards.

5. High-K types having relatively poor power factors and indeterminate temperature coefficients.

The first group is intended for use in the tuned circuits of radio receivers, in which their special temperature versus capacitance characteristics are used to reduce frequency drift during warming-up and running. A condenser having a negative temperature coefficient may be used to compensate the positive temperature coefficient of the tuned circuit alone. For standard specifications see Chapter 38 Sect. 3(vi).

The second group is intended for use in tuned circuits requiring nearly constant capacitance, such as in i-f transformers.

All except High-K types have high stability and values of Q from 335 to 1000, except for capacitances less than 30 $\mu\mu$F. The insulation resistance is not less than 7 500 megohms (R.M.A.).

The High-K types are not suitable for compensation purposes, but are used for by-passing and other non-critical applications. They are manufactured with capacitances up to 15 000 $\mu\mu$F, with capacitance tolerances of $\pm$ 20%. The value of Q is from 30 to 100.

(g) Gang condensers

Gang condensers usually have air dielectric, and are available in 1, 2 and 3 (occasionally 4) gang units. Some are fitted with trimmer condensers—see (h) below—while others are not. The shape of the plates may be designed to provide any desired capacitance characteristic (Refs. B5a, B5b) among which are :

(1) **Straight line capacitance** : Each degree of rotation should contribute an equal increment in capacitance.

(2) **Straight line frequency** : Each degree of rotation should contribute an equal increment in frequency.

(3) **Logarithmic Law** : Each degree of rotation should contribute a constant percentage change of frequency.

(4) **Square Law** : The variation in capacitance should be proportional to the square of the angle of rotation.

Some gang condensers have all sections identical, while others have the oscillator section with specially shaped plates to give correct tracking without the use of a padder condenser. For standard specifications, see Chapter 38 Sect. 3(vii).

The normal construction incorporates an earthed rotor, but condensers with insulated rotors are also available. Condensers with split stators and either earthed or insulated rotors are available for special applications.

(h) Trimmer condensers (" compensators ")

These are available in innumerable forms, and can only be briefly mentioned.

The compression mica type is the least satisfactory of all since it tends to suffer from drift in capacitance, and is far from being linear in its characteristic. It is used in receivers for the medium frequency broadcast band and in the less expensive dual-wave receivers.

The concentric or vane air-dielectric types are more expensive but have greater stability, are easier to adjust, and the better types are more satisfactory under tropical conditions. Ceramic trimmers are also obtainable.

Trimmers are available in a wide range of capacitances, but for ordinary use with gang condensers should preferably have a minimum capacitance not greater than 2 $\mu\mu$F, and a capacitance change not less than 15 $\mu\mu$F (R.M.A. REC-101, REC-106-A).

See also Chapter 38 Sect. 3(vii and xi) for standard specifications.

(iii) Combination units

Combinations of one or more capacitors with one or more resistors are becoming common, and are very convenient. Some popular combinations are

(a) Diode filters incorporating one resistor and two capacitors with a common earth return.

(b) Cathode bias units incorporating one resistor shunted by a capacitor.

(c) Plate and grid decoupling units incorporating one capacitor and one resistor.

(d) Audio frequency coupling unit incorporating a plate load resistor, coupling capacitor and grid resistor with also (in one example) a grid stopper resistor and capacitor.

(iv) Practical inductors

Iron-cored inductors are covered in Chapter 5. Radio frequency inductors, both air-cored and iron-dust cored, are covered in Chapter 11. The calculation of inductance of air-cored inductors at all frequencies is covered in Chapter 10.

(v) References to practical resistors and condensers

(A) REFERENCES TO PRACTICAL RESISTORS

A1. Pender, H., & K. McIlwain (book) " Electrical Engineers' Handbook—Electrical Communication and Electronics " (John Wiley & Sons, New York ; Chapman & Hall Ltd., London, 4th edit. 1950) Section 3.
A2. American R.M.A. Standards (see Chapter 38 Sect. 3(i) and (ii)).
A3. Spratt, H. G. M. " Resistor ratings," W.W. 54.11 (Nov. 1948) 419.
A4. British Standard BS/RC.G/110 " Guide on fixed resistors " (Issue 1) Aug. 1944. To be superseded by RC.G./110—see Chapter 38 Sect. 3.
A5. G.W.O.H. " Behaviour of high resistances at high-frequencies," W.E. 12.141 (June 1935) 291.
A6. Puckle, O. S. " Behaviour of high resistance at high-frequencies " W.E. 12.141 (June 1935) 303.
A7. G.W.O.H. " A further note on high resistance at high-frequency " W.E. 12.143 (Aug. 1935) 413.
A8. Hartshorn, " The behaviour of resistances at high-frequency " W.E. 15.178 (July 1938) 363.
A9. G.W.O.H. " The behaviour of resistances at high-frequencies " W.E. 17.206 (Nov. 1940) 470.
A10. Wilton, R. W. " Cracked carbon resistors " FM & T. 9.2 (Feb. 1949) 29.

