CHAPTER 2.

VALVE CHARACTERISTICS

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Section		Page
1.	Valve coefficients	13
2.	Characteristic curves	15
3.	Resistance-loaded amplifiers	24
4.	Transformer-coupled amplifiers	27
5.	Triode operation of pentodes	34
6.	Conversion factors and the calculation of characteristics other than those published	36
7.	Valve equivalent circuits and vectors	45
8.	Valve admittances	49
9.	Mathematical relationships	57
10.	References	66

SECTION 1 : VALVE COEFFICIENTS

(Otherwise known as Constants, Parameters or Factors)

The triode or multigrid radio valve is a device which allows, under certain operating conditions, an amplified replica of a voltage applied between grid and cathode to appear across an impedance placed between plate and cathode.

A valve, in itself, does not provide amplification of the applied grid-to-cathode voltage. The amplified voltage across the load impedance is due to the action of the valve in controlling the power available from the power supply. The amount of power which can be so controlled is determined by the operating conditions and the characteristics of the valve and of its associated circuits.

The maximum voltage amplification which a valve is capable of giving under ideal conditions is called the amplification factor, generally designated by the Greek symbol μ (mu). This is not truly constant under all conditions (except for an imaginary "ideal valve") and varies slightly with grid bias and plate voltage in the case of a triode, and is very far from being constant with most multi-electrode valves.

The **amplification factor** (μ) is the ratio of the incremental* change in plate voltage to the incremental change in control grid voltage in the opposite direction, under the conditions that the plate current remains unchanged, and all other electrode voltages are maintained constant.

There are two other principal Valve Coefficients, known as the mutual conductance and the plate resistance (or anode resistance), the values of these also being somewhat dependent upon the applied voltages.

*An incremental change of voltage applied to an electrode may be taken as indicating a change so small that the curvature of the characteristics may be neglected. For the mathematical treatment of rate of change, see Chapter 6 Sect. 7(i) and (ii). For treatment of valve coefficients as partial differentials see Chapter 2 Sect. 9(ix).

The **mutual conductance** (or grid-plate transconductance) is the incremental change in plate current divided by the incremental change in the control-grid voltage producing it, under the condition that all other voltages remain unchanged.

The **plate resistance**† is the incremental change in plate voltage divided by the incremental change in plate current which it produces, the other voltages remaining constant.

There is a relationship between these three principal valve coefficients, which is exact provided that all have been measured at the same operating point,

$$\mu = g_m \cdot r_p,$$

$$\text{or } g_m = \frac{\mu}{r_p},$$

$$\text{or } r_p = \frac{\mu}{g_m}.$$

The calculation of these "valve coefficients" from the characteristic curves is given in Section 2 of this Chapter, while their direct measurement is described in Chapter 3 Sect. 3. The mathematical derivation of these coefficients and their relationship to one another are given in Section 9 of this Chapter, as is also the representation of valve coefficients in the form of partial differentials.

The reciprocals of two of these coefficients are occasionally used—

$\frac{1}{\mu} = D$ where D is called the Durchgriff (or Penetration Factor) and which may be expressed as a percentage.

$\frac{1}{r_p} = g_p$ where g_p is called the Plate Conductance (see also below).

Other valve coefficients are described below:—

The **Mu-Factor,** of which the amplification factor is a special case, is the ratio of the incremental change in any one electrode voltage to the incremental change in any other electrode voltage, under the conditions that a specified current remains unchanged and that all other electrode voltages are maintained constant. Examples are

$$\mu_{g1 \cdot g2}, \ \mu_{g2 \cdot p}.$$

The **Conductance** (g) is the incremental change in current to any electrode divided by the incremental change in voltage to the same electrode, all other voltages remaining unchanged.

Examples are grid conductance (g_g), plate conductance (g_p).

Transconductance is the incremental change in current to any electrode divided by the incremental change in voltage to another electrode, under the condition that all other voltages remain unchanged. A special case is the grid-plate transconductance which is known as the mutual conductance. Another example is the plate-grid transconductance (g_n).

Conversion transconductance (S_c) is associated with mixer (frequency changing) valves, and is the incremental change in intermediate-frequency plate current divided by the incremental change in radio-frequency signal-grid voltage producing it.

The **Resistance** (r) of any electrode is the reciprocal of the conductance; for example plate resistance is the reciprocal of plate conductance,

$$r_p = 1/g_p.$$

Perveance (G) is the relation between the space-charge-limited cathode current and the three-halves power of the anode voltage. It is independent of the electrode voltages and currents, so long as the three-halves law holds:

$$G = \frac{i_k}{e_b^{3/2}}$$

The measurement of perveance is covered in Chapter 3 Sect. 3(vi)E.

†This is strictly the "variational plate resistance" and must be distinguished from the d.c. plate resistance.

SECTION 2 : CHARACTERISTIC CURVES

(i) Plate characteristics (ii) Mutual characteristics (iii) Grid current characteristics (iv) Suppressor characteristics (v) Constant current curves (vi) " G " curves (vii) Drift of characteristics during life (viii) Effect of heater-voltage variation.

It is convenient to set down the measured characteristics of a valve in the form of curves. These are thus a record of the actual currents which flow in a given valve when the specified voltages are applied.

The curves published by the valve manufacturers are those of an " average " valve, and any one valve may differ from them within the limits of the manufacturing tolerances.

FIG 2.1

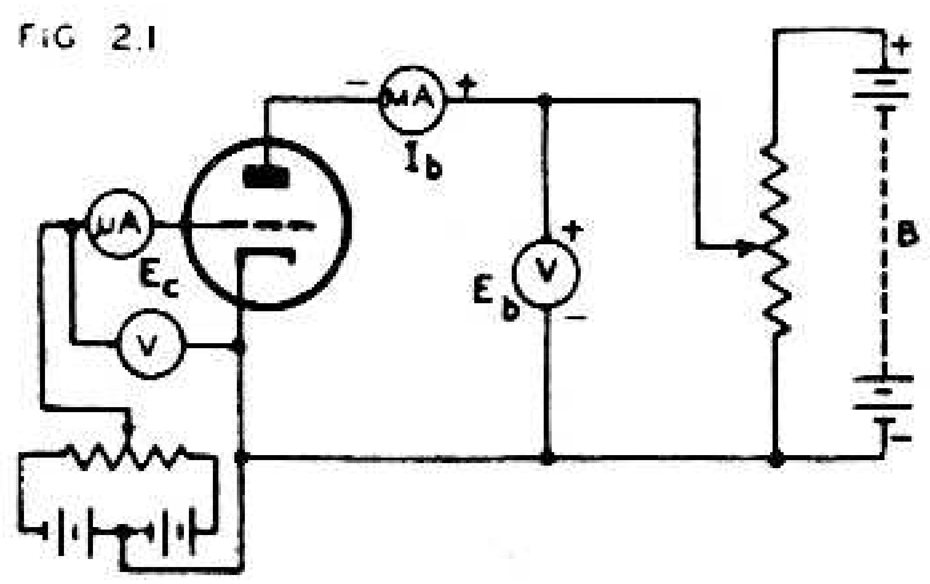


Fig. 2.1. Method of measuring the plate and grid currents of a triode valve.

The method of measuring the plate and grid currents of a triode valve is shown in Fig. 2.1 in which a tapping on the grid bias battery is returned to the cathode so as to permit either positive or negative voltages to be connected to the grid. The grid microammeter and voltmeter should be of the centre-zero type, or provision made for reversal of polarity. For more elaborate testing see Chapter 3 Sect. 3.

(i) Plate characteristics

The Plate Characteristic may be drawn by maintaining the grid at some constant voltage, varying the plate voltage step-by-step from zero up to the maximum available, and noting the plate current for each step of plate voltage. These readings may then be plotted on graph paper with the plate voltage horizontal and plate current vertical. This procedure may be repeated for other values of grid voltage to complete the Plate Characteristic Family.

The Plate Characteristic Family for a typical triode is shown in Fig. 2.2. It is assumed that the plate voltage has been selected as 180 volts, and the grid bias −4 volts. By drawing a vertical line from 180 volts on the E_b axis (point K), the quiescent operating point Q will be determined by its intersection with the " $E_c = -4$ " curve. By referring Q to the vertical scale (I_b) the plate current is found to be 6mA. The plate resistance at the point Q is found by drawing a tangent (EF) to the curve for $E_c = -4$ so that it touches the curve at Q.

The plate resistance (r_p) at the point Q is then EK in volts (65) divided by QK in amperes (6 mA = 0.006 A) or 10 800 ohms.

The amplification factor (μ) is the change of plate voltage divided by the change of grid voltage for constant plate current. Line CD is drawn horizontally through Q, and represents a line of constant plate current. Points C and D represent grid voltages of −2 and −6, and correspond to plate voltages of 142 and 218 respectively. The value of μ* is therefore (218 − 142) plate volts divided by a change of 4 grid volts, this being 76/4 or 19.

The mutual conductance (g_m) is the change of plate current divided by the change of grid voltage for constant plate voltage. Line AB, which is drawn vertically through Q, represents constant plate voltage. Point A corresponds to 9.6 mA, while point B corresponds to 2.6 mA, giving a difference of 7 mA. Since points A and B also differ by 4 volts grid bias, the mutual conductance* is 7 mA divided by 4 volts, which is 1.75 mA/volt or 1750 micromhos.

*The value so determined is not exactly the value which would be obtained with a very small swing, but is sufficiently accurate for most practical purposes.

In these calculations it is important to work with points equidistant on each side of Q to reduce to a minimum errors due to curvature.

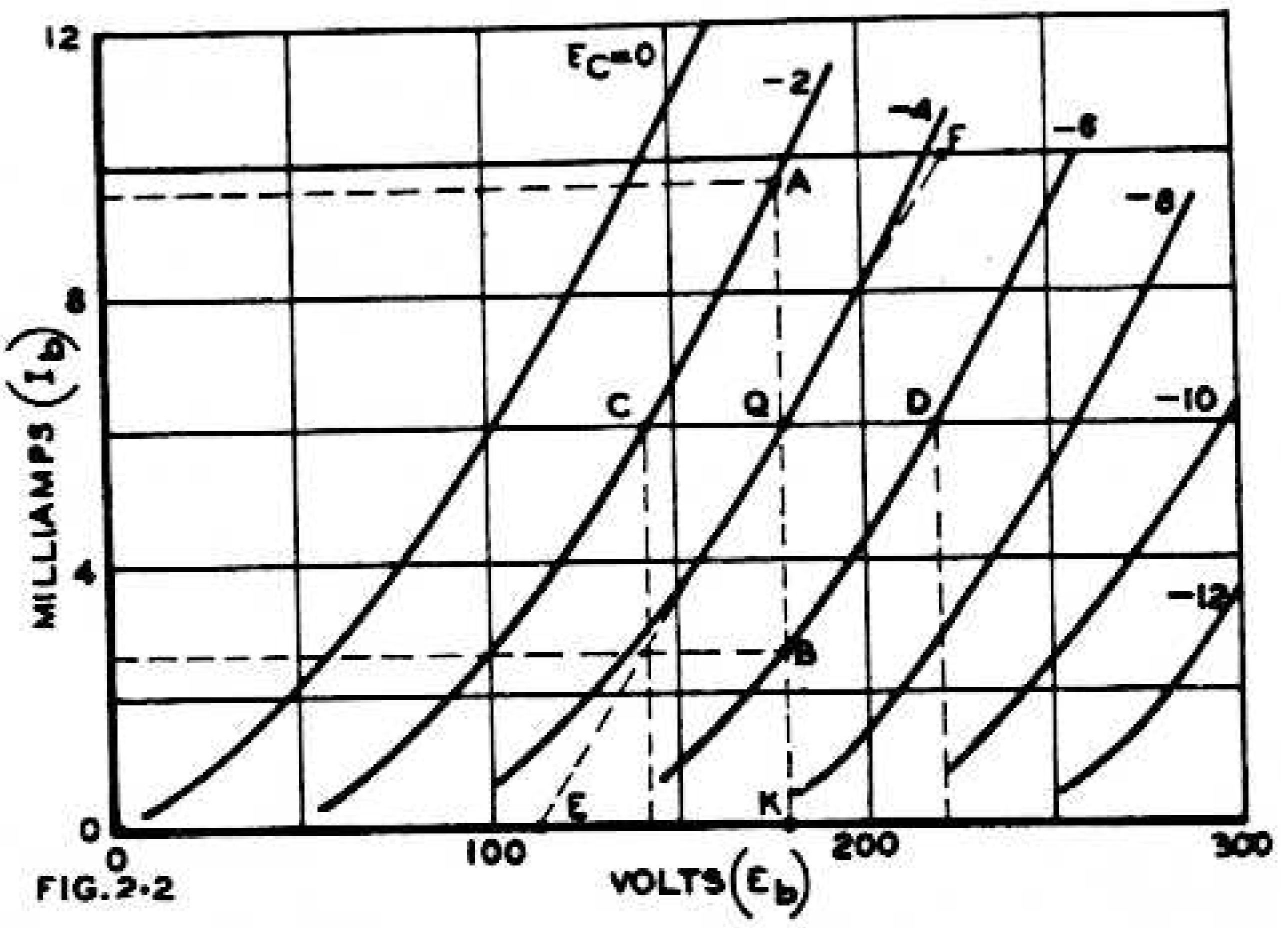


Fig. 2.2. Plate characteristic family of curves for a typical triode.

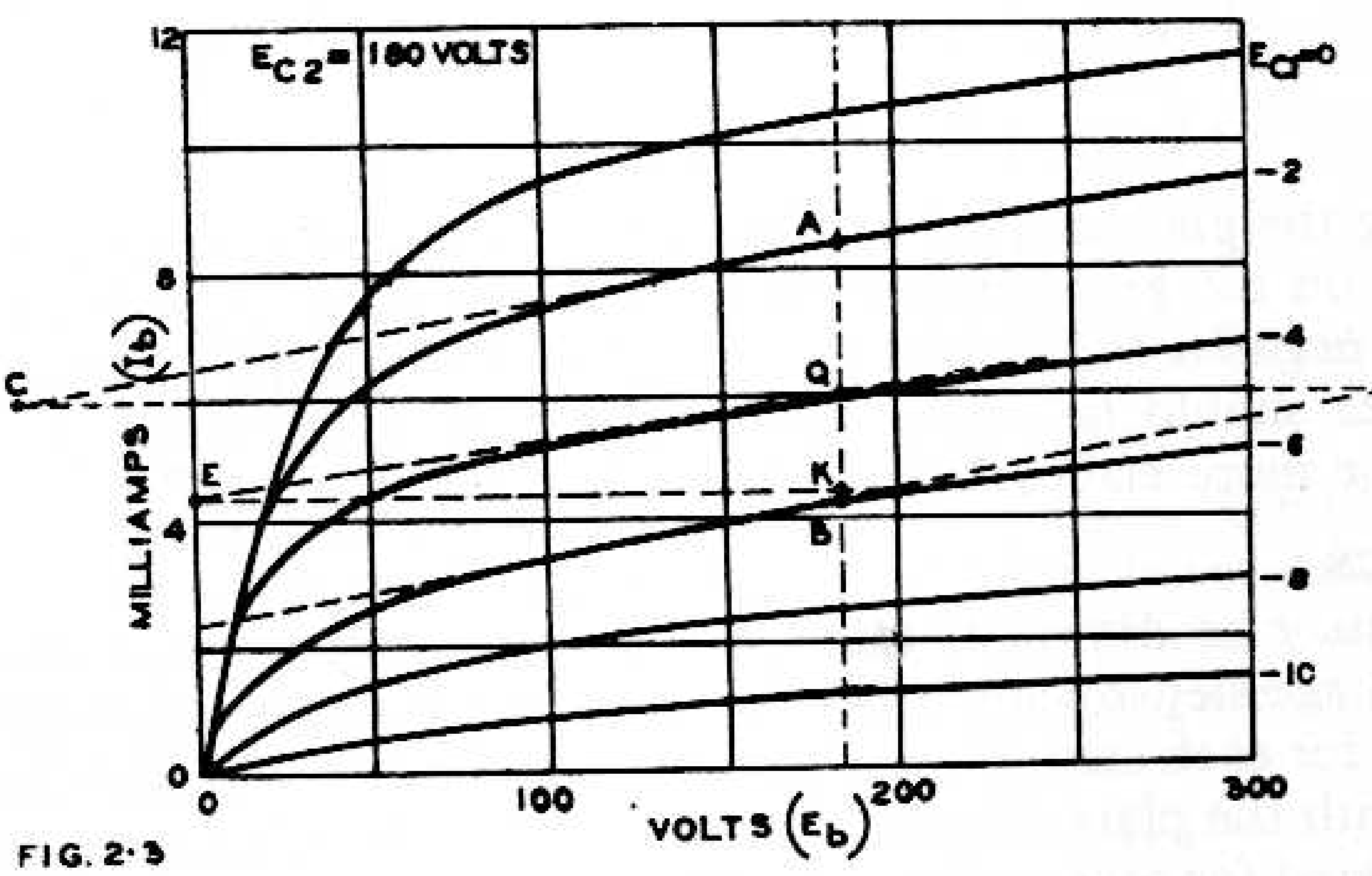


Fig. 2.3. Plate characteristics of a pentode, for one fixed screen voltage.

The plate characteristics of a pentode for one fixed screen voltage are shown in Fig. 2.3. Owing to the high plate resistance of a pentode the slope of the portion of the curves above the " knee " is frequently so flat that it is necessary to draw extended tangents to the curves as at A, B and Q. A horizontal line may be drawn through Q to intersect the tangents at A and B at points C and D. As with a triode, points A and B are vertically above and below Q. The mutual conductance is AB (4.1 mA) divided by 4 volts change of grid bias, that is 1.025 mA/V or 1025 micromhos. The amplification factor is the change of plate voltage (CD = 447 volts) divided by the change of grid voltage (4 volts) or 111.7. The plate resistance is EK/QK, i.e. 180/0.001 65 or 109 000 ohms.

The plate characteristics of a beam tetrode are somewhat similar to those of a pentode except that the " knee " tends to be more pronounced at high values of plate current.

The plate characteristics of a screen-grid or tetrode are in the upper portion similar to a pentode, but the " knee " occurs at a plate voltage slightly greater than the screen voltage and operation below the " knee " is normally inadvisable due to instability.

The plate and screen characteristics of a pentode are shown in Fig. 2.4, from which it will be seen that the total cathode (plate + screen) current for any fixed grid bias is nearly constant, except at low plate voltages, and that the plate current increases at the expense of the screen, and vice versa. A pentode is frequently described as a " constant-current device," but the plate current is not so nearly constant as the combination of plate and screen currents, with fixed grid bias and screen voltage.

(ii) Mutual characteristics†

The Mutual Characteristics may be drawn by maintaining the plate voltage constant, and varying the grid from the extreme negative to the extreme positive voltage desired. For any particular plate voltage, there is a negative grid voltage at which the plate current becomes zero ; this is called the point of plate current cut-off, and any increase of grid voltage in the negative direction has no effect on the plate current, which remains zero. If the mutual characteristic were perfectly straight, the point of plate current cut-off would be at a grid voltage of E_b/μ ; in reality, it occurs at a point slightly more negative, owing to the curved foot of the characteristic.

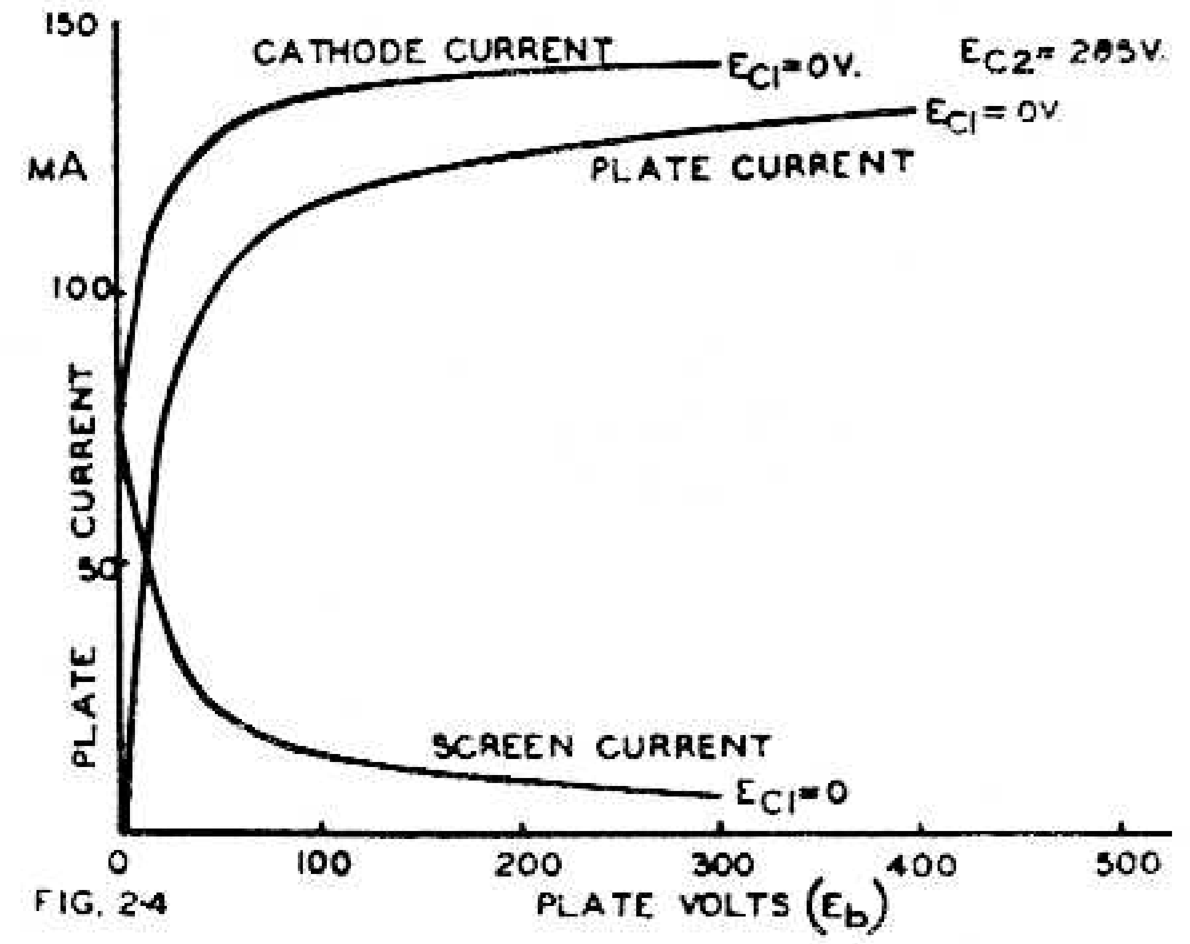


Fig. 2.4. Plate and screen characteristics for a pentode, with fixed screen and grid voltages, showing also the cathode current curve which is the sum of the plate and screen currents at all plate voltages.

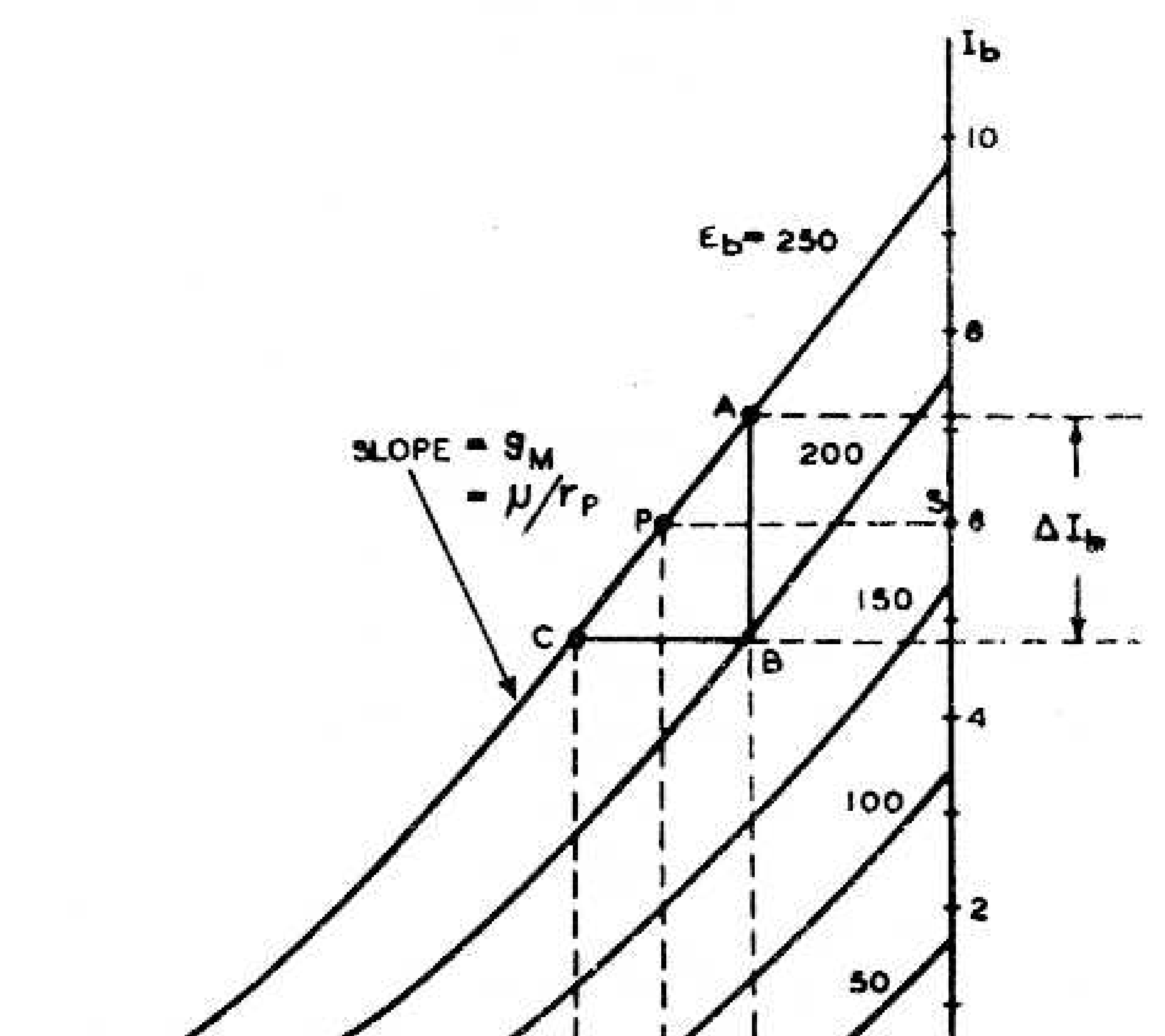


Fig. 2.5. Mutual characteristics of a triode, with one curve for each of five fixed plate voltages.

The Mutual Characteristics of a triode are shown in Fig. 2.5. Each curve corresponds to a constant plate voltage. Let P be a point on the $E_b = 250$ curve, and let us endeavour to find out what information is available from the curves. The bias corresponding to P is given by R (−6 volts) and the plate current is given by S (6 mA). Let now a triangle ABC be constructed so that AP = PC, AB is vertical, CB is horizontal and point B comes on the $E_b = 200$ curve.

The mutual conductance is given by AB/BC or 2·32 mA/4 volts, which is 0·580 mA/volt or 580 micromhos. Thus the slope of the characteristic is the mutual conductance.**

†Also known as Transfer Characteristics.

**This simple construction assumes that A P C is a straight line. In practice it is slightly curved but the construction gives a very close approximation to the slope at point P because the slope of the tangent at P is approximately the slope of the chord joining A and C.

The amplification factor is given by the change of plate voltage divided by the change of grid voltage for constant plate current, that is

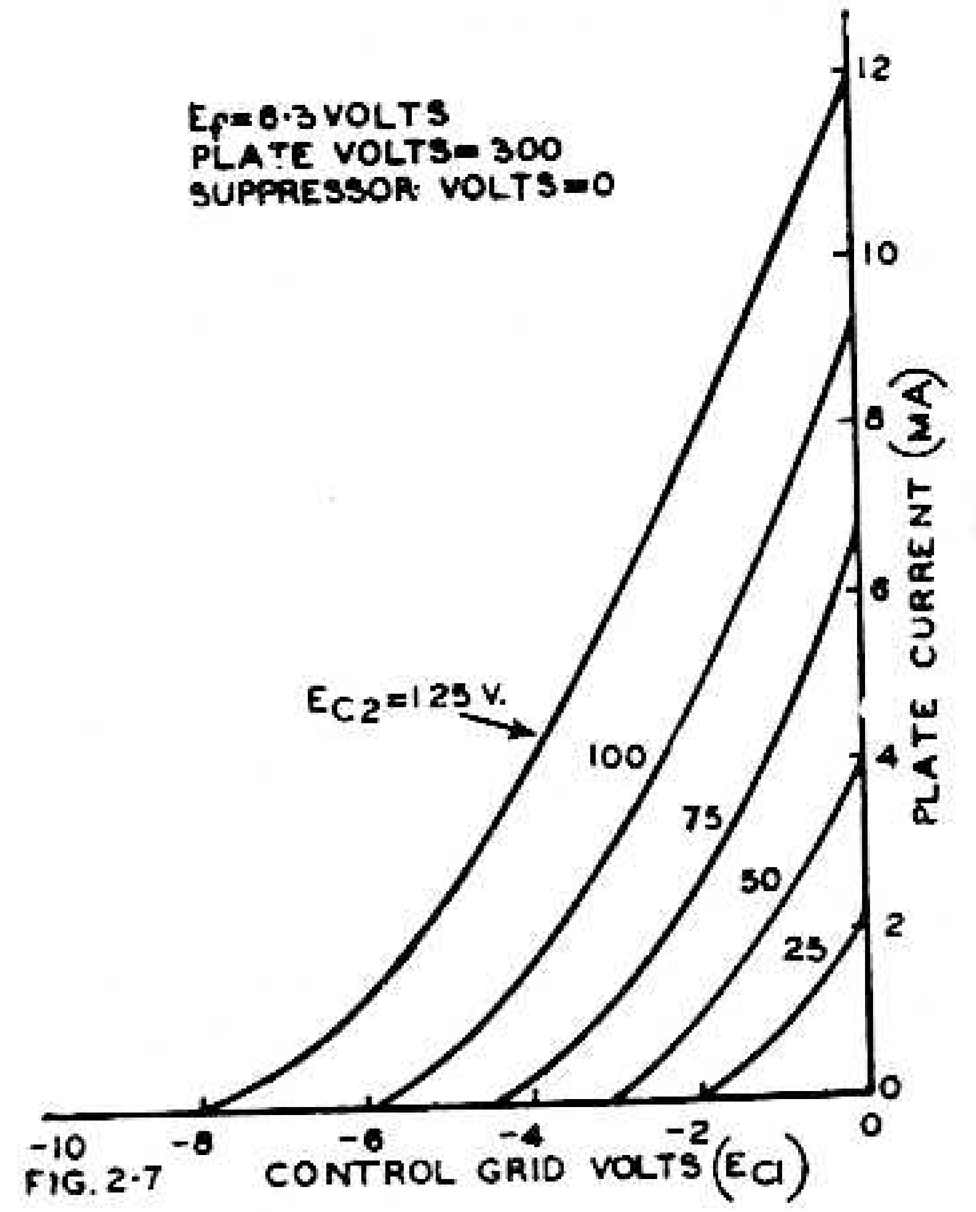
$$\mu = \frac{E_{b1} - E_{b2}}{\text{CB}} = \frac{E_{b1} - E_{b2}}{\triangle E_c} = \frac{250 - 200}{4} = 12{\cdot}5.$$

The plate resistance is given by the change of plate voltage divided by the change of plate current for constant grid voltage; that is

$$r_p = \frac{E_{b1} - E_{b2}}{\text{AB}} = \frac{E_{b1} - E_{b2}}{\triangle I_b} = \frac{250 - 200}{2{\cdot}32 \times 10^{-3}} = 21\ 600 \text{ ohms.}$$

These curves hold only if there is no series resistance in the plate circuit. They could therefore be used for a transformer-coupled amplifier provided that the primary of the transformer had negligible resistance. In the present form they could not be used to predict the operation under dynamic conditions. The static operation point P may, however, be located by their use.

The mutual characteristics of a pentode, for a fixed screen voltage, are very similar to those of a triode except that each curve applies to a different value of screen (instead of plate) voltage. The plate voltage of pentodes having high plate resistance has only a very minor effect on the plate current, provided that it does not come below the screen voltage.



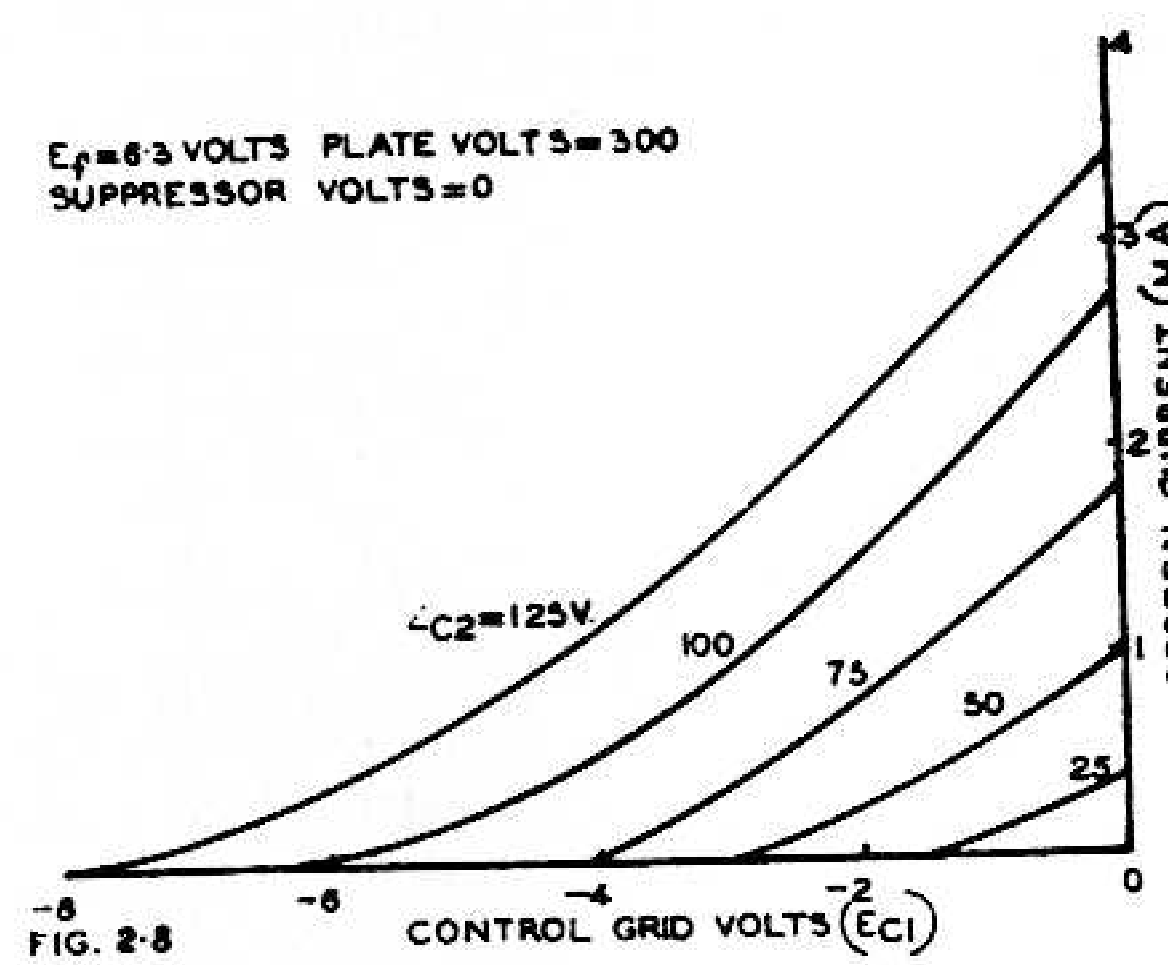


Fig. 2.7. Mutual characteristics of a pentode, with constant plate voltage, and five fixed screen voltages.

Fig. 2.8. Screen current mutual characteristics of a pentode (same as for Fig. 2.7).

The Mutual Characteristic Family for a typical pentode is shown in Fig. 2.7, and the corresponding screen current characteristics in Fig. 2.8.

The resemblance between the shapes of the plate and screen characteristics is very close, and there is an almost constant ratio between the plate and screen currents along each curve.

(iii) Grid current characteristics

Positive grid current in a perfectly hard indirectly-heated valve usually commences to flow when the grid is slightly negative (point Y in Fig. 2.9) and increases rapidly as the grid is made more positive (Curve A). The position of point Y is affected both by the contact potential between grid and cathode and also by the initial electron velocity of emission; the latter is a function of the plate and grid voltages and the amplification factor of the valve, and will therefore vary slightly as the electrode voltages are changed. The grid current commencement point in perfectly hard battery valves is usually slightly positive, so that they may be operated at zero bias with negligible positive grid current (Curve B).

A typical valve at its normal negative bias will have negative (or reverse) grid current which is the sum of gas (ionization) current, leakage current and grid primary emission current. If the two latter are negligibly small, negative grid current (i.e. *gas current*) will be roughly proportional to the plate current, and will increase with the pressure of gas in the valve. If the plate current is maintained constant, the gas current varies approximately as the square of the plate voltage; reduced cathode temperature has little effect on this relationship (Ref. A12). See Chapter 1 Sect. 1 for general information regarding gas current and Chapter 3 Sect. 3(iv)A for the measurement of reverse grid current.

References to grid current characteristics—A12, H1, H2.

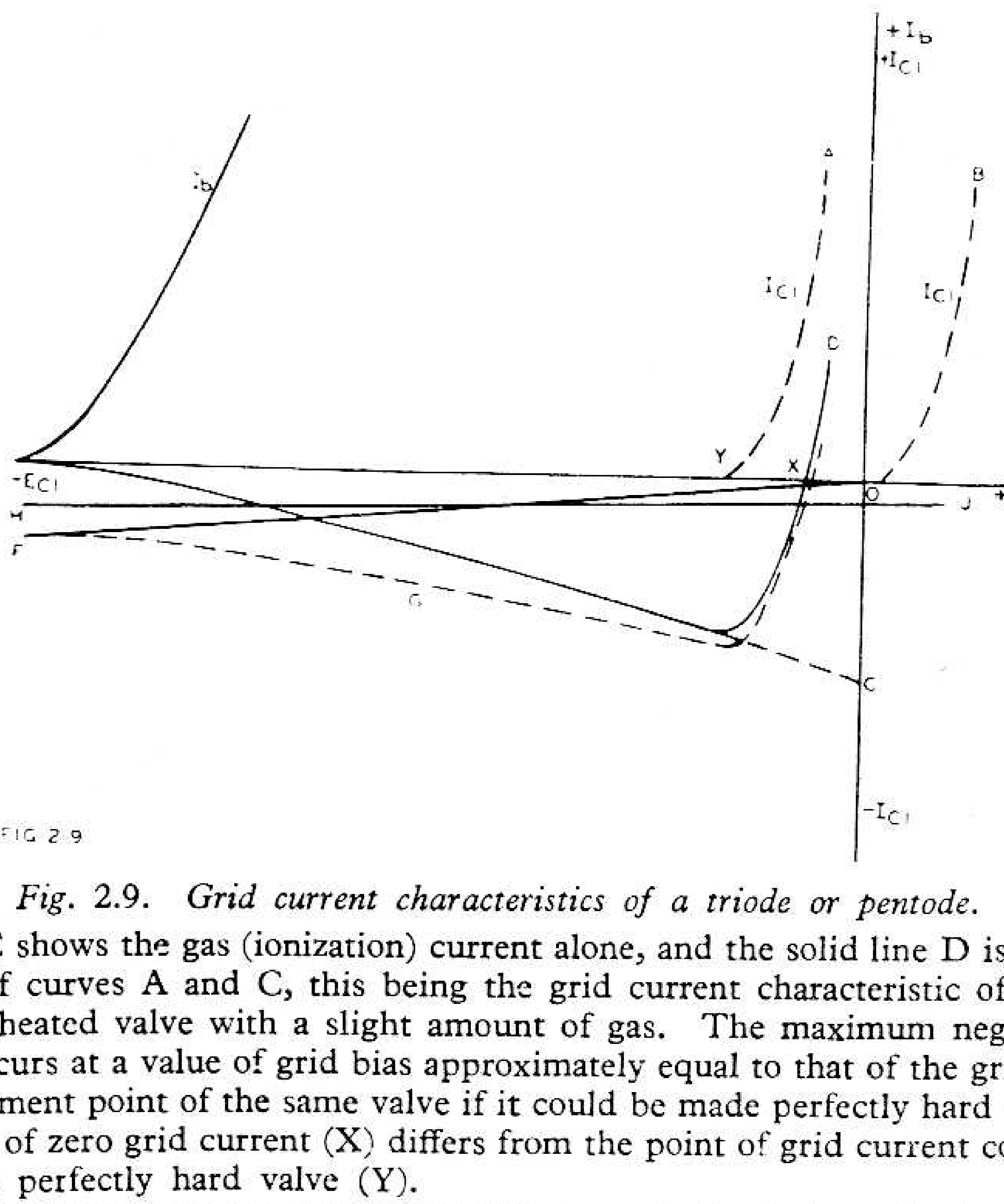


Fig. 2.9. Grid current characteristics of a triode or pentode.

Curve C shows the gas (ionization) current alone, and the solid line D is the combination of curves A and C, this being the grid current characteristic of a typical indirectly heated valve with a slight amount of gas. The maximum negative grid current occurs at a value of grid bias approximately equal to that of the grid current commencement point of the same valve if it could be made perfectly hard (point Y). The point of zero grid current (X) differs from the point of grid current commencement in a perfectly hard valve (Y).

The grid current cross-over point (X) in a new indirectly heated valve is usually between zero and −1·0 volt, and some slight variations in the value are to be expected during life. Change of contact potential between grid and cathode results in a corresponding shift of the mutual characteristics; a change of contact potential in the direction which makes the grid current cross-over point move in the positive direction during the life of the valve will result in decreased plate current, which may be quite serious in a high-mu triode operating on a low plate supply voltage. This is one reason why grid leak bias (with a grid resistor of about 5 or 10 megohms) is often used with such valves, so that the operating point is maintained in the same relation to the mutual characteristic.

In battery type valves the grid current cross-over point may be either positive or negative and designers should allow for some valves with negative values, particularly in cases of low screen voltage operation.

Contact potential is only one of several effects acting on the grid to change the cross-over point (X)—the others include gas current, grid (primary) emission, leakage, and the internal electron velocity of emission.

The grid variational conductance is equal to the slope of the grid characteristic at the operating point. The conductance increases rapidly as the grid voltage is made less than that corresponding to point Y, irrespective of the value of ionization current, so that input circuit damping due to the flow of electrons from cathode to grid (i.e. the positive component of the grid current) occurs in a typical valve even when the grid current is zero or negative (grid voltages between X and Y in Fig. 2.9). **It is possible for the damping on the positive peaks of applied input voltage to be quite serious even when the microammeter reads zero.** This point is applied in connection with r.c. triodes in Chapter 12 Sect. 2(iv).

If the valve has a leakage path between grid and cathode, the leakage current is given by the line OF, which must be added to the gas current to give the grid current characteristic G. If it has leakage between grid and plate (or screen) the leakage current is given by the line HJ, which intersects the horizontal axis at a positive voltage equal to the plate (or screen) voltage ; this also must be added to the other components to provide the grid current characteristic. The combined leakage currents may be measured by biasing the grid beyond the point of plate current cut-off provided that the grid emission is negligibly small—otherwise see below.

Grid emission with a negative grid is the primary emission of electrons due to grid heating from both cathode and plate (or screen) ; it gradually increases as the valve becomes warmer during operation. It increases the total negative grid current and is included with leakage currents in the total negative grid current indicated by a valve tester. For methods of testing to discriminate between the various components of negative grid current, see Chapter 3 Sect. 3(iv)A.

Negative-grid load lines

When a valve is operated with a fixed negative grid bias, but has a total grid circuit resistance R_g, the actual voltage on the grid may differ from the applied bias due to grid current. If negative grid current is present the condition will be as shown in Fig. 2.10 in which OA represents the applied bias. The plate current operating point with no grid current will obviously be Q but if the grid current characteristic is as shown, the grid operating point will be B and the plate operating point Q′. Point B is determined by the intersection of the grid current characteristic and a load line having a slope of $-1/R_g$. The shift in grid bias due to voltage drop across R_g will be $\triangle E_{c1}$ or $R_g.I_{c1}$. The operating point can obviously never be swung beyond the grid-current cross-over point C, so that the static plate current can never go beyond D (Fig. 2.10) due to negative grid current.

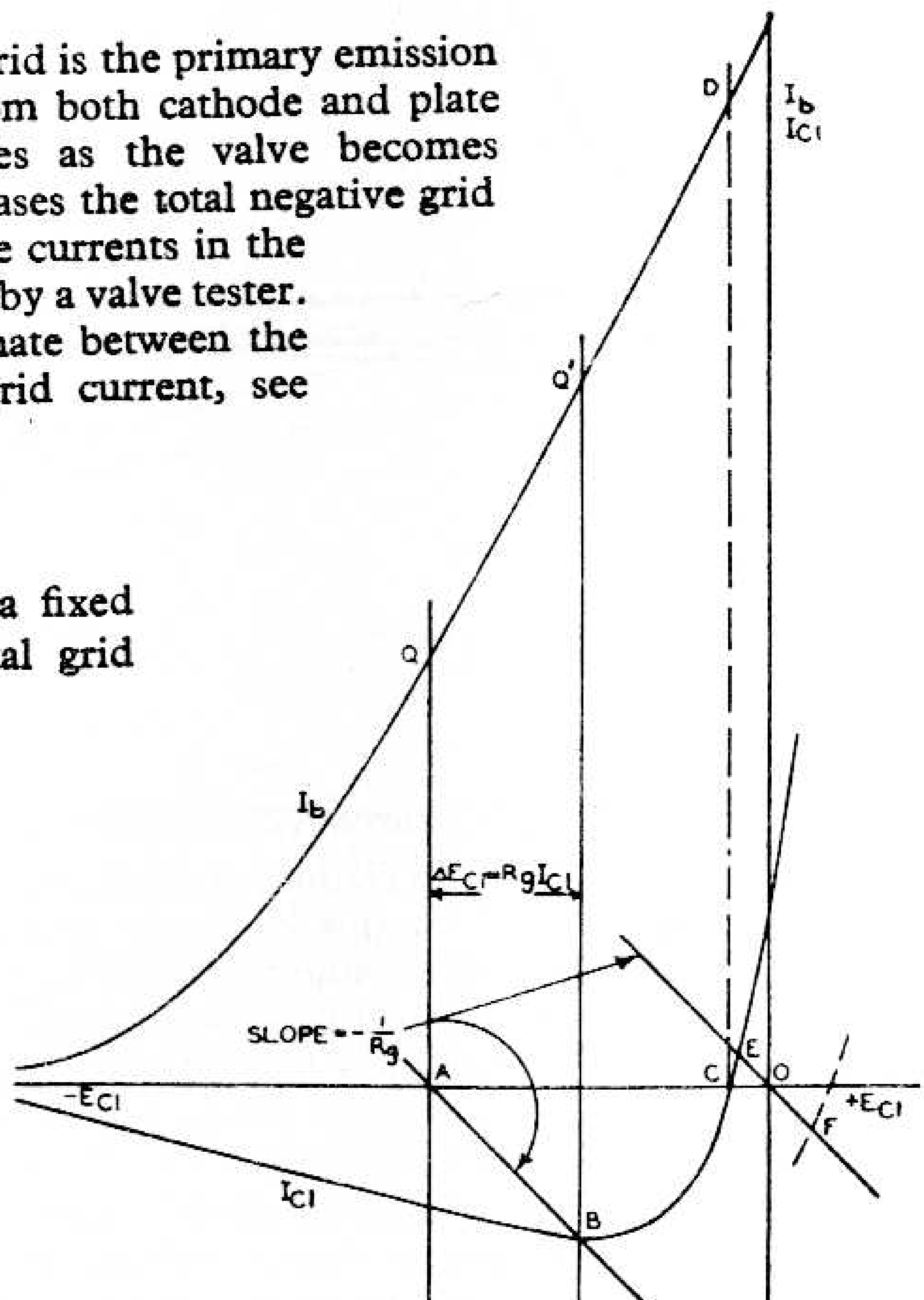


Fig. 2.10. *Grid current characteristics with grid loadlines.*

If valve is operated with its grid completely open-circuited, the operating point will be at D, since this is the only point corresponding to zero grid current, unless the grid characteristic has a second point of zero grid current at a positive grid voltage (see under grid blocking).

If the valve is operated with zero bias, that is with the grid resistor returned to cathode, the grid static operating point will be at E, the intersection of the grid current characteristic and the grid loadline through O. If the valve is one with positive cross-over point, operating at zero bias, the grid static operating point will occur at F.

In all cases considered above, the operating points are for static conditions, and any large signal voltages applied to the grid may have an effect in shifting the operating point. If the signal voltage swings the grid sufficiently to draw positive grid current, the operating point will shift as the result of rectified current flowing through R_g.

The effect of negative grid current on the maximum grid circuit resistance and the operation of a-f amplifiers is described in Chapter 12 Sect. 2(iii) and (iv); Sect. 3(iv)C and (v); also Chapter 13 Sect. 10(i).