A12. Cooper, W. H., & R. A. Seymour " Temperature dependent resistors " W.E. 24.289 (Oct. 1947) 298.
A13. Fixed composition resistors (draft specification not complete) B.R.M.F. Bulletin 1.7 (July 1948) 8-10. Superseded by Refs. A26, A27.
A14. Blackburn, J. F. (Editor) " Components Handbook " (1st edit. 1949, McGraw-Hill Book Co., New York & London, for Massachusetts Institute of Technology). Also gives extensive bibliography.
A15. American JAN-R-11 Specification with Amendment No. 3 (see Chapter 38 Sect. 3 for details).
A16. Catalogues and reports from resistor manufacturers.
A17. Valley, G. E., & H. Wallman (Editors) " Vacuum tube amplifiers " (McGraw-Hill Book Co., New York & London, 1948).
A18. Data Sheet XIX " Circuit noise due to thermal agitation " Electronic Eng. 14.167 (Jan. 1942) 591.
A19. Johnson, J. B. " Thermal agitation of electricity in conductors " Phys. Rev. 32.1 (July 1928) 97.
A20. Nyquist, H. " Thermal agitation of electric charge in conductors " Phys. Rev. 32.1 (July 1928) 110.
A21. Campbell, R. H., & R. A. Chipman " Noise from current-carrying resistors 20 to 500 Kc " Proc. I.R.E. 37.8 (Aug. 1949) 938.
A22. " Properties and uses of negative coefficient resistors—thermistors " W.W. 55.10 (Oct. 1949) 405.
A23. " Deposited-carbon resistors " Elect. 22.10 (Oct. 1949) 182.
A24. " Radio Components Handbook " (1st ed., 1948, Technical Advertising Associates, Cheltenham, Pennsylvania, U.S.A.).
A25. Oakes, F. " Noise in fixed resistors " Electronic Eng. 22.264 (Feb. 1950) 57 ; letters 22.267 (May 1950) 207.
A26. The Radio Industry Council, London, Specification No. RIC/112 (Issue 1, May 1950) Resistors, fixed, composition, Grade 1.
A27. The Radio Industry Council, London, Specification No. RIC/113 (Issue 1, June 1950) Resistors, fixed, composition, Grade 2.
A28. Coursey, P. R. " Fixed resistors for use in Communication equipment, with special reference to high stability resistors " Proc. I.E.E. 96. Part 3.41 (May 1949) 169. Discussion 96. Part 3.44 (Nov. 1949) 482.
A29. Arthur, G. R., & S. E. Church " Behaviour of resistors at high frequencies " TV Eng. 1.6 (June 1950) 4.
A30. Oakes, F. " Noise in variable resistors and potentiometers " Electronic Eng. 22.269 (July 1950) 269.
A31. Rosenberg, W. " Thermistors " Electronic Eng. 19.232 (June 1947) 185.
A32. Oakes, F. " The measurement of noise in resistors " Electronic Eng. 22.273 (Nov. 1950) 464. See also References Chapter 18 Sect. 2(ii)b.
A33. Pavlasek, T. J. F., & F. S. Howes " Resistors at radio frequency—characteristics of composition type " W.E. 29.341 (Feb. 1952) 31.

(B) REFERENCES TO PRACTICAL CONDENSERS (books)

B1. Brotherton, M. " Capacitors—their use in electronic circuits " (D. Van Nostrand Co. Inc. New York, 1946).
B2. Coursey, P. R. " Electrolytic Condensers " (Chapman & Hall Ltd. London, 2nd edit. 1939).
B3. Deeley, P. McK. " Electrolytic Capacitors " (Cornell-Dubilier Electric Corp., South Plainfield, N.J. 1938).
B4. Georgiev, A. M. " The Electrolytic Capacitor " (Murray Hill Books Inc. New York and Toronto, 1945).
B5a. Blakey, R. E. " The Radio and Telecommunications Design Manual " (Sir Isaac Pitman & Sons Ltd , London, 1938) Section 3.
B5b. Henney, K. " Radio Engineering Handbook " (McGraw-Hill Book Co. New York & London, 4th ed. 1950).

OTHER REFERENCES

B6. Brotherton, M. " Paper capacitors under direct voltages," Proc. I.R.E. 32.3 (March 1944) 139.
B7. American and English Standard specifications—see Chapter 38 Sect. 3.
B8. Roberts, W. G. " Ceramic capacitors " Jour. Brit. I.R.E. 9.5 (May 1949) 184.
B9. Gough, Kathleen A. " Choosing capacitors " W.W. 55.6 (June 1949) S.5.
B10. Cornell, J. I. " Metallized paper capacitors " Comm. (Jan. 1947) 22.
B11. Cozens, J. H. " Plastic film capacitors " W.W. 55.6 (June 1949) S. 11.
B12. Trade catalogues and data.
B13. Bennett, A. E., & K. A. Gough " The influence of operating conditions on the construction of electrical capacitors " Proc. I.E.E. Part III 97.48 (July 1950) 231.
B14. McLean, D. A. " Metallized paper for capacitors " Proc. I.R.E. 38.9 (Sept. 1950) 1010.
B15. Weeks, J. R. " Metallized paper capacitors " Proc. I.R.E. 38.9 (Sept. 1950) 1015.
B16. Fisher, J. H. " Metallized paper capacitors " Elect. 23.10 (Oct. 1950) 122.
B17. Whitehead, M. " New electrolytic capacitors—use of tantalum for electrodes " FM-TV 11.2 (Feb. 1951) 26.

Additional references will be found in the Supplement commencing on page 1475.