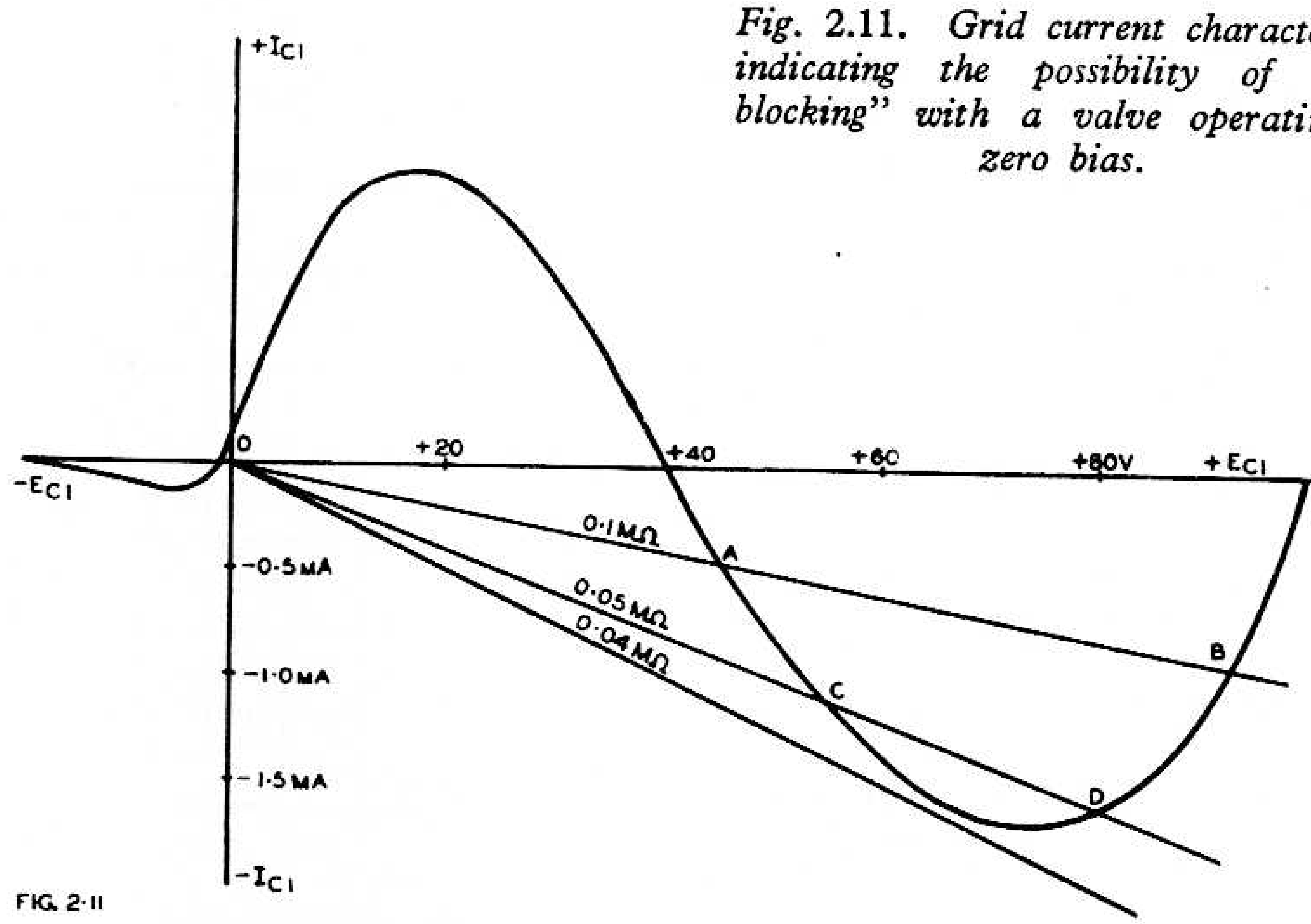


Fig. 2.11. Grid current characteristics indicating the possibility of "grid blocking" with a valve operating at zero bias.

Positive grid voltages and grid blocking

When the grid is made positive, it is bombarded by electrons which cause it to increase in temperature, and it may have both **primary and secondary electron emission.** This current is in a direction opposite to that of positive grid current flow, and may result in a slight kink in the grid characteristic, or may be severe enough to cause the grid current in this region to become negative. A typical case of the severe type is shown in Fig. 2.11, in which the greatest negative grid current occurs at a positive grid voltage of 70 or 80 volts. Such a valve is capable of " grid blocking " if the grid is swung sufficiently positive, and if the grid circuit resistance is high enough. Grid blocking can only occur if the grid loadline cuts the negative loop of grid current. In Fig. 2.11 the 0·1 megohm loadline cuts it at points A and B, but point A is unstable and the grid will jump on to point B and remain there until the valve is switched off, or the grid circuit resistance decreased until the grid loadline no longer cuts the curve (e.g. 0.04 megohm in Fig. 2.11).

(iv) Suppressor characteristics

In some pentodes, the suppressor is brought out to a separate pin, and may be used for some special purposes. Fig. 2.12 shows the mutual characteristics of a pentode suitable for suppressor modulation, although typical of any pentode. The curves are in the shape of a fan pivoted at the cut-off point, with the slope controlled by the suppressor voltage. The curves of electrode currents versus suppressor voltage are given in Fig. 2.13, and indicate that the plate current curve rises fairly steadily from the point of cut-off at a high negative voltage but flattens out while still at a negative suppressor voltage. The screen current falls as the plate current rises, as would be expected, and the suppressor current commences at a slight positive voltage, although in this case it becomes negative at high voltages due to secondary emission.

The suppressor is occasionally used as a detector in receivers, instead of a diode, but its rectification efficiency is low, since the internal resistance is of the order of 20 000 ohms.

In remote cut-off r-f pentodes the suppressor is sometimes used to provide a more rapid cut-off characteristic. A family of mutual conductance and plate resistance curves for a typical remote cut-off pentode are given in Fig. 2.14.

It will be seen the mutual conductance for any fixed control grid voltage (say $E_{c1} = -3$) may be reduced by making the suppressor voltage negative. This has the additional effect, however of decreasing the plate resistance from 0.8 megohm (at $E_{c3} = 0$) to 35 000 ohms at $E_{c3} = -37$, for $E_{c1} = -3$ volts. The initial rate of reduction is very steep, and occurs with all values of control grid voltage.

FIG. 2.12

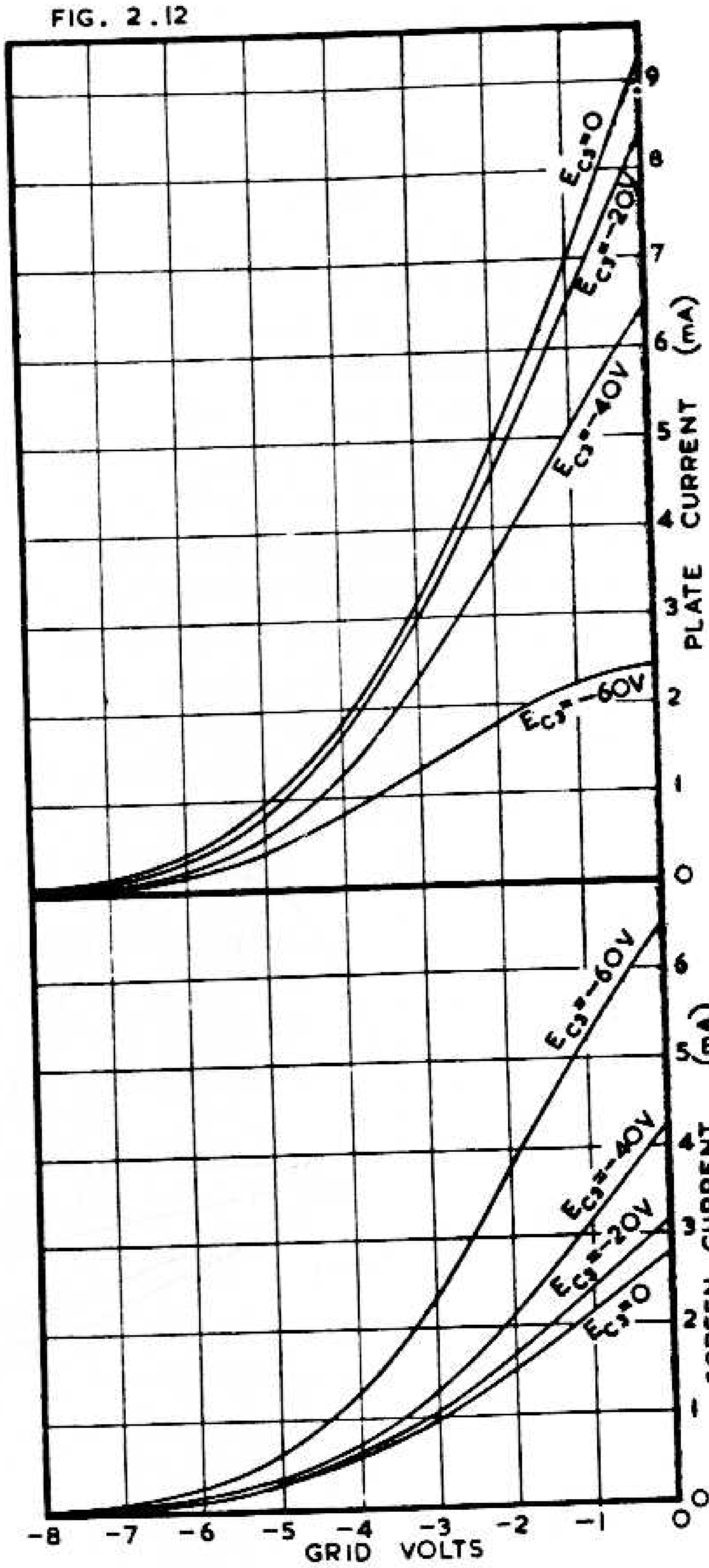


Fig. 2.12. *Mutual characteristics of a pentode* (6SJ7) *for various suppressor voltages.*

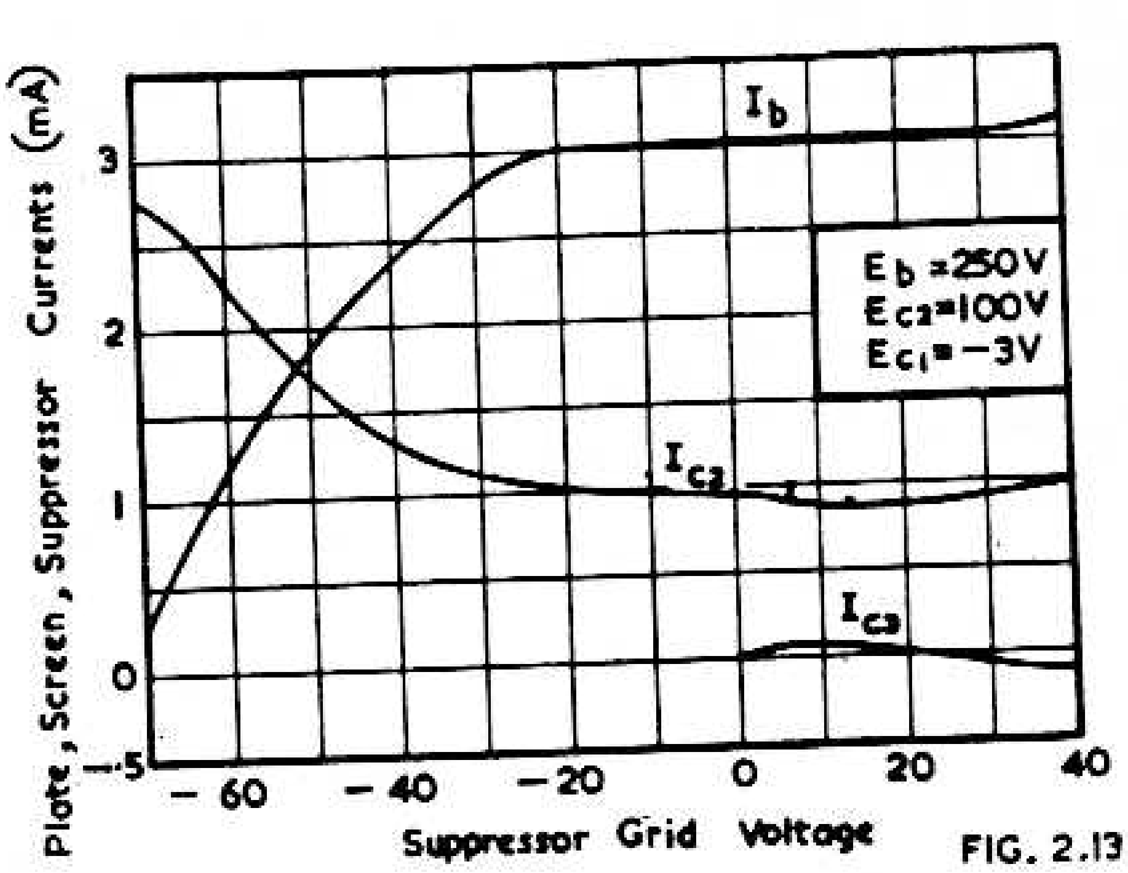


FIG. 2.13

Fig. 2.13. *Suppressor characteristics of a pentode* (6SJ7) *for fixed control grid, screen and plate voltages.*

If the suppressor grid has the same bias control voltage as the control grid, the control characteristic will be as shown by the curve marked "$E_{c1} = E_{c3}$," but in this case the plate resistance, although initially slightly lower for $E_{c3} = -3$ than for $E_{c3} = 0$, rises rapidly as $E_{c1} = E_{c3}$ is made more negative.

(v) Constant current curves

The third principal type of valve characteristic is known as the "Constant Current" Characteristic. A typical family of Constant Current Curves is shown in Fig. 2.15, these being for a typical triode (type 801). The slope of the curves indicates the amplification factor, and the slope of the loadline indicates the stage voltage gain. The operating point is fixed definitely by a knowledge of plate and grid supply voltages, but the loadline is only straight when both plate and grid voltages follow the same law (e.g., both sine wave). Distortion results in curved characteristics, so that this form of representation is not very useful except for tuned-grid tuned-plate or "tank-circuit" coupled r-f amplifiers. Constant Current Curves may be drawn by transferring points from the other published characteristics. For a full treatment the reader is referred to

FIG. 2.14

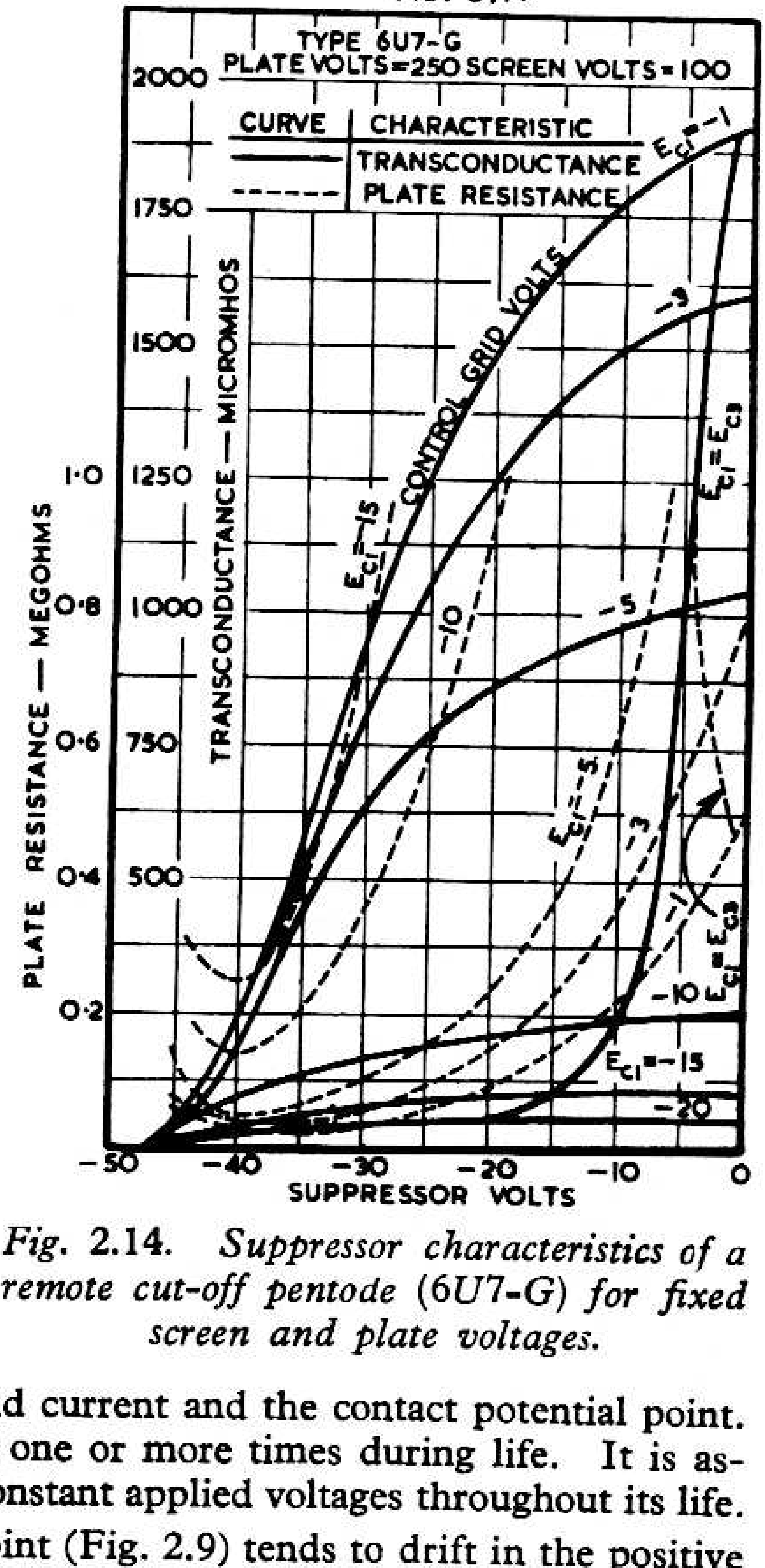


Fig. 2.14. Suppressor characteristics of a remote cut-off pentode (6U7-G) for fixed screen and plate voltages.

(1) Mouromtseff, I. E., and H. N. Kozanowski "Analysis of the operation of vacuum tubes as Class C Amplifiers" Proc. I.R.E. 23.7 (July, 1935) 752 : also 24.4 (April, 1936) 654.

(2) Everest, F. A., "Making life more simple" Radio 221 (July, 1937) 26.

(3) "Reference Data for Radio Engineers" (2nd edition, Federal Telephone and Radio Corporation, New York, 1946).

(vi) "G" curves

Curves of constant g_m and g_p are plotted for a typical triode in Fig. 13.9B. These are helpful in calculating the voltage gain of resistance-coupled triodes and, to a less extent, pentodes, and in other applications. (Refs. B14, B22, B31, B32).

(vii) Drift of characteristics during life

During the life of a valve there is always a slow drift which is particularly apparent in the plate and screen currents, mutual conductance, negative grid current and the contact potential point. The direction of drift sometimes reverses one or more times during life. It is assumed here that the valve is operated at constant applied voltages throughout its life.

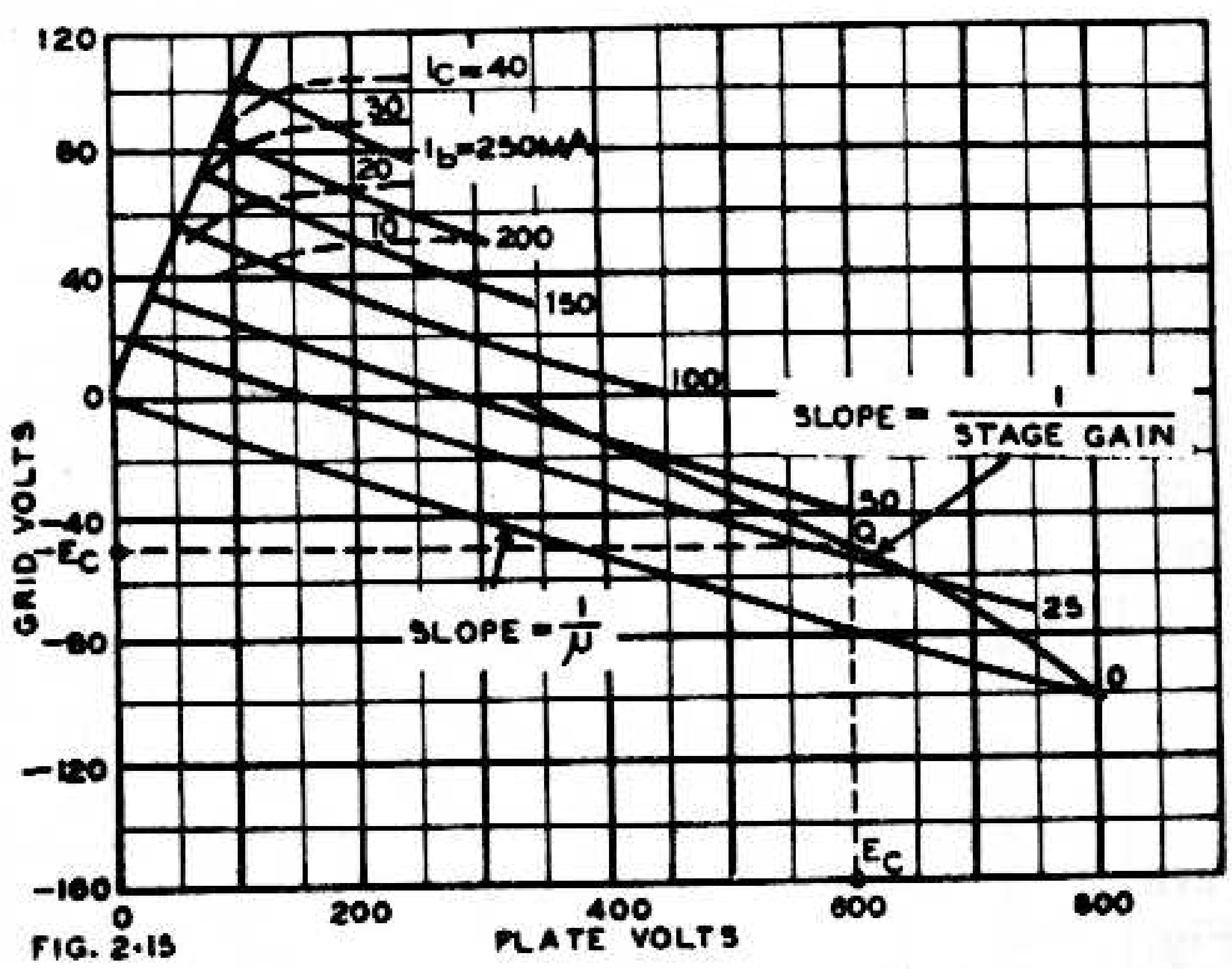
In general, the grid current crossover point (Fig. 2.9) tends to drift in the positive direction during life. The movement of the contact potential point results in a shift of the mutual characteristics which in turn has the effect of reducing the plate current which flows at a fixed grid bias.

Life tests have been carried out (Ref. A12) for a period of 3200 hours on type 6SL7 high-mu twin triodes. The recorded characteristic was the grid voltage to give a plate current of 0·1 mA with a plate voltage of 75 volts. The maximum drift was 0·6 volt (from −1·65 to −1·05 volts), but the majority of the valves did not go outside the limits −1·5 to −1·1 volt (0·4 volt drift). In most cases the drift was generally in a positive direction, but there were two exceptions (out of a total of twelve units) which showed a general tendency to drift in the negative direction for the first hundred hours or so and then to drift in the positive direction, ending up at approximately the same values where they began. However, even those having a positive general direction showed rapid changes in the rate of change, and usually at least one temporary reversal of direction.

It was found that minimum drift occurred for plate currents between 0·1 and 1·0 mA for indirectly-heated types, or between 10 and 100 μA for small filament types.

This drift occurs in diodes and all types of amplifying valves, being particularly noticeable in its effects on high-mu triodes (on account of the short grid base) and on power amplifiers (on account of the decrease in maximum power output). In direct-coupled amplifiers this drift becomes serious, the first stage being the one most affected.

Most of the drift usually occurs during the first hundred hours of operation. If stability is required it is advisable to age the valves for at least 2 days, but in some cases this does not cure the rapid drift. Reference A12, pp. 730-733.



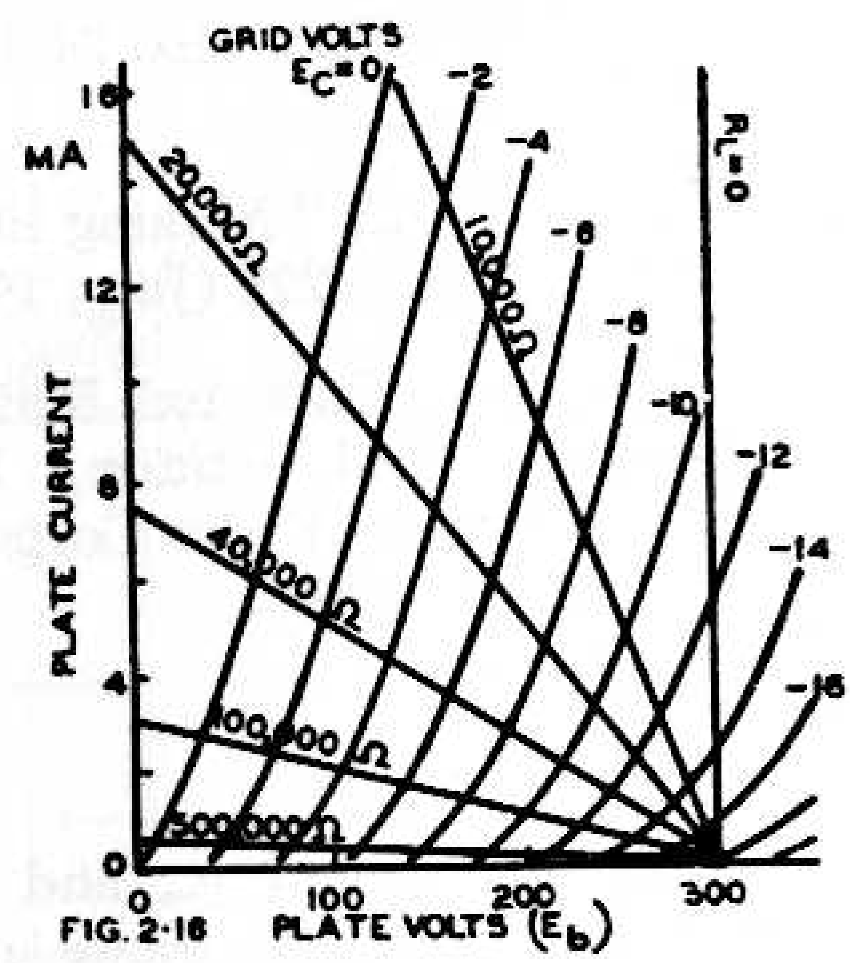


Fig. 2.15. *Constant current characteristics for a typical small transmitting triode* (801).

Fig. 2.16. *Triode plate characteristics with loadlines for five values of load resistance.*

(viii) Effect of heater-voltage variation

When a valve is being operated so that the plate current is small compared with the total cathode emission, an increase in heater voltage normally causes an increase in plate current, which may be brought back to its original value by an increased negative bias. With indirectly-heated cathodes the increase in negative bias is approximately 0·2 volt for a 20% increase in heater voltage, whether the valve is a diode, triode or multi-grid valve (Ref. A12, p. 421).

This effect is serious in d-c amplifiers ; there are methods for cancelling the effect (Ref. A12, p. 458).

SECTION 3 : RESISTANCE-LOADED AMPLIFIERS

(*i*) *Triodes* (*ii*) *Pentodes.*

(i) Triodes

When there is a resistance load in the plate circuit, the voltage actually on the plate is less than that of the supply voltage by the drop in the load resistor,

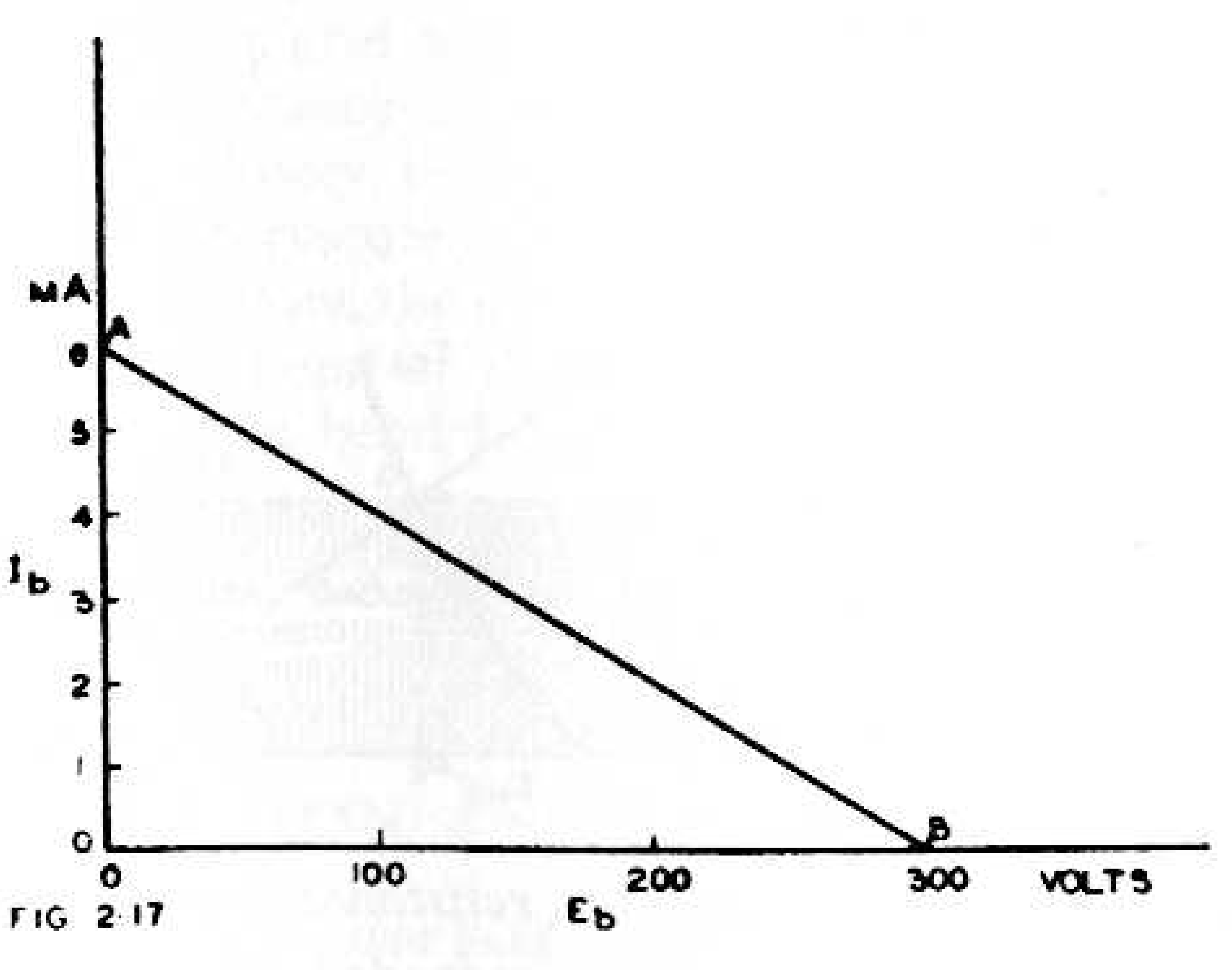
$$E_b = E_{bb} - R_L I_b.$$

This equation may be represented by what is known as a **Load Line** on the plate characteristics. Since the load is a pure resistance it will obey Ohm's Law, and the relationship between current and voltage will be a straight line ; the loadline will therefore be a straight line.

The static operating point is the intersection of the loadline and the appropriate characteristic curve. Fig. 2.16 shows several loadlines, corresponding to different load resistors, drawn on a plate characteristic family. Zero load resistance is indicated by a vertical loadline, while a horizontal line indicates infinite resistance.

A loadline may be drawn quite independently of the plate characteristics, as in Fig. 2.17. The point B is the plate supply voltage E_{bb} (in this case 300 V); the slope* of the loadline AB = $-1/R_L$ and therefore AO = E_{bb}/R_L (in this case 300/50 000 = 0·006 A = 6 mA). The voltage actually on the plate can only be equal to E_{bb} when the current is zero (point B). At point A the voltage across the valve is zero and the whole supply voltage is across R_L; this is what happens when the valve is short-circuited from plate to cathode. The plate current (E_{bb}/R_L) which flows under these conditions is used as a reference basis for the correct operation of a resistance coupled amplifier (Chapter 12). As the plate voltage, under high level dynamic conditions, must swing about the operating point, the latter must be somewhere in the region of the middle of AB; the plate current would then be in the region of 0·5 E_{bb}/R_L and the plate voltage 0·5 E_{bb}—in other words, the supply voltage is roughly divided equally between the valve and the load resistance. Actually, the operating point may be anywhere within the limits 0·4 and 0·85 times E_{bb}/R_L—see Chapter 12 Sect. 2(vi) and Sect. 3(vi).

In most resistance-loaded amplifiers, the plate is coupled by a capacitor to the grid of the following valve, which has a grid resistor R_g to earth. This resistor acts as a load on the previous valve, but only under dynamic conditions. In Fig. 2.18 the loadline AB is drawn, as in Fig. 2.17, and the operating point Q is fixed by selecting the grid bias (here −6 volts). Through Q is then drawn another line CD having a slope of $-(1/R_L + 1/R_g)$; this is the dynamic loadline, and is used for determining the voltage gain, maximum output voltage and distortion (Chapter 12).



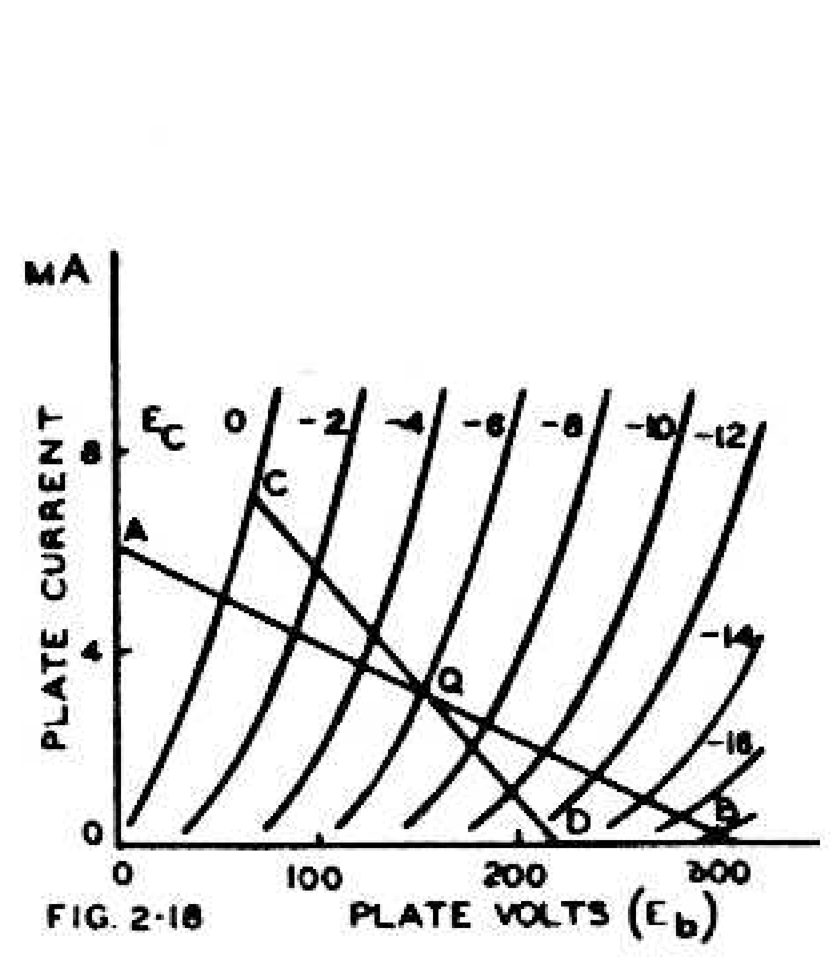


Fig. 2.17. *Loadline is independent of valve curves.*

Fig. 2.18. *Loadlines of resistance loaded triode; AQB is without any following grid resistor, CQD allows for the grid resistor.*

The dynamic characteristic is the effective mutual characteristic when the valve has a resistive load in the plate circuit†. While the slope of the mutual characteristic is g_m or μ/r_p, the slope of the dynamic characteristic is $\mu/(r_p + R_L)$. Owing to $(r_p + R_L)$ being more nearly constant than r_p, the dynamic characteristic is more nearly straight than the mutual characteristic.

*The slope of AB is negative since the plate voltage is the difference between the supply voltage and the voltage drop in R_L, and the inverted form ($1/R_L$) is due to the way in which the valve characteristics are drawn with current vertically and voltage horizontally. The slope of AB is often loosely spoken of as being the resistance of R_L, the negative sign and inverted form being understood.

†It does not make allowance for the following grid resistor, and does not therefore correspond to the dynamic loadline. It is, of course, possible to derive from the dynamic loadline a modified dynamic characteristic which does make allowance for the grid resistor.

74197

A typical dynamic characteristic is shown in Fig. 2.19 applying to a supply voltage of 250 volts and load resistance 0·1 megohm ; the mutual characteristics are shown with dashed lines.

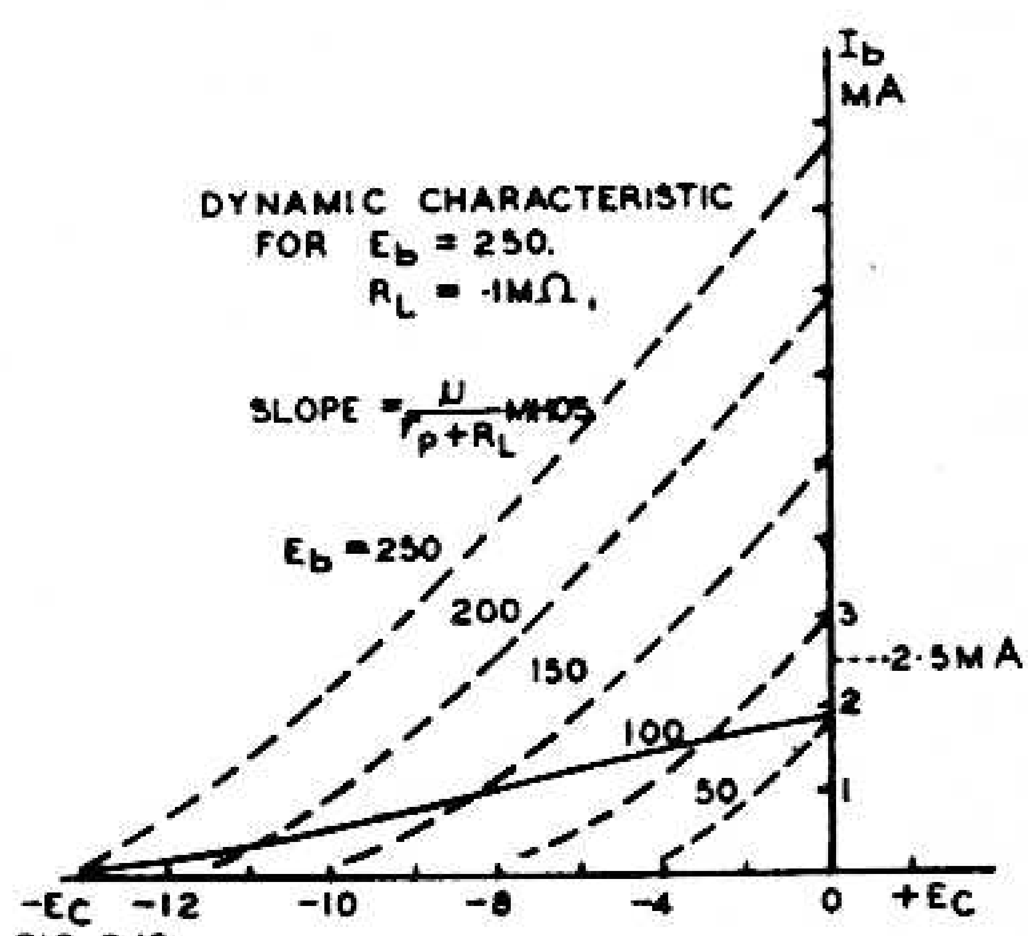
The dynamic characteristic may be drawn by transferring points from along the loadline in the plate characteristic to the mutual characteristic. An alternative method making use of the mutual characteristic is as follows—

When the plate current is zero, the voltage drop in the load resistance is zero, and the plate voltage is equal to the supply voltage (250). For the plate voltage to be 200 volts, there must be a drop of 50 volts in the load resistor (100 000 ohms) and the plate current must therefore be 50/100 000 or 0·5 mA, and so on. A table may be prepared for ease of calculation :

Plate Voltage	Voltage Drop in Load Resistor	Plate Current (= volts drop/R_L)
250	0	0
200	50	0·5 mA.
150	100	1·0 mA.
100	150	1·5 mA.
50	200	2·0 mA.

It will be seen that this table is not affected by the shape of the valve characteristics. The dynamic characteristic may then be plotted by taking the intersections of the various plate voltage curves with the plate current values given in the table.

The dynamic characteristic of a triode is very nearly straight along the central portion, with curves at both ends, the " upper bend " being always in the positive grid current region.



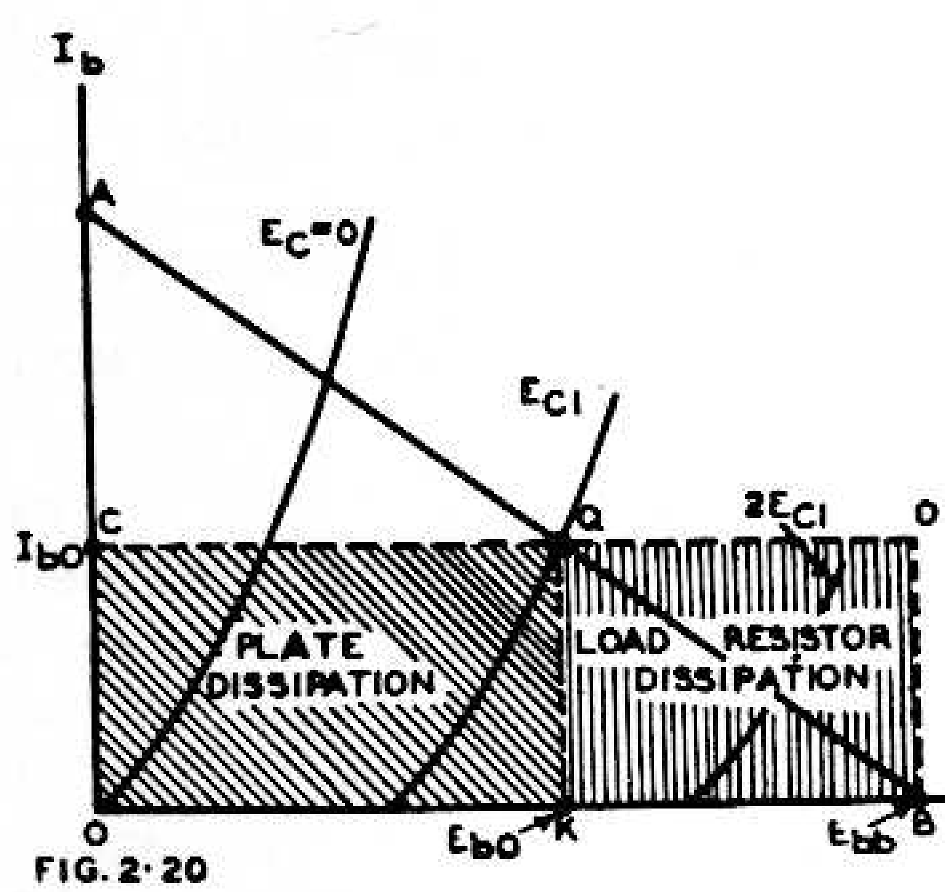


Fig. 2.19. *Triode dynamic characteristic (solid line) for resistance loading.*

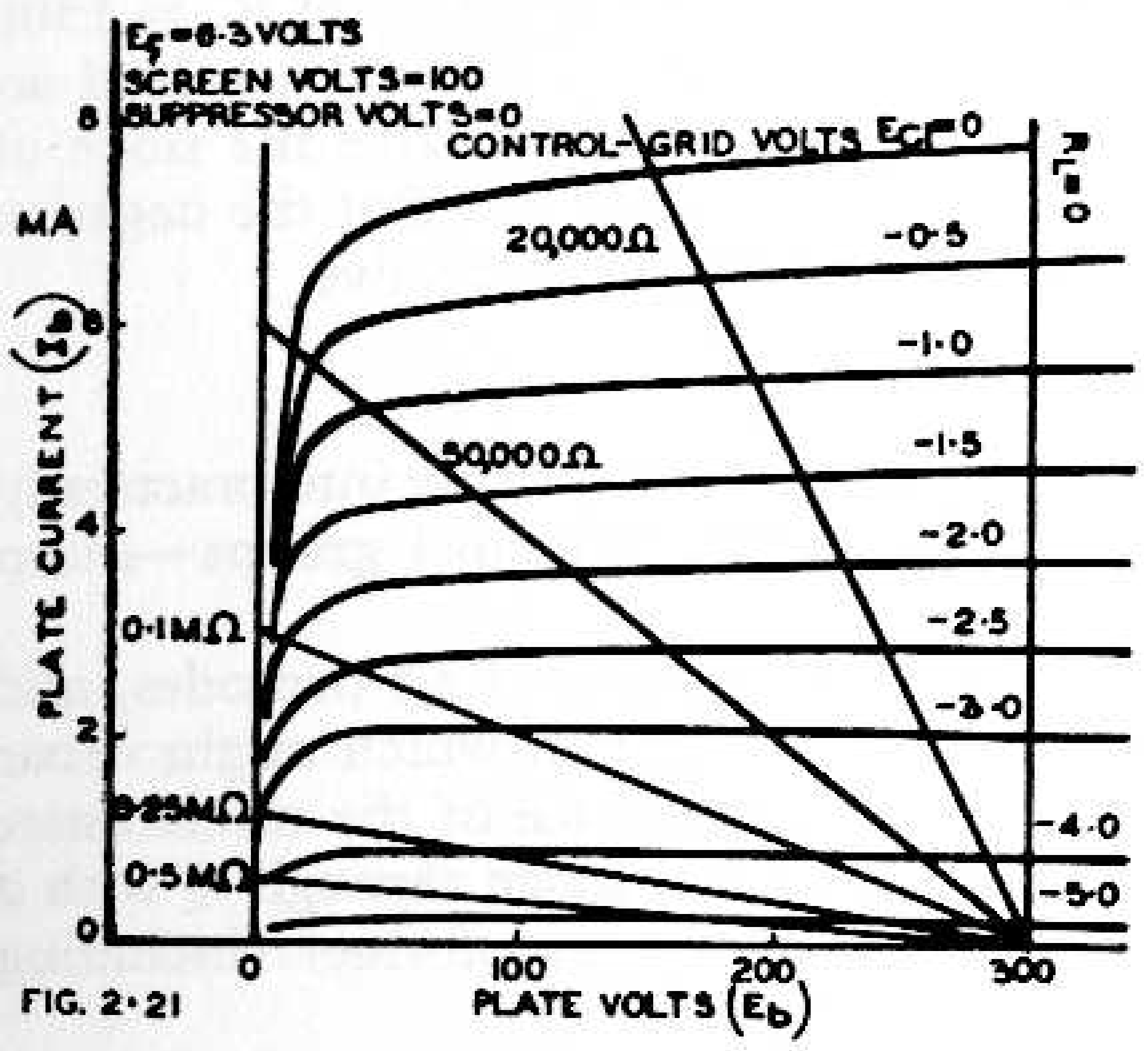
Fig. 2.20. *Illustrating power dissipation in a resistance-loaded triode.*

Refer to Chapter 12 for further information on resistance coupled amplifiers.

When a resistance-loaded triode is operated under steady conditions, the **power dissipation** is indicated by Fig. 2.20. The area of the rectangle OCDB represents the total power ($E_{bb} I_{bo}$) drawn from the plate supply. The area of the rectangle OCQK represents the plate dissipation of the valve ($E_{bo} I_{bo}$) and the area of the rectangle KQDB represents the dissipation in the load resistor $(E_{bb} - E_{bo}) I_{bo} = I_{bo}^2 R_L$. Under dynamic conditions the plate dissipation decreases by the amount of power output, and the load resistor dissipation increases by the same amount, provided that there is no a.c. shunt load and that there is no distortion. The case of transformer-coupled loads is treated in Chapter 13.

(ii) Pentodes

Pentodes with resistive loads are treated in the same manner as triodes, the only complication being the screen voltage which must be selected at some suitable value and maintained constant (Fig. 2.21). The operating point as an amplifier will normally, as with a triode, be in the region of the middle of the loadline so that the voltage across the valve and that across R_L will be approximately the same. The only special case is with very low values of R_L (e.g. 20 000 ohms) where grid current occurs at approximately $E_{c1} = 0$, thereby limiting the useful part of the loadline.



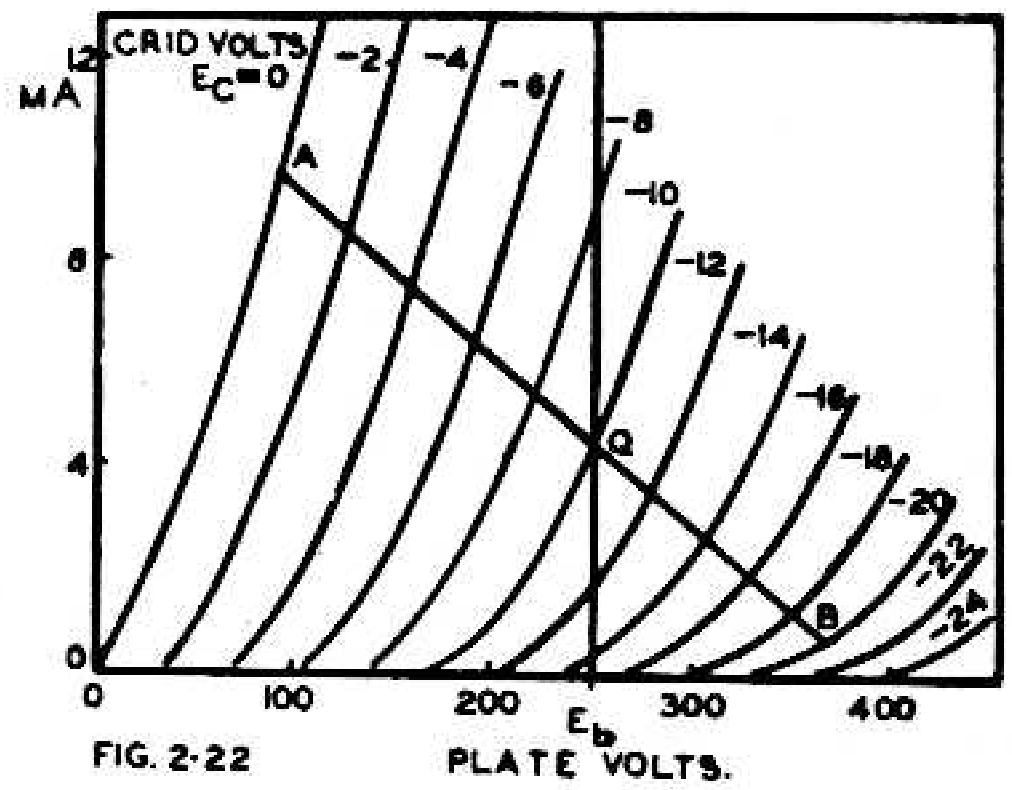


Fig. 2.21. Loadlines of resistance-loaded pentode.

Fig. 2.22. Triode plate characteristics and loadline with transformer-coupled load.

With any value of screen voltage, and any value of load resistance, it is possible to select a grid bias voltage which will give normal operation as an amplifier. With load resistance of 0·1 megohm and above, pentodes give dynamic characteristics which closely resemble the shape of triode dynamic characteristics with slightly greater curvature at the lower end; at the upper end, provided that the screen voltage is not too low, the pentode has a curved portion where the triode runs into grid current. The top bend of the pentode dynamic characteristic is often used in preference to the bottom bend for plate detection—see Chapter 27 Sect. 1(ii)C.

For further information on resistance coupled pentode amplifiers, reference should be made to Chapter 12 Sect. 3.

SECTION 4 : TRANSFORMER-COUPLED AMPLIFIERS

(*i*) *With resistive load.* (*ii*) *Effect of primary resistance* (*iii*) *With i-f voltage amplifiers* (*iv*) *R-F amplifiers with sliding screen* (*v*) *Cathode loadlines* (*vi*) *With reactive loads.*

(i) With resistive load

When the load resistance is coupled to the valve by an ideal transformer, there is no direct voltage drop between the supply voltage and the plate. The slope of the loadline, as before, is $-1/R_L$ but the loadline must be lifted so that it passes through the operating point. Fig. 2.22 shows a typical triode with $E_b = 250$ volts, and $E_c = -10$ volts, thus determining the static operating point Q. The loadline AQB is then drawn through Q with a slope corresponding to a resistance of 30 000 ohms. It is not taken beyond point A ($E_c = 0$) because in this case it is intended to be a Class A amplifier, operating without grid current. It is not taken beyond B because this is the limit of swing in the downward direction corresponding to A in the upward direction and having twice the bias of point Q (i.e. -20 volts). Of course, AB could be projected upwards and downwards if it were desired to increase the grid swing without regard to grid current or distortion.

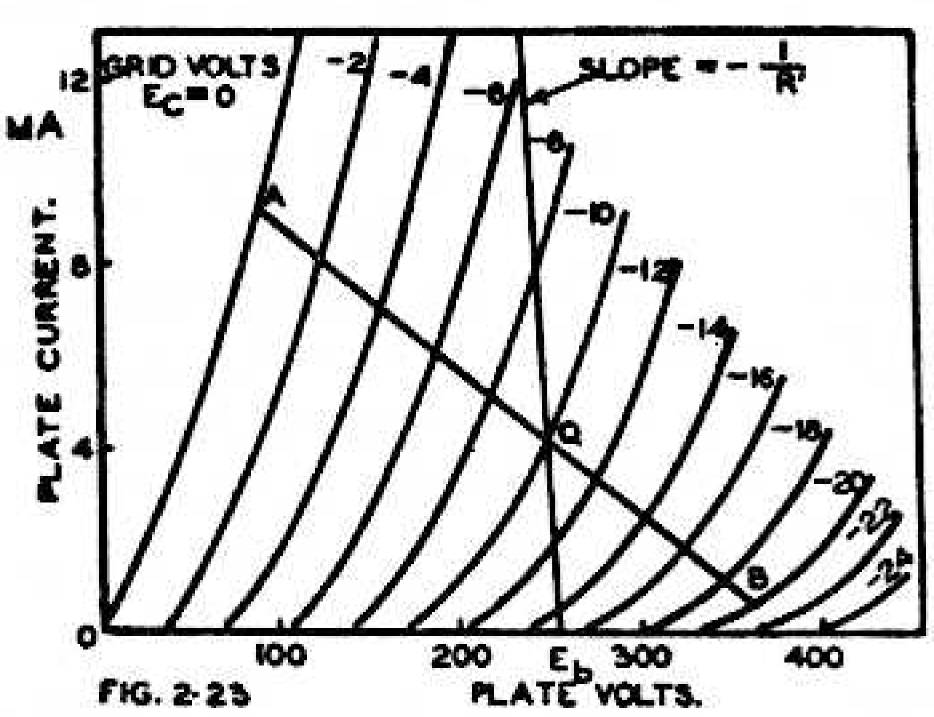
(ii) Effect of primary resistance

If the primary circuit includes resistance, the point Q must be determined by drawing through E_b a straight line with a slope of $-1/R'$, where R' includes all resistances in the primary circuit other than the plate resistance of the valve. R' will include the d.c. resistance of the transformer primary winding and any equivalent internal resistance of the plate supply source. Fig. 2.23 is a typical example, with $R' = 1500$ ohms, from which point Q can be determined as previously. The total a.c. load on the valve is then $(R_L + R')$, in this case 31 500 ohms, which will give the slope of AQB. In these examples it is assumed that fixed bias is used, and that the negative side of the supply voltage is applied directly to the cathode of the valve.

(iii) With i-f voltage amplifiers

I-F amplifiers, when correctly tuned, operate with the valve working into practically a resistive load. I-F and r-f amplifier valves are in two principal groups—sharp cut-off and remote cut-off.

Sharp cut-off r-f pentodes operate in much the same manner as a-f pentodes, and the tuned transformer in the plate circuit reduces any distortion which might occur through non-linearity of the characteristics. The d.c. resistance of the transformer is usually so small that it may be neglected and the loadline drawn through Q with a slope corresponding to the dynamic load resistance of the transformer (including its secondary load, if any, referred to the primary).



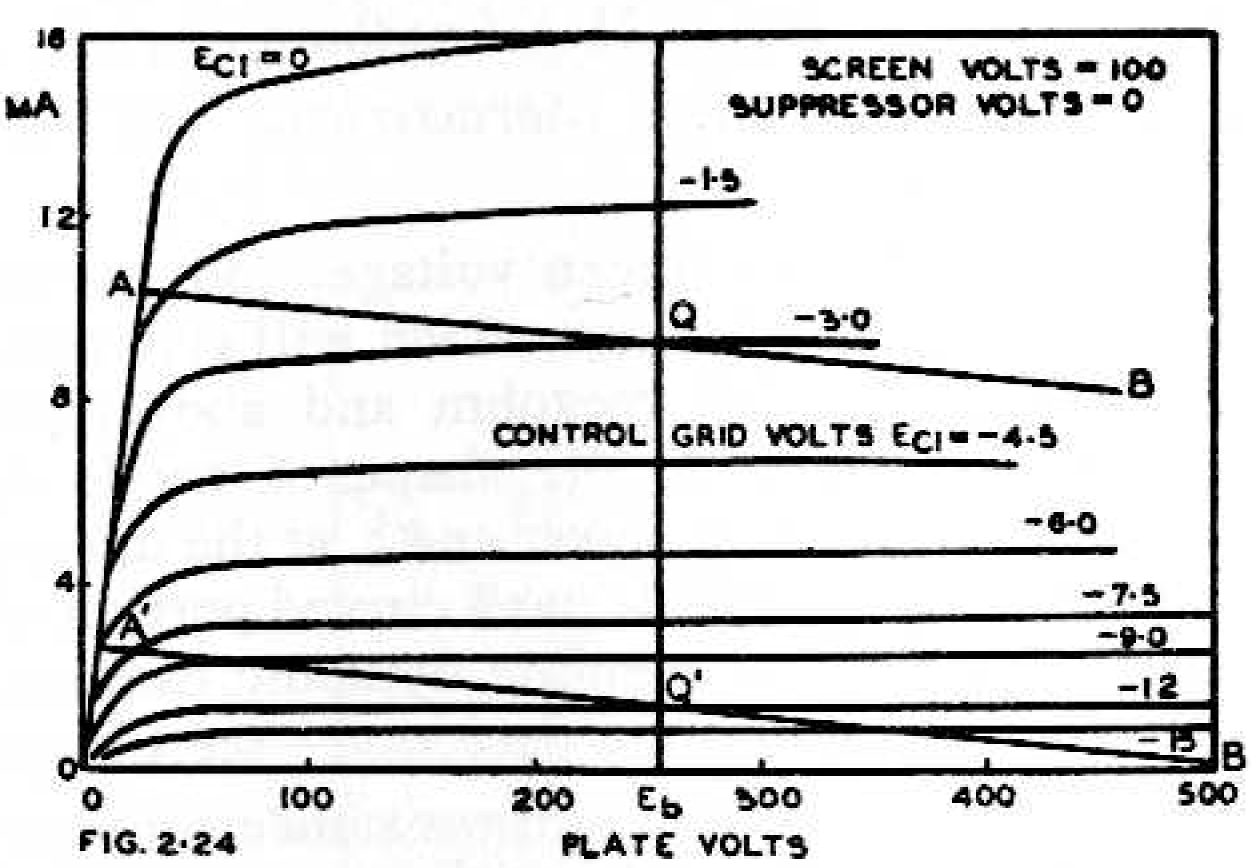


Fig. 2.23. Triode plate characteristics and loadlines with transformer-coupled load, allowing for the resistance of the primary winding.

Fig. 2.24. Plate characteristics of typical remote cut-off pentode with fixed screen and suppressor voltages.

Remote cut-off r-f pentodes are similar, except that the mutual characteristics are curved, and the distortion is greater. Fig. 2.24 shows the plate characteristics of a typical remote cut-off pentode, with $E_b = 250$ volts. Two loadlines (AQB, A′ Q′ B′) have been drawn for grid bias voltages of -3 and -12 volts respectively, with a slope corresponding to a load resistance of 200 000 ohms, as for an i-f amplifier. This application of the loadline is not entirely valid, although it gives some useful information, since the tuned plate circuit acts as a "flywheel" to improve the linearity and reduce the distortion. This is a case in which constant current curves could be used with advantage. However, the ordinary plate characteristics at least indicate the importance of a high Q (high dynamic resistance) second i-f transformer if it is desired to obtain high output voltages at even moderately high negative bias voltages; a steeper loadline would reach plate current cut-off at the high voltage peak.

(iv) R-F Amplifiers with sliding screen

Remote cut-off pentodes may have their cut-off points made even more remote by supplying the screen from a higher voltage (generally the plate supply) through a

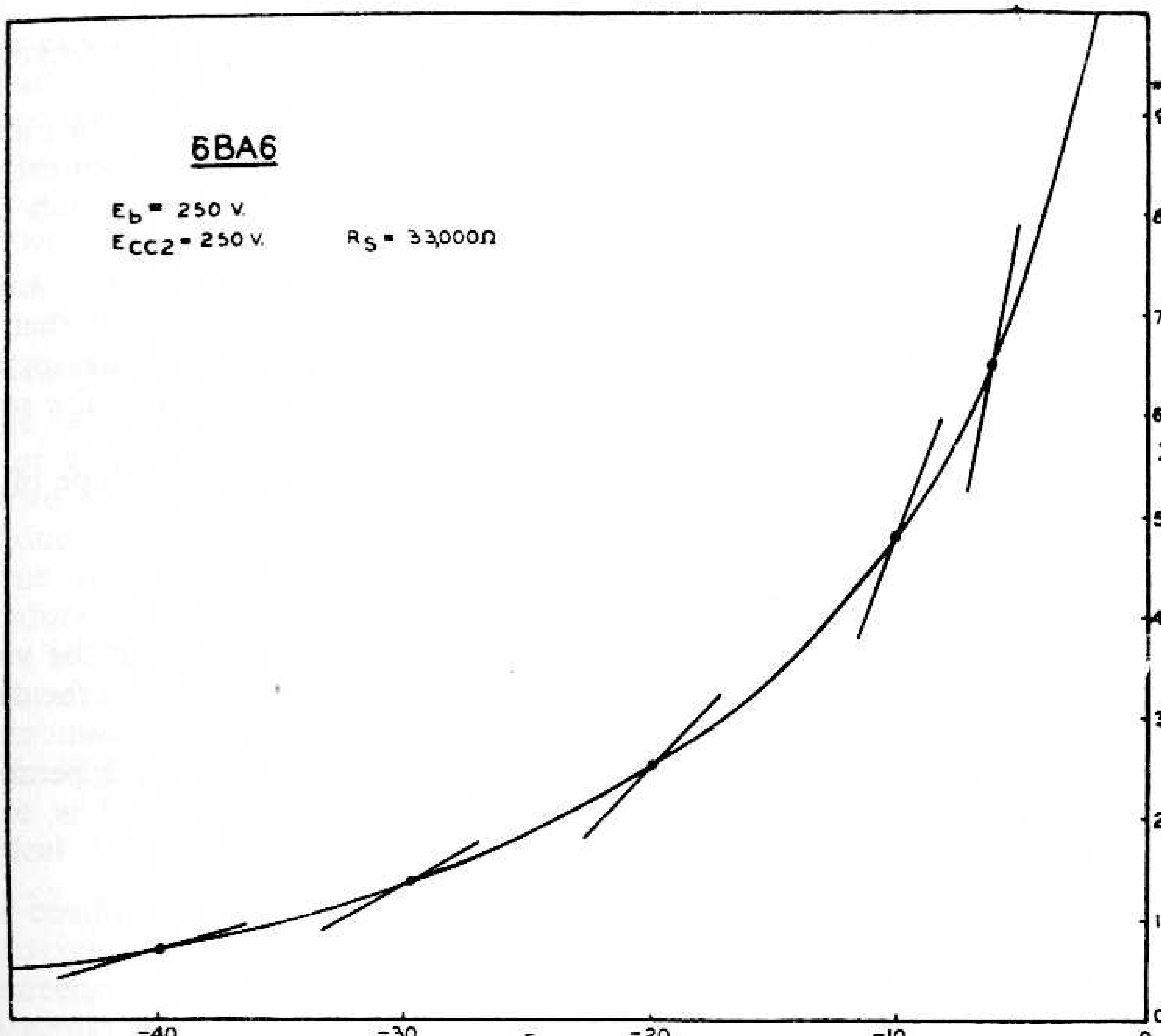


FIG. 2·25

Fig. 2.25. *Plate current characteristic of remote cut-off pentode with "sliding screen." The straight lines indicate the mutual conductances at several points.*

resistor designed to provide the correct screen voltage for the normal (minimum bias) operating condition. The screen requires to be by-passed to the cathode.

The same method may be used with a sharp cut-off pentode to provide a longer grid base. This does not make it possible to obtain the same results as with a properly designed remote cut-off pentode, although it does increase the maximum input voltage which can be handled with a limited distortion. It is important to remember that the extended plate current characteristic curve obtained by this method cannot be used to determine the dynamic slope, since the latter is higher than would be calculated from the characteristic. This is demonstrated in Fig. 2.25 which shows the "sliding screen" plate current characteristic, with straight lines drawn to indicate the mutual conductance at several points.

The procedure for deriving the "sliding screen" plate current characteristic from the fixed voltage data is as follows—

Let plate and screen current curves be available for screen voltages of 50, 75, 100 and 125 volts (Fig. 2.26) and take the case with a series screen resistor (R_s) of 250 000 ohms from a supply voltage of 300.

E_{c2}	E_{drop}*	I_{c2}**	Point	E_{c1}†	Point	I_b‡
50 V	250 V	1·0 mA	A	−0·1	E	3·7 mA
75	225	0·9	B	−1·7	F	3·15
100	200	0·8	C	−3·3	G	2·6
125	175	0·7	D	−5·2	H	2·1

*The voltage drop in the screen resistance = $300 - E_{c2}$.
**$I_{c2} = E_{drop}/R_S$.
†Derived from the screen characteristics and transferred to the plate characteristics.
‡Derived from the plate characteristics.

(v) Cathode loadlines

The static operating point with cathode self bias may be determined graphically by the use of the mutual characteristic. The mutual characteristic of a triode shown in Fig. 2.27 applies to the voltage between plate and cathode—the total supply voltage will be greater by the drop in the cathode resistor R_k.

Through O should be drawn a straight line OD, having a slope of $-1/R_k$ ohms. The point P where OD intersects the curve corresponding to the plate-to-cathode voltage (here 250 V) will be the static operating point, with a bias $-E_{c1}$ and plate current I_{b1}.

In the case of pentodes, with equal plate and screen voltages, the " triode " mutual characteristic should be used, if available. With the plate voltage higher than the screen voltage, the triode mutual characteristic may be used as a fairly close approximation, provided that the triode curve selected is for a voltage the same as the screen voltage.

Alternatively, pentodes may be treated as for triodes, except that the slope of OD should be

$$-\frac{1}{R_k} \cdot \frac{I_b}{I_b + I_{c2}}$$

where I_b and I_{c2} may be taken to a sufficient degree of accuracy as being the values under published conditions. The plate current (I_b') may then be read from the curve, and the screen current calculated from the ratio of screen to plate currents.

For the use of cathode loadlines with resistance coupled triodes and pentodes, refer to Chapter 12.

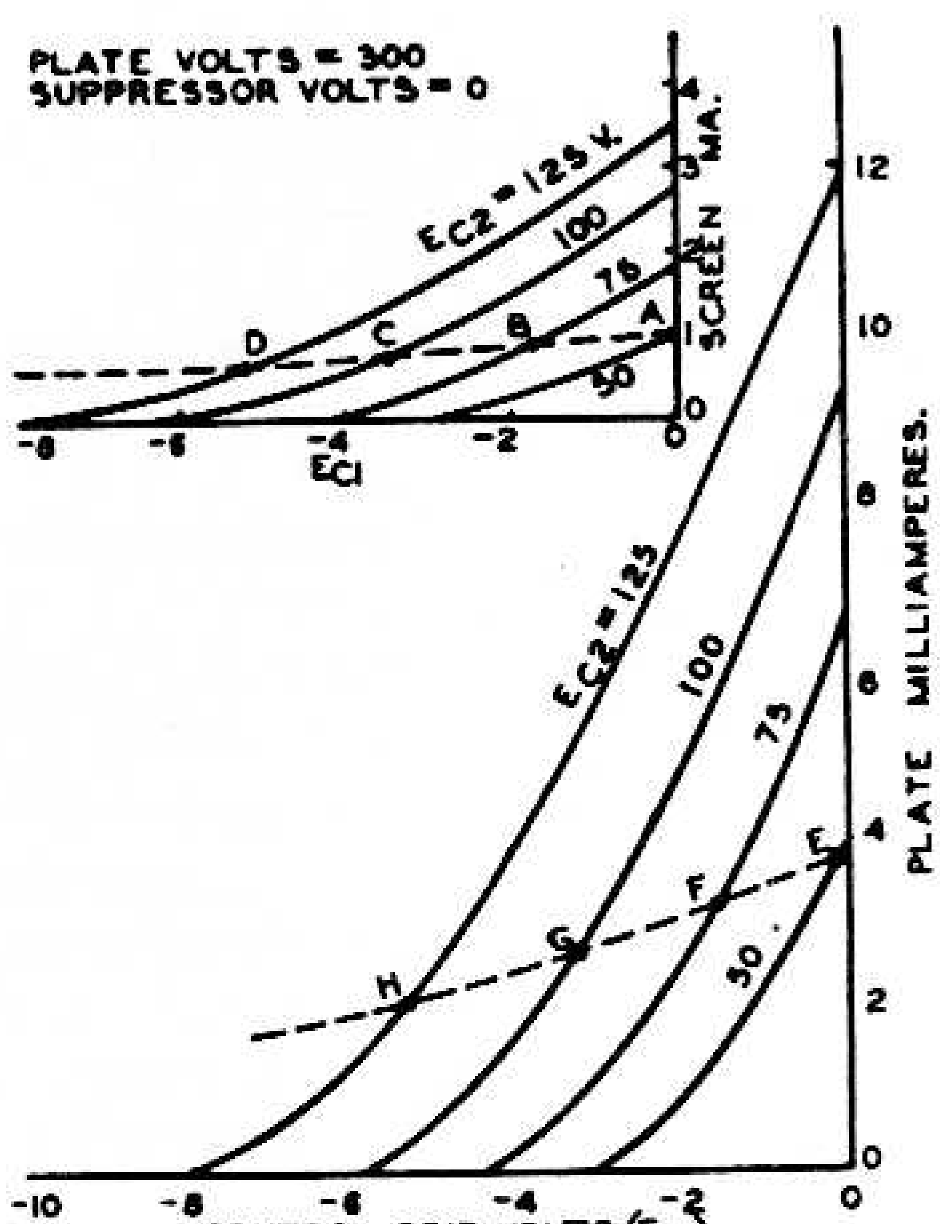


FIG. 2·26

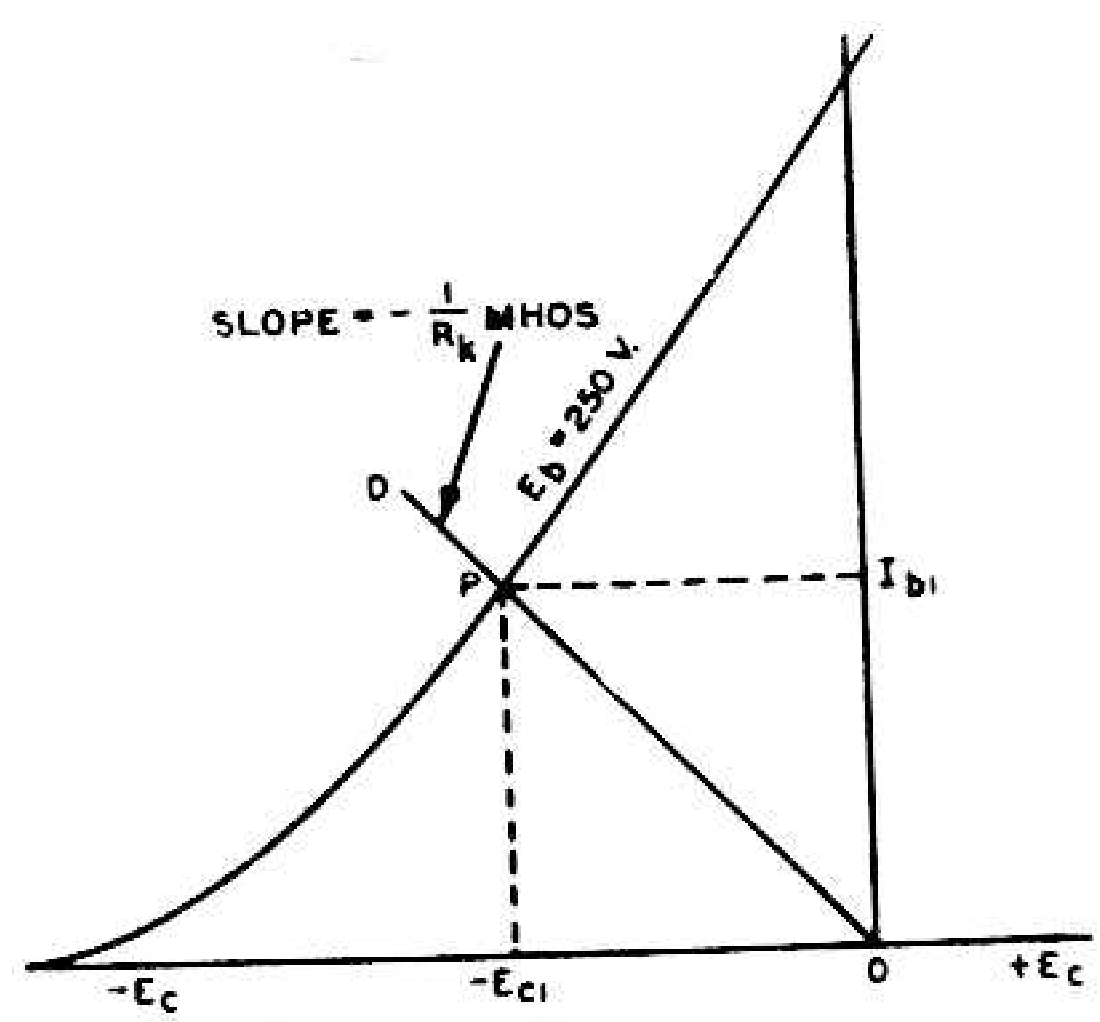


FIG. 2·27

Fig. 2.26. Plate and screen current characteristics of pentode illustrating procedure for deriving "sliding screen" characteristics.

Fig. 2.27. Triode mutual characteristics with cathode bias loadline OD.

(vi) With reactive loads

When the load on the secondary of the transformer is not purely resistive, the loadline is normally in the form of an ellipse instead of a straight line. Fig. 2.28 shows three different examples of elliptical loadlines for purely reactive loads. A purely capacitive load has exactly the same shape of loadline as a purely inductive one, but the direction of rotation of the point is opposite, as indicated by the arrows. Curve A is for a high reactance, curve B for an intermediate value of reactance, and curve C for a low reactance. In each case the maximum current is E_o/X_o where E_o is the peak voltage across the reactance and $X_o = \omega L$ for the inductive case, and $X_o = 1/\omega C$ for the capacitive case. The voltage E_o is shown as negative to the right of O, so as to be suitable for applying directly to the plate characteristics of the valve.

For convenience in application, the horizontal and vertical scales should be the same as in the valve characteristics to which the loadline is to be applied. For example, if on the plate characteristics one square represents 1 mA in the vertical direction and 25 V in the horizontal direction, the same proportion should be maintained for the elliptical loadline. Having drawn the ellipse for any convenient value of E_o, it may be expanded or contracted in size, without changing its shape (that is the ratio of the major to the minor axis when both are measured in inches).

(a) Resistance and inductance in series

The load is more commonly a combination of resistance and reactance. When the load is a resistance R_L in series with an inductive reactance ωL, the maximum current through both will be I_o and the procedure is to draw both the straight resistive loadline for R_L (AB in Fig. 2.29) and the elliptical loadline for ωL, and then to combine them in series. It will be seen that in Fig. 2.29 the peak current of the ellipse and of the resistive loadline are identical (I_o).

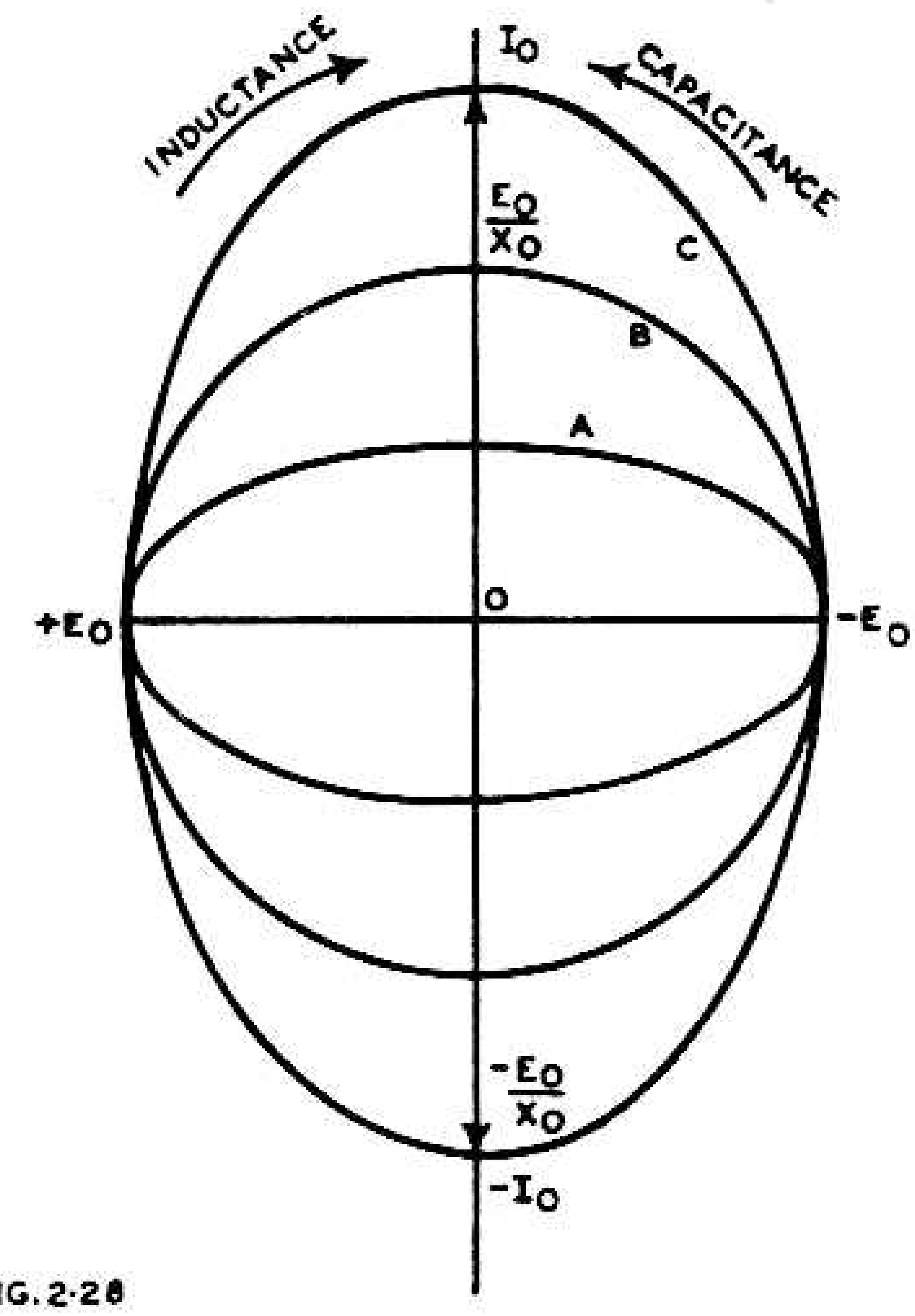


FIG. 2·28

Fig. 2.28. Three examples of elliptical loadlines for purely reactive loads.

To combine these in series, it is necessary to consider the phase relations. When the current is a maximum (OE), the voltage drop across R_L is a maximum (AE) and that across L is zero, because there is 90° phase difference between the voltage and current : the total voltage drop across R_L and L in series is therefore AE and point A is on the desired loadline. When the current is zero, the voltage drop across R_L is zero, and that across L is OC ; the total voltage drop is therefore OC, and point C is on the desired loadline. At any intermediate point (OF) with current increasing the voltage drop across R_L is FG, and that across L is FH, so that the total drop is

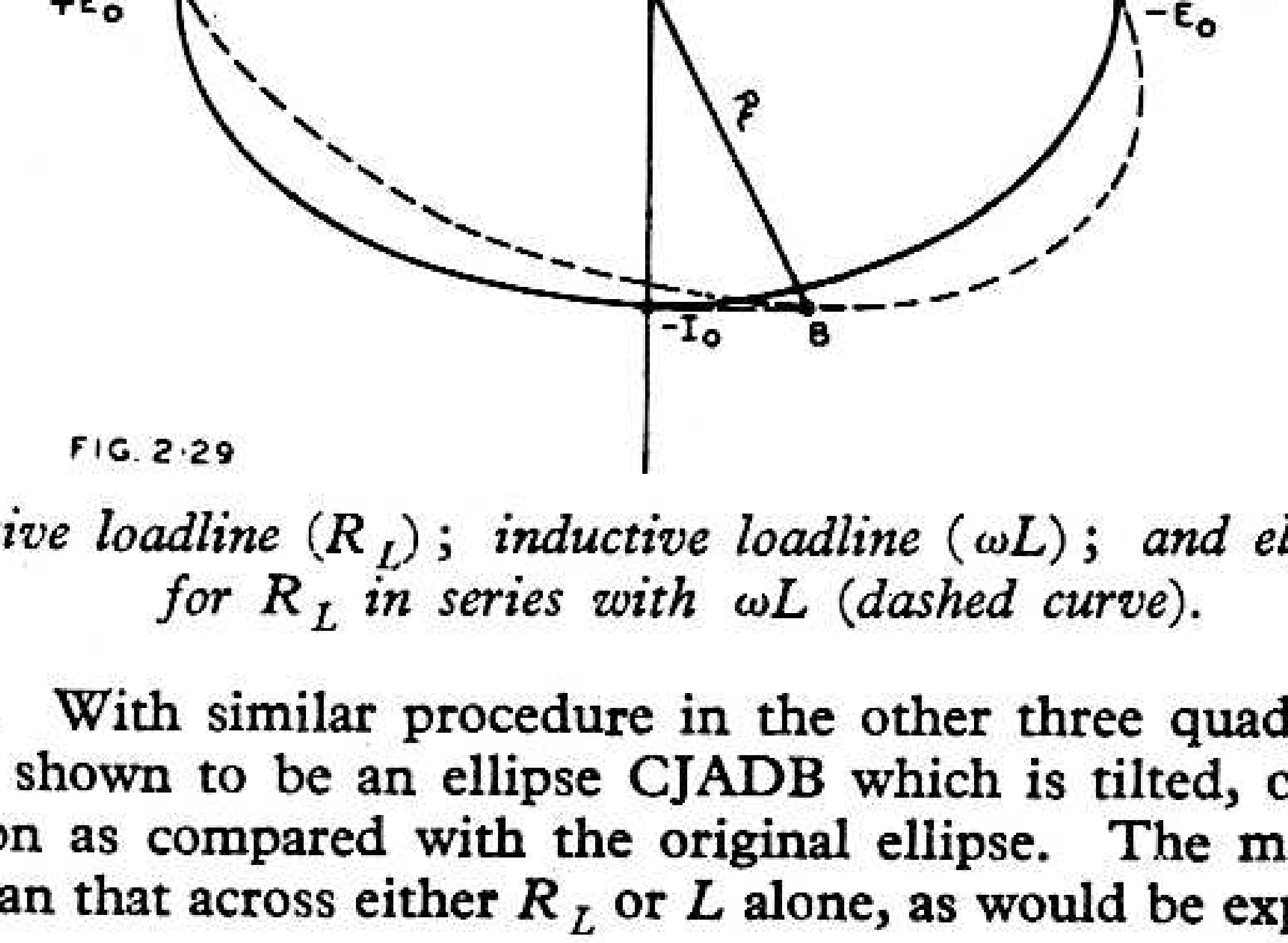


FIG. 2·29

Fig. 2.29. Resistive loadline (R_L) ; inductive loadline (ωL) ; and elliptical resultant for R_L in series with ωL (dashed curve).

FJ = FG + FH. With similar procedure in the other three quadrants, the combined loadline is shown to be an ellipse CJADB which is tilted, or rotated in the clockwise direction as compared with the original ellipse. The maximum voltage drop is greater than that across either R_L or L alone, as would be expected.

If an elliptical loadline is known, as for example the dashed ellipse of Fig. 2.29, its series components may readily be determined. Mark points A and B where the ellipse reaches its maximum and minimum current values, then draw the line AB; the slope of AB gives R_L. Mark O as the centre of the line AB; draw COD horizontally to cut the ellipse at points C and D.

Then $\omega L = E_o/I_o$ ohms,

where E_o = voltage corresponding to length OD

and I_o = current (in amperes) corresponding to max. vertical height of ellipse above line COD.

Alternatively

$$\omega L = \frac{\text{Length of horizontal chord of ellipse through O, in volts}}{\text{Maximum vertical extent of ellipse, in amperes}}.$$

(b) **Resistance and inductance in parallel**

When the load is a resistance R_L in parallel with an inductive reactance ωL, the maximum voltage across both will be E_o, and the resistive loadline and reactive ellipse may be drawn as for the series connection. In this case, however, the currents have to be added. In Fig. 2.30 the maximum current through R_L is CK (corresponding to $+E_o$), while the maximum current through L is OE. When the voltage is zero and increasing, the current through R_L is zero, and that through L is the minimum value OP; point P is therefore on the desired loadline. When the voltage is its positive maximum (OC), the current through R_L is CK and that through L is zero; point K is therefore on the desired loadline. Similarly with points E and M. At an intermediate value, when the voltage is negative and approaching zero (OR), the current through R_L is RS, and that through L is RT; the total current is therefore RT + RS = RW, and W is on the desired loadline. The loadline is therefore the ellipse PKEMW.

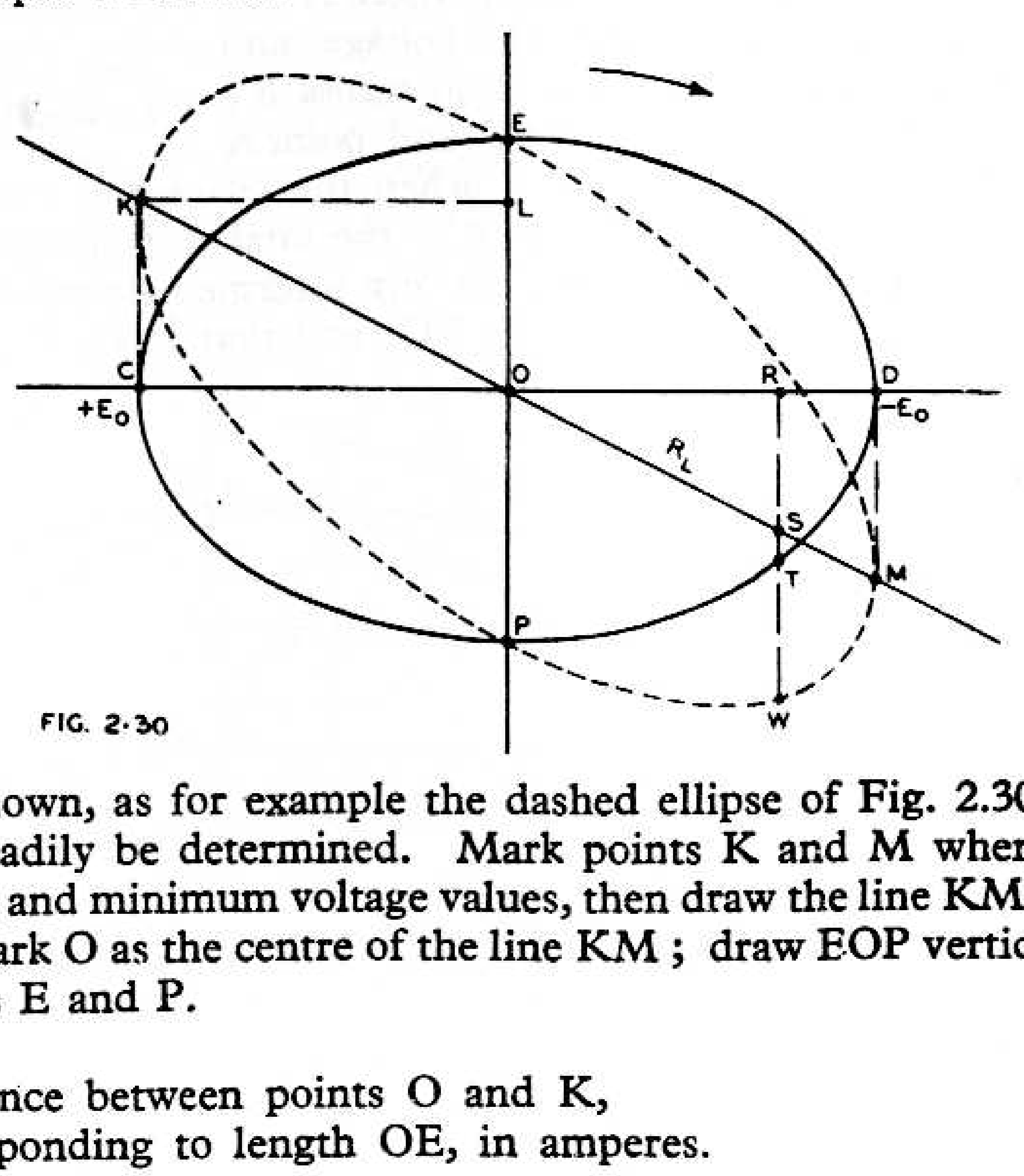


Fig. 2.30. *Resistive loadline* (R_L); *inductive loadline (solid ellipse); and elliptical resultant for* R_L *in parallel with* ω_L *(dashed curve).*

If an elliptical loadline is known, as for example the dashed ellipse of Fig. 2.30, its parallel components may readily be determined. Mark points K and M where the ellipse reaches its maximum and minimum voltage values, then draw the line KM; the slope of KM gives R_L. Mark O as the centre of the line KM; draw EOP vertically to cut the ellipse at points E and P.

Then $\omega L = E_o/I_o$ ohms,

where E_o = voltage difference between points O and K,

and I_o = current corresponding to length OE, in amperes.

Alternatively

$$\omega L = \frac{\text{Maximum horizontal length of ellipse, in volts}}{\text{Length of vertical chord of ellipse through O, in amperes}}.$$

(c) **Resistance and capacitance**

A similar shape of loadline is obtained when the inductance is replaced by a capacitance of equal reactance, except that the direction of rotation is opposite.

(d) Applying elliptical loadlines to characteristics

The elliptical loadlines derived by the methods described above may be applied to the plate characteristics of a valve, but it is first necessary to enlarge or reduce their size until they just fit between grid voltage curves corresponding to extreme swing

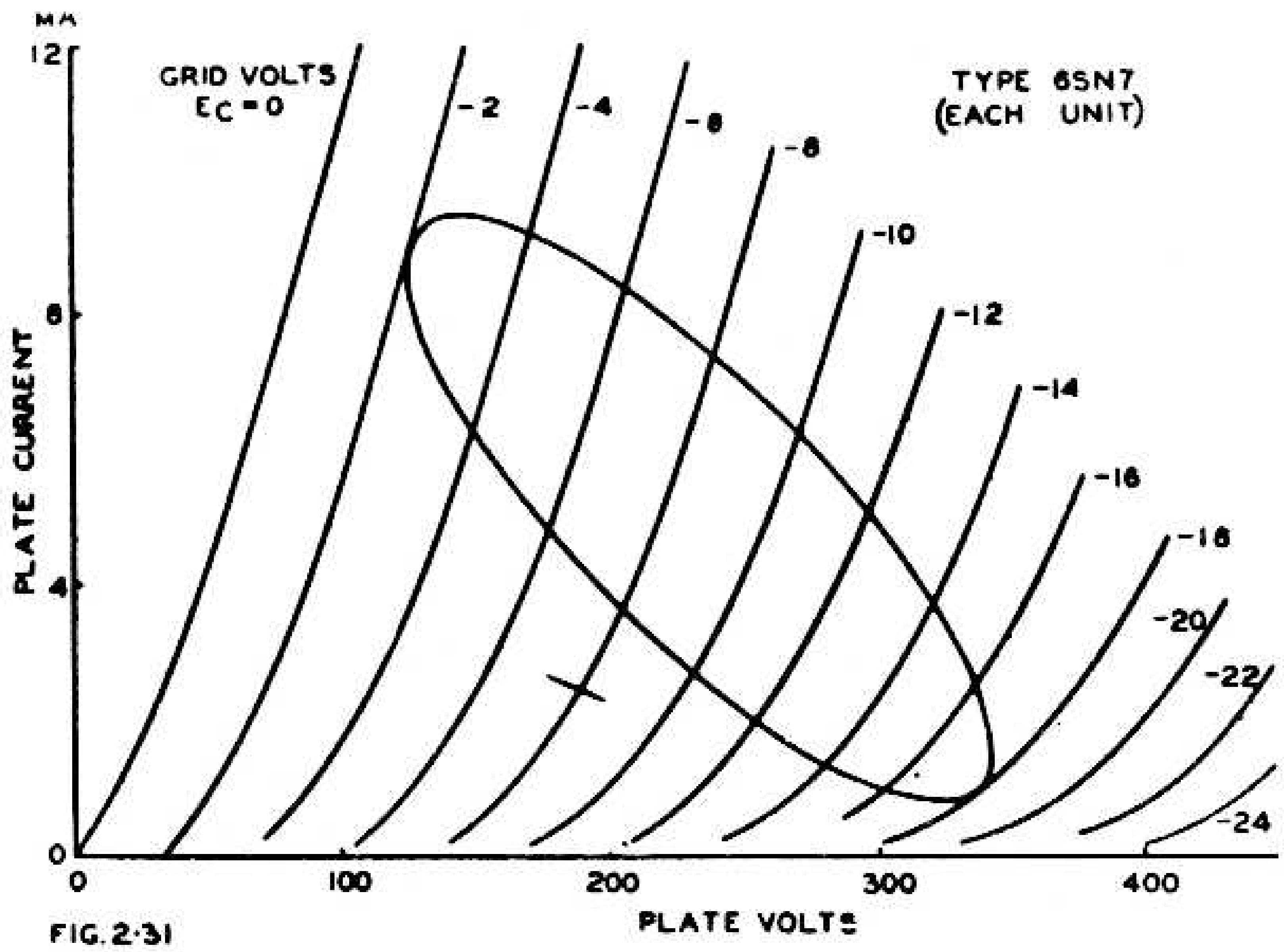


Fig. **2.31.** *Triode plate characteristics with elliptical loadlines corresponding to resistance* 25 000 *ohms in series with reactance of* 18 000 *ohms.*

in each direction. The examples taken have all been based on an arbitrary current (I_o) or voltage (E_o), which may be made larger or smaller as desired. In Fig. 2.31 there is shown the elliptical loadline corresponding to a resistance of 25 000 ohms in series with a reactance of 18 000 ohms, on triode plate characteristics with $E_b = 250$ volts, $E_c = -10$ volts and peak grid amplitude $E_{gm} = 8$ volts.

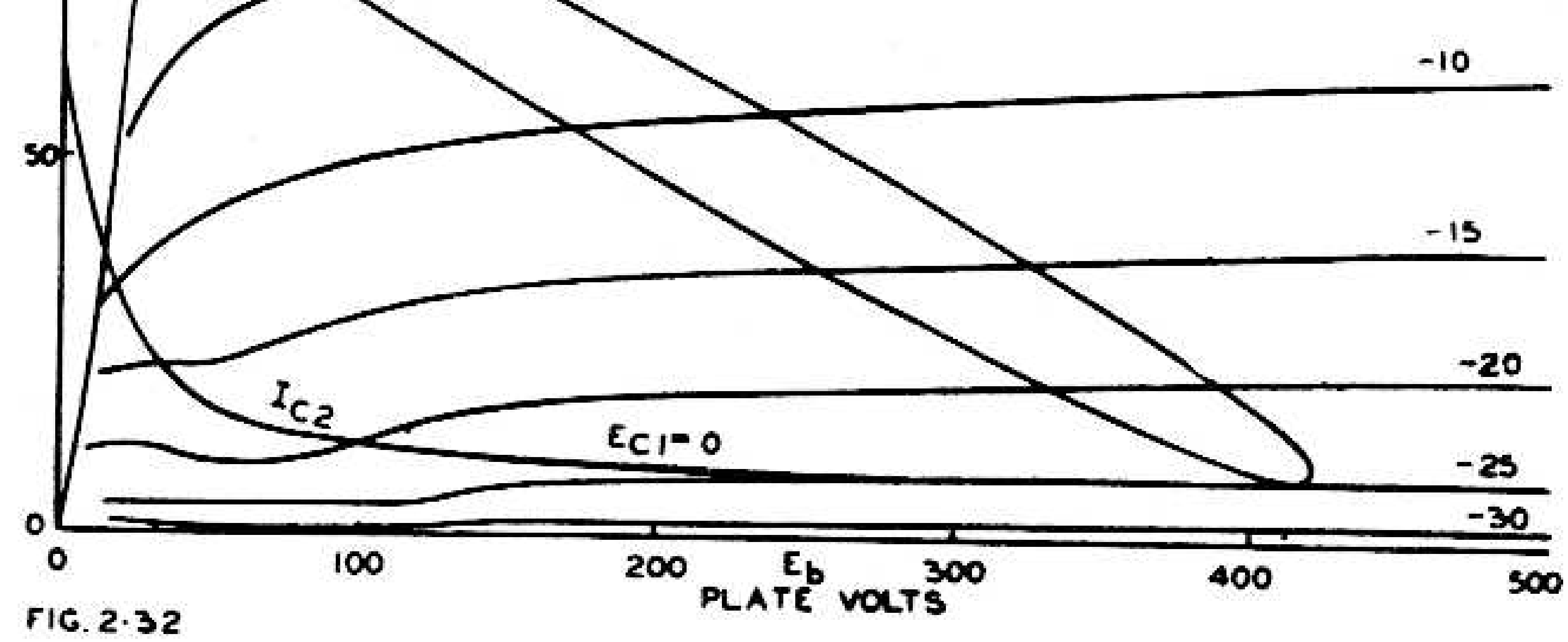


Fig. **2.32.** *Beam power amplifier plate characteristics with elliptical loadline corresponding to a resistance of* 4750 *ohms in parallel with a reactance of* 23 000 *ohms.*

Fig. 2.32 shows a typical beam power amplifier with an elliptical loadline with a resistive load of 4750 ohms shunted by a reactance of 23 000 ohms. The plate voltage is 250 volts, grid bias −12.5 volts, and grid swing from 0 to −25 volts.

In all applications of elliptical loadlines to characteristics, the shape of the ellipse (i.e. the ratio of its major to its minor axis) and the slope of the major axis are determined solely by the nature of the load. The ellipse can be imagined as being slowly blown up, like a balloon, until it just touches without cutting the two curves of extreme voltage swing. If there is no distortion, the centre of the ellipse will coincide with the quiescent working point, but in the general case the centre of the ellipse will be slightly displaced.

SECTION 5 : TRIODE OPERATION OF PENTODES

(i) Triode operation of pentodes (ii) Examples of transconductance calculation (iii) Triode amplification factor (iv) Plate resistance (v) Connection of suppressor grid.

(i) Triode operation of pentodes

Any pentode may be operated as a triode, provided that none of the maximum ratings is exceeded, and the characteristics may readily be calculated if not otherwise available.

When the cathode current of a valve is shared by two collecting electrodes (e.g. plate and screen) the mutual conductance of the whole cathode stream (i.e. the " triode g_m ") is shared in the same proportion as is the current.

Let I_k = cathode current
I_{c2} = screen current
I_b = plate current
g_m = pentode transconductance (to the plate)
g_t = triode transconductance (with screen and plate tied together)
and g_s = screen transconductance (with pentode operation).

Then $I_k = I_{c2} + I_b$ (1)
$g_t = g_m + g_s$ (by definition) (2)
and $g_m/g_t = I_b/I_k$ (3)

If it is desired to find the screen transconductance, this can be derived from the expression

$g_s/g_m = I_{c2}/I_b$ (4)
or $g_s/g_t = I_{c2}/I_k$ (5)

(ii) Examples of transconductance calculation

Example 1 : Type 6J7-G as a pentode with 100 volts on both screen and plate, and with a grid bias of −3 volts, has the following characteristics :—

Transconductance	1185 micromhos
Plate Current	2·0 mA
Screen Current	0·5 mA

It is readily seen that the cathode current (see equation 1 above) is given by

$$I_k = 0{\cdot}5 + 2{\cdot}0 = 2{\cdot}5 \text{ mA.}$$

The triode transconductance is calculated by inverting equation (3) above,

$$g_t/g_m = I_k/I_b \ .$$

Therefore $g_t/1185 = 2{\cdot}5/2{\cdot}0$
and g_t = 1482 micromhos.

The example selected was purposely chosen so as to have equal plate and screen voltages. Under these conditions the method is exact, and the calculated triode mutual conductance applies to the same conditions of plate and grid voltages as for the pentode operation (in this example 100 volts and −3 volts respectively).

Example 2 : Type 6J7-G as a pentode with 250 volts on the plate, 100 volts on the screen, and −3 volts grid bias.

In this case a similar method may be used, but it is necessary to make an assumption which is only approximately correct. Its accuracy is generally good enough for most purposes, the error being within about 5% for most conditions.

The assumption (or approximation) which must be made is—*That the plate current of a pentode valve does not change as the plate voltage is increased from the same voltage as that of the screen up to the voltage for pentode operation.*

This assumption means, in essence, that the plate resistance is considered to be infinite—a reasonable approximation for most r-f pentodes, and not seriously in error for power pentodes and beam power valves.

In this typical example we can take the published characteristics, and assume that the plate current and transconductance are the same for 100 as for 250 volts on the plate. From then on the procedure is exactly as in the previous example. It is important to note that the calculated triode characteristics only apply for a triode plate voltage of 100 volts and a grid bias of −3 volts.

Example 3 : To find the screen transconductance under the conditions of Example 1.

From eqn. (2) we may derive the expression—

$$g_s = g_t - g_m = 1482 - 1185$$
$$= 297 \text{ micromhos.}$$

This could equally well have been derived from eqn. (4) or (5).

(iii) Triode amplification factor

The triode amplification factor (if not available from any other source) may be calculated by the following approximate method.

Let μ_t = triode amplification factor
E_{co} = negative grid voltage at which the plate current just cuts off
and E_{c2} = screen voltage.
Then $\mu_t = E_{c2}/E_{co}$ approx. (6)

For example, with type 6J7-G having a screen voltage of 100 volts, the grid bias for cut-off is indicated on the data sheet as being −7 volts approx. This is the normal grid bias for complete plate current cut-off, but it is not very suitable for our purpose since equation (6) is based on the assumption that the characteristic is straight, whereas it is severely curved as it approaches cut-off. The preferable procedure is to refer to the plate current-grid voltage characteristic, and to draw a straight line making a tangent to the curve at the working point—in this case with a screen voltage of 100 volts and grid bias −3 volts. When this is done, it will be seen that the tangent cuts the zero plate current line at about −5 volts grid bias. If this figure is used, as being much more accurate than the previous value of −7 volts, the triode amplification factor will be

$$\mu_t = 100/5 = 20.$$

Alternatively, if only the plate characteristics are available, much the same result may be obtained by observing the grid bias for the lowest curve, which is generally very close to plate current cut-off.

In the case of remote cut-off characteristics it is essential to adopt the tangent method, and the result will only apply to the particular point of operation, since the triode amplification factor varies along the curve.

The amplification factor of the screen grid in a pentode valve with respect to the control grid is almost exactly the same as the triode amplification factor.

The amplification factor of the plate of a pentode valve with respect to its screen grid may be calculated from the expression—

$$\mu_{g1 \cdot p} = \mu_{g1 \cdot g2}\,\mu_{g2 \cdot p} \qquad (7)$$

where $\mu_{g1 \cdot p}$ = pentode amplification factor
and $\mu_{g2 \cdot p}$ = screen grid-plate mu factor.

This expression can only be used when the pentode amplification factor is known. If this is not published, it may be determined from a knowledge of the plate resistance and mutual conductance. If the former is not published, it may be derived graphically ; this derivation is only very approximate in the case of sharp cut-off r-f pentodes, since the characteristics are nearly horizontal straight lines.

For example type 6AU6 has the following published values—

$$\left.\begin{aligned} r_p &= 1{\cdot}5 \text{ megohms} \\ \text{and} \quad g_m &= 4450\ \mu\text{mhos} \end{aligned}\right\} \text{at } E_b = 250, E_{c2} = 125, \ E_{c1} = -1\text{ V}$$

from which $\mu = 6675$.
But $\mu_{g1 \cdot g2} = 36$ approx.
Therefore $\mu_{g2 \cdot p} = 6675/36 = 185$ approx.

(iv) Plate resistance

The "plate resistance" of each electrode (plate or screen) in the case of pentode operation, and the "triode plate resistance" when plate and screen are tied together, may be calculated from the corresponding values of μ and g_m.

(v) Connection of suppressor grid

The suppressor may be connected either to cathode or to the screen and plate, with negligible effect on the usual static characteristics. Some valves have the suppressor internally connected to the cathode, so that there is no alternative. In other cases, connection to cathode slightly increases the output capacitance. In low level amplifiers, connection of the suppressor to cathode may give lower noise in certain cases if there is a high resistance leakage path from suppressor to cathode; similarly its connection to screen and plate will give lower noise if there is leakage to the latter electrodes.

SECTION 6 : CONVERSION FACTORS, AND THE CALCULATION OF CHARACTERISTICS OTHER THAN THOSE PUBLISHED

(i) The basis of valve conversion factors (ii) The use of valve conversion factors (iii) The calculation of valve characteristics other than those published (iv) The effect of changes in operating conditions.

Conversion Factors provide a simple approximate means of calculating the principal valve characteristics when all the voltages are changed by the same factor. It is possible to make certain additional calculations so as to allow for the voltage of one electrode differing from this strict proportionality.

(i) The basis of valve conversion factors

Valve Conversion Factors are based on the well-known mathematical expression of valve characteristics

$$I_b = A(E_b - \mu E_c)^x \qquad (1)$$

where I_b = plate current
E_b = plate voltage
E_c = grid voltage
A = a constant depending upon the type of valve
μ = amplification factor
and x = an exponent, with a value of approximately 1·5 over the nearly straight portion of the characteristics.

If we are concerned merely with changes in the voltages and currents, then we can reduce the expression to the form

$$I_b \propto (E_b - \mu E_c)^x. \qquad (2)$$

Now if we agree to change the grid voltage in the same proportion as the plate voltage, we obtain the very simple form

$$I_b \propto E_b^x. \qquad (3)$$

Finally, if we take x as 1·5 or 3/2, we have the approximation

$$I_b \propto E_b^{3/2}. \qquad (4)$$

Put into words, this means that the plate current of a valve varies approximately as the three-halves power of the plate voltage, provided that the grid voltage is varied in the same proportion as the plate voltage.

The same result may be obtained with pentodes, provided that both the grid and screen voltages are varied in the same proportion as the plate voltage. This result is the basis of Valve Conversion Factors, so that we must always remember that their use is restricted to cases in which all the electrode voltages are changed in the same proportion.

Let F_e be the factor by which all the voltages are changed (i.e. grid, screen, and plate), and let I_b' be the new plate current.

Then $I_b' \propto (F_e.E_b)^{3/2}$. (5)

But $I_b' = F_i.I_b$

where F_i is the factor by which the plate current is changed.

Therefore $F_i.I_b \propto (F_e.E_b)^{3/2}$. (6)

From the combination of (4) and (6) it will be seen that

$$F_i = F_e^{3/2} \quad (7)$$

Now the power output is proportional to the product of plate voltage and plate current so that

$$P_o \propto E_b.I_b \quad (8)$$

and $$P_o' \propto (F_e.E_b)(F_i.I_b) \quad (9)$$

so that $$P_o' \propto F_e.F_i\,(E_b.I_b) \quad (10)$$

$$\propto F_e.F_i\,(P_o). \quad (11)$$

We may therefore say that the power conversion factor F_p is given by the expression

$$F_p = F_e.F_i. \quad (12)$$

Therefore $$F_p = F_e^{5/2}. \quad (13)$$

The mutual conductance is given by

$$g_m = \frac{\text{change of plate current}}{\text{change of grid voltage}}$$

Therefore $$F_{gm} = F_i/F_e = F_e^{3/2}/F_e = F_e^{\frac{1}{2}}. \quad (14)$$

The Plate Resistance is given by

$$r_p = \frac{\text{change of plate voltage}}{\text{change of plate current}}$$

Therefore $$F_r = F_e/F_i = F_e/F_e^{3/2} = F_e^{-\frac{1}{2}}. \quad (15)$$

This also applies similarly to the load resistance and cathode bias resistance.

We may therefore summarize our results so far :—

$$F_i = F_e^{3/2} \quad (7)$$
$$F_p = F_e^{5/2} \quad (13)$$
$$F_{gm} = F_e^{\frac{1}{2}} \quad (14)$$
$$F_r = F_e^{-\frac{1}{2}} \quad (15)$$

These are shown in graphical form on the Conversion Factor Chart (Fig. 2.32A).

(ii) The use of valve conversion factors

It is important to remember that the conversion factors may only be used when all the voltages (grid, screen and plate) are changed simultaneously by the same factor. If it is required to make any other adjustments, these may be carried out before or after using conversion factors, by following the method given under (iii) below.

Conversion factors may be used on any type of valve whether triode, pentode or beam tetrode, and in any class of operation whether class A, class AB1, class AB2 or class C.

The use of conversion factors is necessarily an approximation, so that errors will occur which become progressively greater as the voltage factor becomes greater. In general it may be taken that voltage conversion factors down to about 0·7 and up to about 1·5 times will be approximately correct. When the voltage factors are extended beyond these limits down to 0·5 and up to 2·0, the accuracy becomes considerably less, and any further extension becomes only a rough indication.

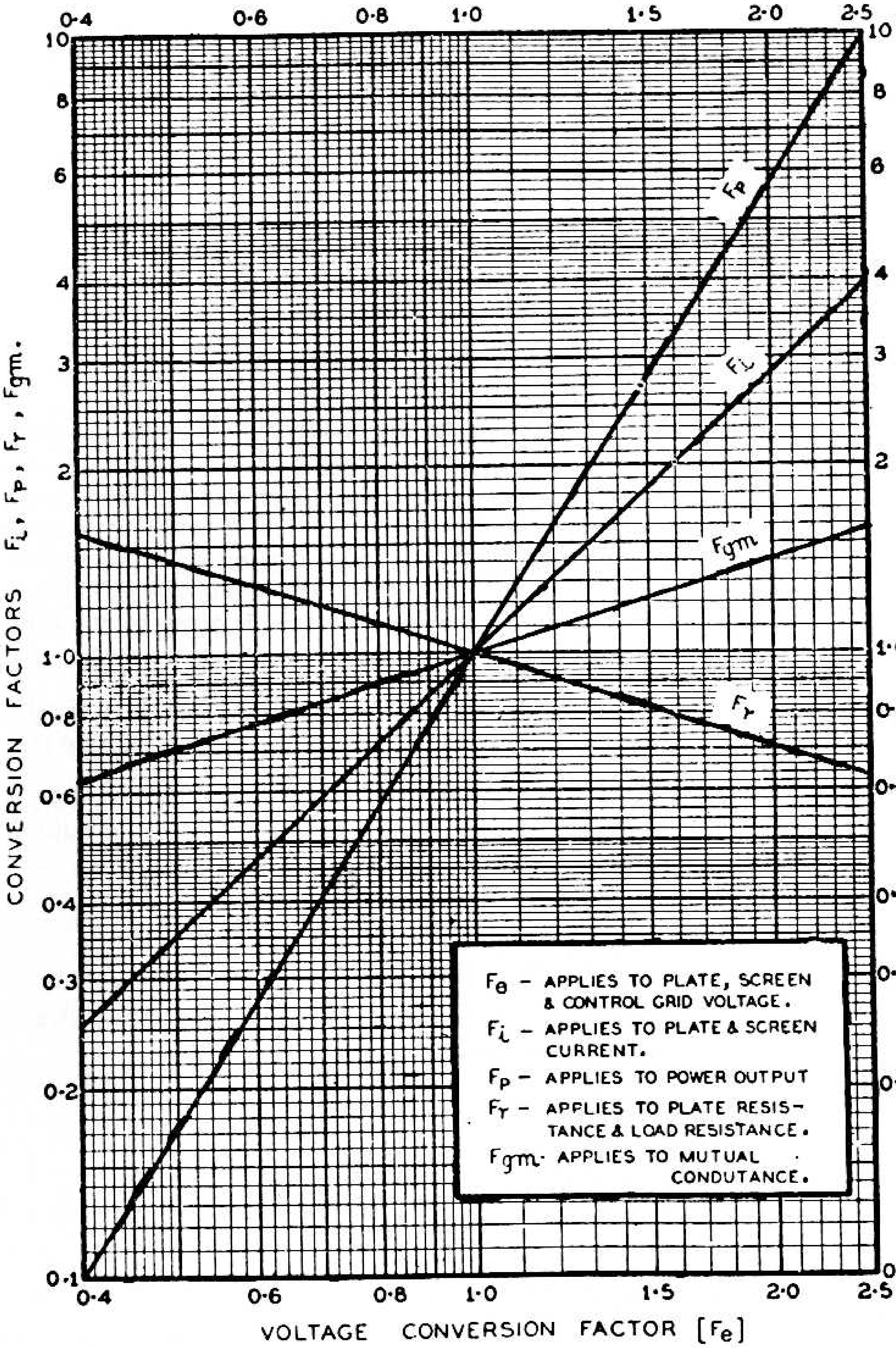


Fig. 2.32A. *Conversion factor chart (by courtesy of R.C.A.).*

The example given below is a straightforward case of a pentode valve whose characteristics are given for certain voltages and which it is desired to operate at a lower plate voltage.

Plate and screen voltage	250 volts
Control grid voltage	−15 volts
Plate current	30 mA

Screen current	6 mA
Mutual conductance	2,000 μmhos
Power Output	2·5 watts.

It is required to determine the optimum operating conditions for a plate voltage of 200 volts.

The Voltage Conversion Factor (F_e) = 200/250 = 0·8.

The new screen voltage will be 0·8 × 250 = 200 volts.

The new control grid voltage will be −(0·8 × 15) = −12 volts.

Reference to the chart then gives the following :

Current Conversion Factor (F_i)	0·72
Mutual Conductance Conversion Factor (F_{gm})	0·89
Power Output Conversion Factor (F_p)	0·57

The new plate current will be 0·72 × 30 = 21·6 mA.

The new screen current will be 0·72 × 6 = 4·3 mA.

The new mutual conductance will be 0·89 × 2000 = 1780 μmhos.

The new power output will be 0·57 × 2·5 = 1·42 watts.

There are two effects not taken into account by conversion factors. The first is contact potential, but its effects only become serious for small grid bias voltages. The second is secondary emission, which occurs with the old type of tetrode at low plate voltages ; in such a case the use of conversion factors should be limited to regions of the plate characteristic in which the plate voltage is greater than the screen voltage. With beam power amplifiers the region of both low plate currents and low plate voltages should also be avoided for similar reasons.

The application of conversion factors to resistance-capacitance-coupled triodes and pentodes is covered in Chapter 12 Sect. 2(x) and Sect. 3(x) respectively.

FIG. 2.33

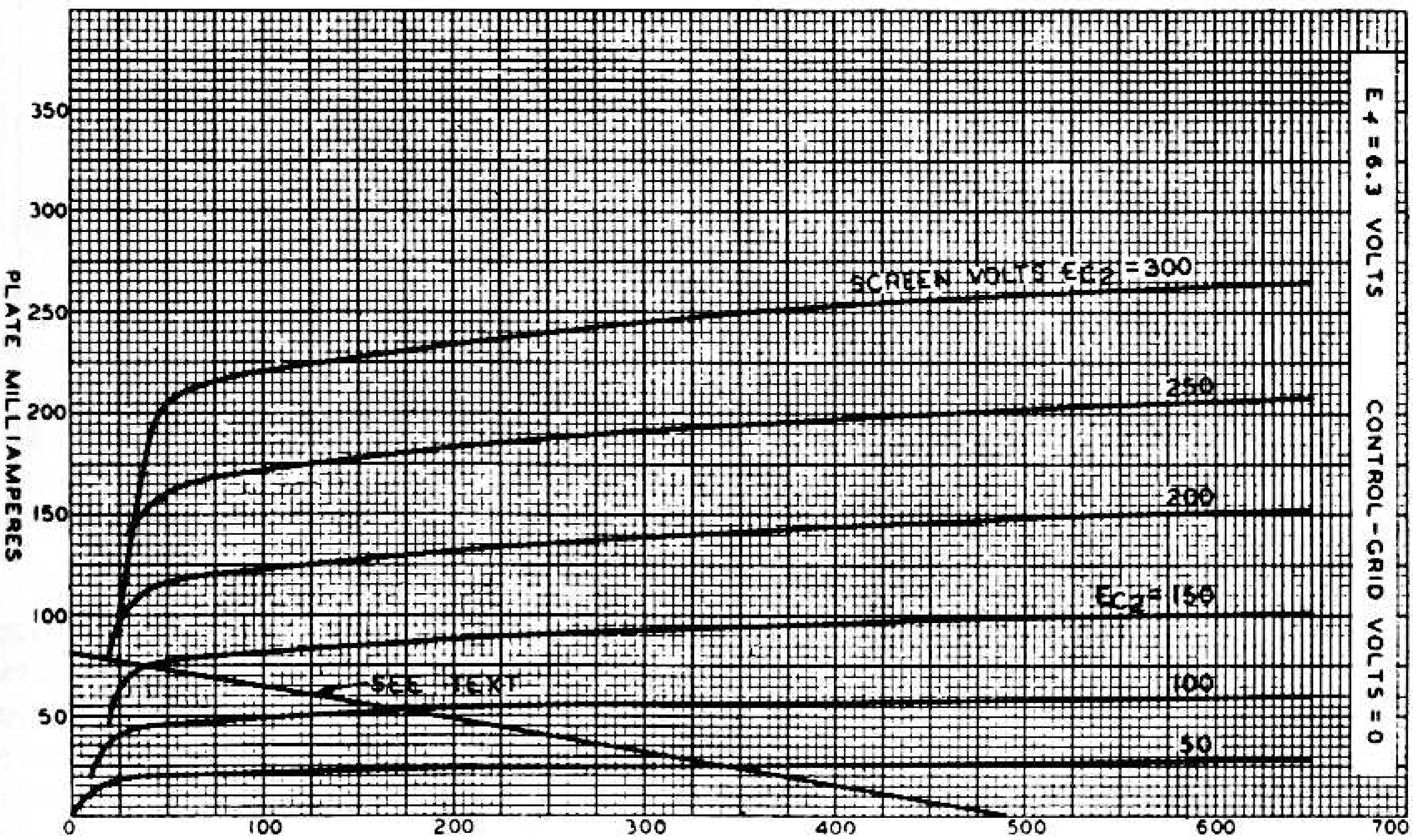


Fig. 2.33. *Zero bias plate characteristics for type* 807 *beam power amplifier with six values of screen voltage* (*Ref. E*2).

Greater accuracy in the use of conversion factors over a wide range of screen voltages may be obtained, if curves are available for zero bias at a number of different screen voltages as in Fig. 2.33 (Ref. E2).

When the plate, screen, and grid voltages of a pentode or beam power amplifier are multiplied by the same voltage conversion factor, the ratio of the plate current at a given grid bias to that at zero bias does not change. In order to convert a given family of plate characteristics to a new screen voltage condition, it is therefore only necessary to have a zero-bias plate characteristic for the screen voltage of interest.

Example

Suppose that the family of plate characteristics shown in Fig. 2.34, which obtains for a screen voltage of 250 volts, is to be converted for a screen voltage of 300 volts. The zero-bias plate characteristic for E_{c2} = 300 volts, which is shown in Fig. 2.33, is replotted as the upper curve in Fig. 2.35.

Since all bias values shown in Fig. 2.34 must be multiplied by 300/250 = 1·2, corresponding plate characteristics for the new family obtain for bias values that are 20 per cent. higher than those shown in Fig. 2.34. Consider the conversion of −10-volt characteristic of Fig. 2.34. At a plate voltage (E_b) of 250 volts in Fig. 2.34, AB/AC = 100/187 = 0·535. On the new characteristic in Fig. 2.35 which corresponds to a bias of −12 volts, A′B′/A′C′ must also equal 0·535 at E_b = 300 volts. Therefore, A′B′ = 0·535 × A′C′. From the given zero-bias characteristic of Fig. 2.35, A′C′ = 244 at E_b = 300 volts; hence A′B′ = 131 milliamperes. At E_b = 200 volts in Fig. 2.34 DE/DF = 98/183 = 0·535. Therefore, at E_b = 200 × 1·2 = 240 volts in Fig. 2.35, D′E′ = 0·535 × 238 = 127 milliamperes. This process is repeated for a number of plate voltages and a smooth curve is drawn through the points on the new characteristic.

FIG. 2.34

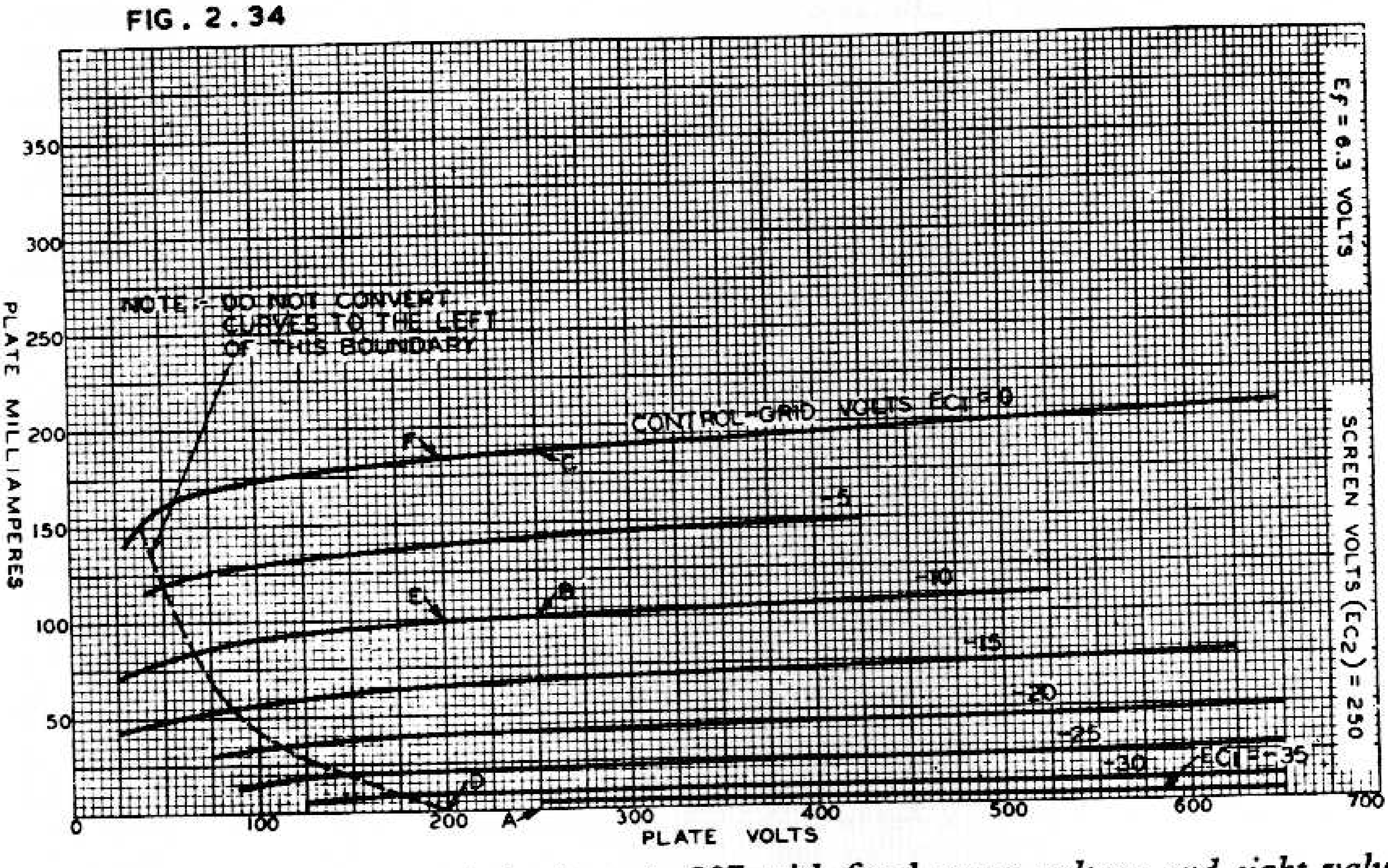


Fig. 2.34. *Plate characteristics for type* 807 *with fixed screen voltage and eight values of grid voltage* (*Ref. E*2).

The factor 0·535 can be used for the −10-volt characteristic at plate voltages greater than that at which the knee on the zero-bias characteristic of Fig. 2.34 occurs; for plate voltages in the immediate region of the knee, a new factor should be determined for each point. The plate characteristics of Fig. 2.34 should not be converted to the left of the dashed line of Fig. 2.34 because of space-charge effects. This limitation is not a serious one, however, because the region over which the valve usually operates can be converted with sufficient accuracy for most applications. The converted plate characteristic of Fig. 2.35 for E_{c1} = −30 volts was obtained in a similar manner to that for E_{c1} = −12 volts.

The curves of Fig. 2.35 were checked under dynamic conditions by means of a cathode-ray tube and the dotted portions show regions where measured results departed from calculated results.

(iii) The calculation of valve characteristics other than those published

It is frequently desired to make minor modifications in the operating conditions of a valve, such as by a slight increase or decrease of the plate voltage, change in grid bias or load resistance. It is proposed to describe the effects which these changes will have on the other characteristics of the valve.

The procedure to be adopted is summarized below :—

FIG. 2.35

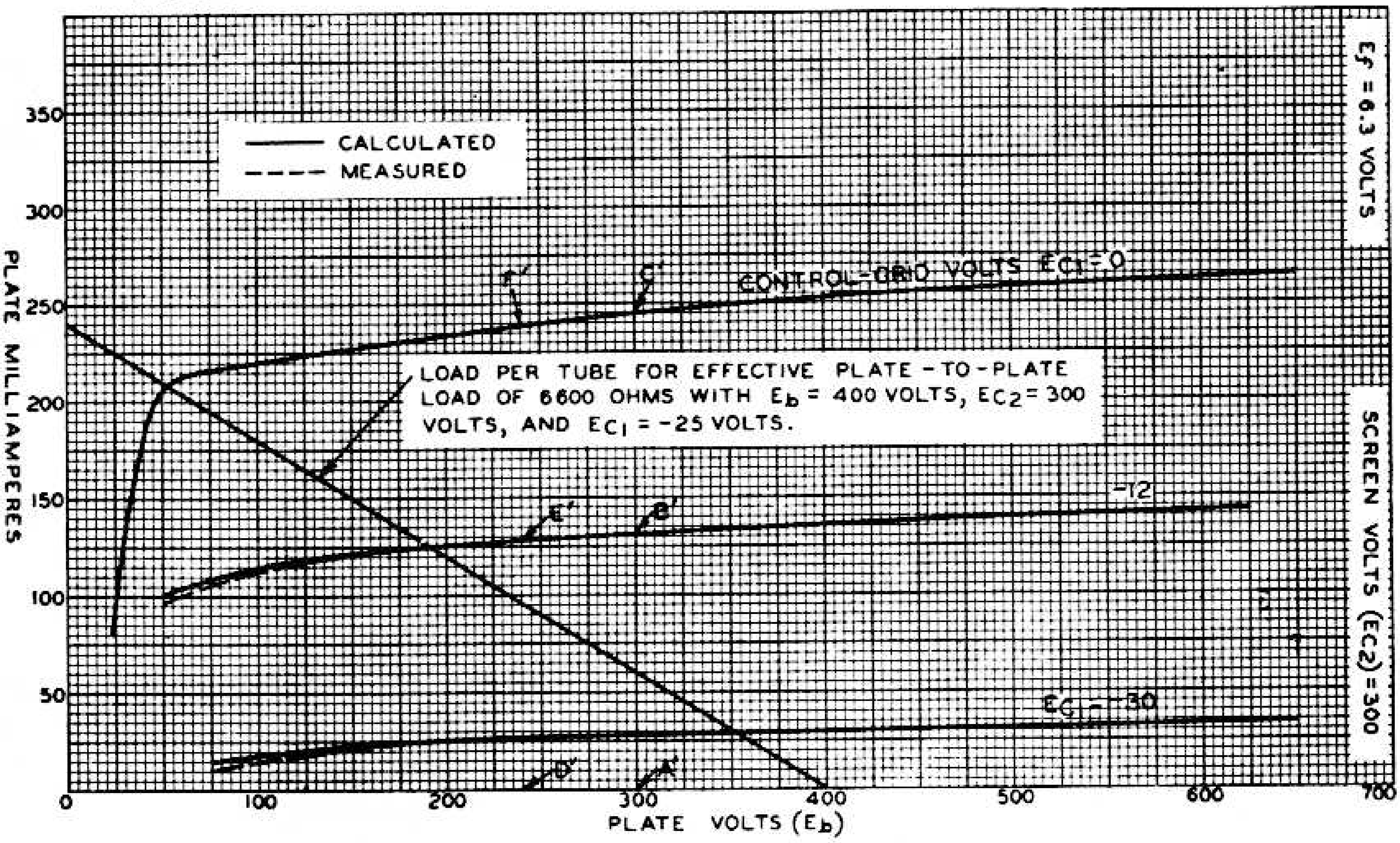


Fig. 2.35. *Derived plate characteristics for type* 807 *with different screen voltage, making use of Figs.* 2.33 *and* 2.34 *with conversion factors* (*Ref. E*2).

(a) In the absence of valve curves

Triode—Use conversion factors to adjust the plate voltage to its new value, and apply the correct conversion factors to all other characteristics ; then adjust the grid bias to its desired new value by the method given below, and finally adjust the load resistance.

Pentode or beam power amplifier—Use conversion factors to adjust the screen voltage to its new value, and apply the correct conversion factors to all other characteristics ; then adjust the plate voltage to the desired new value by the method given below ; then adjust the grid bias to its desired new value, and finally adjust the load resistance.

(b) When valve curves are available

Triode with no d.c. load resistance in the plate circuit : Refer to the published characteristics to find the maximum plate dissipation ; calculate the maximum plate current which can be permitted at the desired new plate voltage ; select a suitable plate current for the particular application (which must not exceed the maximum) ; and refer to the curves to find the grid bias to give the desired plate current.

If the valve is a power amplifier, the load resistance may be determined by one of the methods described in Chapter 13 [e.g. triodes Sect. 2(iii) ; pentodes Sect. 3(iii)A].

Triode with resistor in plate circuit : Use conversion factors, with adjustments as required in accordance with the method given in (iv) below.

Pentode or beam power amplifier : If curves are available for the published value of screen voltage, use the method in (iv) below to obtain the characteristics for a plate voltage such that, when conversion factors are applied, the plate voltage is the desired value. For example, if curves and characteristics are available for plate and screen voltages of 250 volts, and it is desired to determine the characteristics for a plate voltage of 360 volts and screen voltage of 300 volts : firstly determine the characteristics for a plate voltage of 300 and screen voltage of 250 ; then apply voltage conversion factors of 1·2 to the plate, screen and grid voltages so as to provide the desired conditions.

If curves are available for the new value of screen voltage, use conversion factors to bring the screen voltage to the desired value, then apply the method below to adjust the plate voltage, load resistance and grid bias.

(iv) The effect of changes in operating conditions

(A) *Effect of Change of Plate Voltages of Pentodes and Beam Power Amplifiers*

(a) On plate current

The plate current of a pentode or beam power valve is approximately constant over a wide range of plate voltages, provided that the plate voltage is maintained above the "knee" of the curve. The increase of plate current caused by an increase in plate voltage from E_{b1} to E_{b2} is given by the expression

$$\Delta I_b = \frac{\Delta E_b}{r_p} = \frac{E_{b2} - E_{b1}}{r_p}. \quad (16)$$

In many cases the plate characteristic curves are available, and the change in plate current may be read from the curves.

(b) On screen current

In the case of both pentodes and beam power valves the total cathode current (i.e., plate plus screen currents) is approximately constant over a wide range of plate voltages (see Fig. 2.4). The increase in plate current from E_{b1} to E_{b2} is approximately equal to the decrease in screen current over the same range.

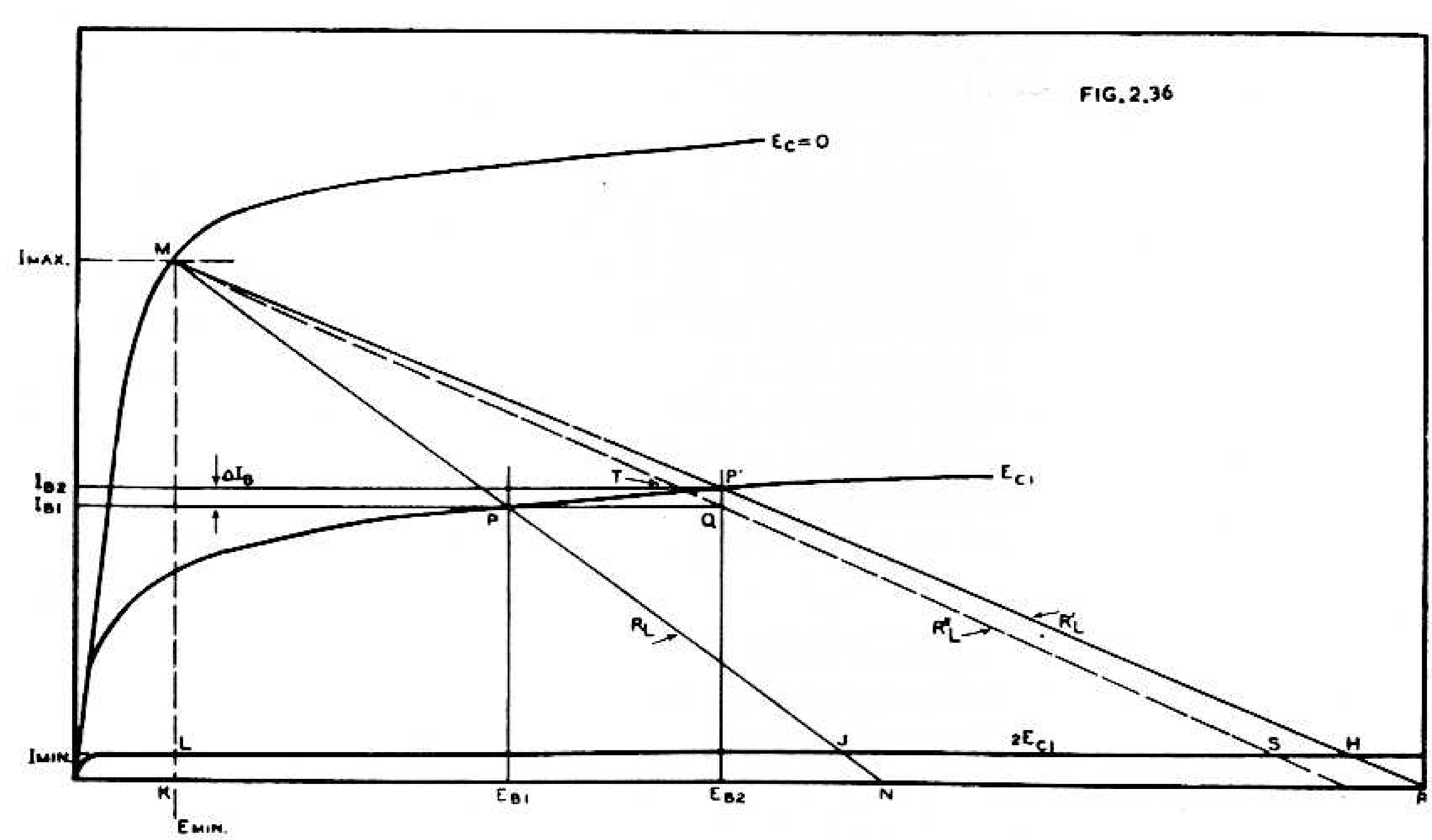


Fig. 2.36. Plate characteristics of power pentode illustrating effect of change of plate voltage.

(c) On load resistance and power output

The plate characteristics of a typical power pentode are shown in Fig. 2.36 in which I_{b1} is the "published" plate current at plate voltage E_{b1} and grid bias $-E_{c1}$. The loadline MPJ swings up to I_{max} at $E_c = 0$ and down to I_{min} at $2E_{c1}$, the assumption being made that the $2E_{c1}$ curve is straight and horizontal over the range of plate voltages in which we are interested.

If the plate voltage is increased to E_{b2}, the new loadline will be MP′H, the point M being common to both, since it is at the knee of the characteristic. The quiescent operating point P′ is at a higher plate current than P, the difference being ΔI_b.

Since the power output is proportional to the area of the triangle under the loadline, it is also proportional to the value of the load resistance, all triangles having ML as a common side. It may readily be shown that

$$R_L = \frac{E_{b1} - E_{min}}{I_{max} - I_{b1}}$$

and

$$R_L' = \frac{E_{b2} - E_{min}}{I_{max} - I_{b2}}.$$

Therefore

$$\frac{R_L'}{R_L} = \frac{E_{b2} - E_{min}}{E_{b1} - E_{min}} \cdot \frac{I_{max} - I_{b1}}{I_{max} - I_{b2}} \qquad (17)$$

which is also the ratio of the output powers. If $I_{b2} = I_{b1}$ or the rise of plate current is neglected as an approximation, then

$$\frac{R_L'}{R_L} = \frac{E_{b2} - E_{min}}{E_{b1} - E_{min}}. \qquad (18)$$

As an example, apply this to type 6V6-GT under the following conditions—

	Published Condition	Desired Condition
Plate voltage	250	300 V
Screen voltage	250	250 V
Grid voltage	−12·5	−12·5 V
Load resistance	5000	(see below) ohms
Plate current (I_{b1})	47	48* mA
Peak plate current (I_{max})	90*	90* mA
Min. plate current (I_{min})	8*	8* mA
Min. plate voltage (E_{min})	35	35 V
Power output	4·5	(see below) W

*From curve.

Using equation (17)—

$$\frac{R_L'}{R_L} = \frac{300 - 35}{250 - 35} \cdot \frac{90 - 47}{90 - 48} = \frac{265}{215} \cdot \frac{43}{42} = 1{\cdot}26$$

whence $R_L' = 1{\cdot}26 \times 5000 = 6300$ ohms.

The increase of power output is in proportion to the increase in load resistance. i.e. $P_o = 4{\cdot}5 \times 1{\cdot}26 = 5{\cdot}66$ watts.

This method is remarkably accurate when there is very small rectification in the plate circuit, as is usually the case with power pentodes. With beam power amplifiers of the 6L6 and 807 class, in which the rectification is considerable (strong second harmonic component), the "corrected" loadline should be used as a basis, and the values of I_{max}, I_{b1} and E_{min} should be those corresponding to the corrected loadline.

If the rise in plate current (ΔI_b) is considerable, the point P′ will be above the centre point of the loadline MH, and there will be an appreciable amount of second harmonic distortion; this may be reduced to zero (if desired) by increasing the load resistance slightly.

(B) Effect of change of load resistance

In a r.c.c. triode the effect of a change in R_L on stage gain is very slight, provided that $R_L \geqslant 5r_p$. In any case where the change cannot be neglected, eqn. (7) of Chapter 12 Sect. 2 may be used to calculate stage gain.

In a r.c.c. pentode the effect of a change in R_L on stage gain is given by eqn. (7) of Chapter 12 Sect. 3, bearing in mind that the mutual conductance at the operating plate current is increased when R_L is decreased. As a rough approximation, the voltage gain is proportional to the load resistance. If optimum operating conditions are to be obtained, conversion factors should be applied to the whole amplifier—see Chapter 12 Sect. 3(x)C.

(C) *Effect of change of grid bias*

In any valve which is being operated with fixed voltages on all electrodes and without any resistance in any of the electrode circuits, a change of grid bias will result in a change of plate current as given by the expression

$$\Delta I_b = \Delta E_c \times g_m \tag{19}$$

where ΔI_b = increase of plate current,
ΔE_c = change of grid bias in the positive direction,
and g_m = mutual conductance of valve at the operating plate current.

In most practical cases, however, the valve is being operated with an impedance in the plate circuit and in some cases also in the screen circuit. The effect of a change in grid bias is therefore treated separately for each practical case.

(a) On resistance-coupled triodes

In this case a plate load resistor is used, resulting in a considerable voltage drop and a decrease in the effective slope of the valve.

The change in plate voltage brought about by a change in grid bias is given by the expression

$$\Delta I_b = \Delta E_c \times \mu/(r_p + R_L) \tag{20}$$

where μ = amplification factor of valve at the operating point,
r_p = plate resistance of valve at the operating point,
and R_L = resistance of plate load resistor.

(b) On resistance-coupled pentodes

The change of plate current with grid bias is given by the expression.

$$\Delta I_b = \Delta E_g \times g_d \tag{21}$$

where g_d = dynamic transconductance at the operating point,
= slope of dynamic characteristic at the operating point.

The change of screen current (with fairly low screen voltages) is approximately proportional to the plate current up to plate currents of 0·6 E_{bb}/R_L and the change in screen current is given by the expression

$$\Delta I_{c2} = \Delta I_b \,(I_{c2}/I_b) \text{ approx.} \tag{22}$$

where I_{c2} = screen current
and E_{bb} = plate supply voltage.

For further information on resistance coupled valves, see Chapter 12, Sects. 2 and 3.

(c) On i-f or r-f amplifier

In this case there is no d.c. load resistor and the full supply voltage reaches the plate of the valve. The change of plate current is given by eqn. (19) while the change in screen current may be calculated from the ratio of screen and plate currents, which remains approximately constant. The voltage gain is proportional to the mutual conductance* of the valve, and is therefore a maximum for the highest plate current at the minimum bias. A decrease in bias will therefore normally result in increased gain, while increased bias will result in decreased gain. The limit to increased gain is set by the plate or screen dissipation of the valve, by positive grid current, and, in some circuits, by instability. In most cases the mutual conductance curves are published so as to enable the change of gain to be calculated.

(d) On power valves

This subject is covered in detail in Chapter 13.

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*The voltage gain is also affected by the plate resistance, but this is quite a secondary effect unless the plate resistance is less than 0.5 megohm. In most remote cut-off pentodes the plate resistance falls rapidly as the bias is decreased towards the minimum bias, but this is more than counterbalanced by the rise in mutual conductance.

SECTION 7 : VALVE EQUIVALENT CIRCUITS AND VECTORS

(i) Constant voltage equivalent circuit (ii) Constant current equivalent circuit (iii) Valve vectors.

Much useful information can be derived from an equivalent circuit of a valve, even though this may only be valid under limited conditions. The equivalent circuit is only a convenient fiction, and it must be remembered that it is the plate supply which, in reality, supplies the power—the valve merely controls the current by its varying d.c. plate resistance. The equivalent circuit is merely a device to produce in the load the same a.c. currents and voltages which are produced by the valve when alternating voltages are applied to its grid.

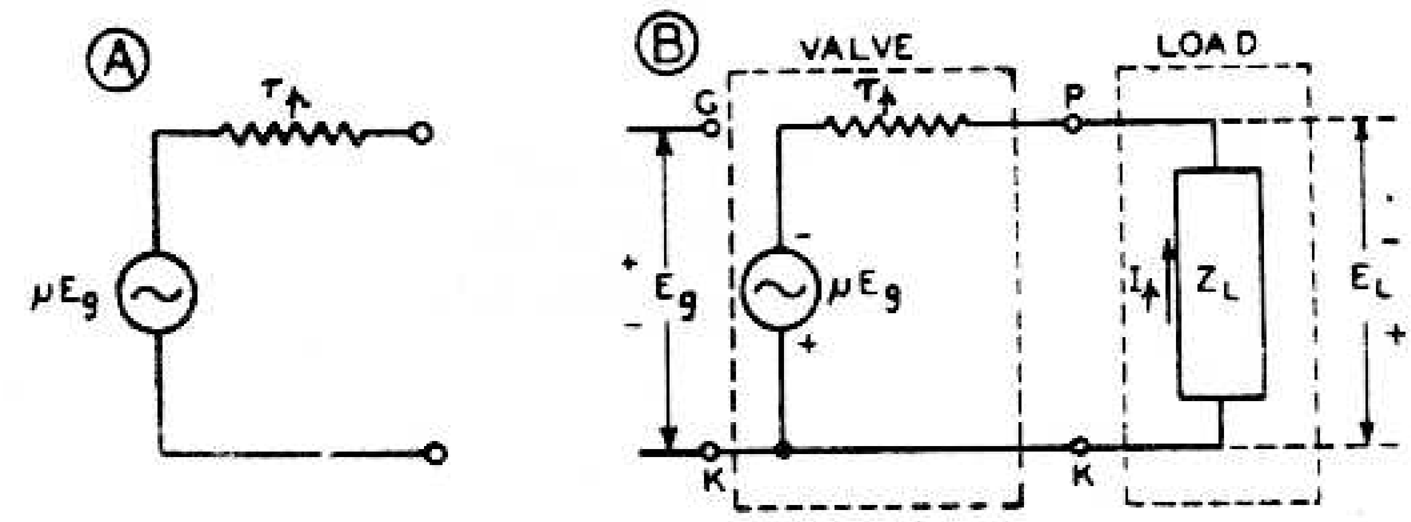

FIG. 2·37

Fig. 2.37. (A) Equivalent circuit of valve using constant voltage generator (B) Equivalent circuit of valve and load.

(i) Constant voltage equivalent circuit

The simplest equivalent circuit treats the valve as an a.c. generator of constant r.m.s. voltage μE_g, which is applied through an internal generator resistance r_p (Fig. 2.37A). This is valid for small alternating voltages (under which conditions the characteristics are practically uniform) but is of no assistance in determining direct currents or voltages, phase angles or operating conditions. It is also limited to frequencies at which the effects of capacitances are negligible.

This may be elaborated, as in Fig. 2.37B, with the inclusion of the input circuit GK and the load Z_L. The input voltage E_g is shown by the $\pm$ signs to be such that the grid is instantaneously positive, and the plate negative (with respect to the cathode) at the same instant. It is assumed that the grid is biased sufficiently to prevent grid current flow.

The current I_p flowing through the load Z_L produces across the load a voltage E_L which is of opposite sign to E_g. It will be noted that the " fictitious " voltage μE_g is opposite in sign to E_g, although μ is positive ; this apparent inversion is a consequence of treating the valve as an a.c. generator.

In the simplest case, Z_L is a resistance R_L. We can then derive the following relationships—

$$\mu E_g = (r_p + R_L)\, I_p \tag{1}$$

$$E_L = -I_p R_L = -\mu E_g \frac{R_L}{r_p + R_L} \tag{2}$$

and

$$\frac{E_L}{E_g} = \text{voltage gain} = -\frac{\mu R_L}{r_p + R_L} \tag{3}$$

If the load is made up of a resistor R_L and an inductor X_L in series—

Complex Values	*Scalar Values*	
$Z_L = R_L + jX_L$	$\sqrt{R_L^2 + X_L^2}$	(4)
$\mu E_g = (r_p + R_L + jX_L)\, I_p$	$\sqrt{(r_p + R_L)^2 + X_L^2}.I_p$	(5)
$\dfrac{E_L}{E_g} = -\dfrac{\mu(R_L + jX_L)}{r_p + R_L + jX_L}$	$= \dfrac{\mu\sqrt{R_L^2 + X_L^2}}{\sqrt{(r_p + R_L)^2 + X_L^2}}$	(6)

and similarly for any other type of load.

The **interelectrode capacitances** are shown in the equivalent circuit of Fig. 2.38, and may be taken as including the stray circuit capacitances. This circuit may be applied at frequencies up to nearly 10 Mc/s, beyond which the inductances of the leads and electrodes become appreciable. It may also be applied to a screen grid (tetrode) or pentode, provided that the screen is completely by-passed to the cathode; in this case C_{gk} becomes the input capacitance ($C_{g1\cdot k} + C_{g1\cdot g2}$) and C_{pk} becomes the output capacitance (C_p to all other electrodes).

(ii) Constant current equivalent circuit

An alternative form of representation is the constant current generator equivalent circuit (Fig. 2.39), this being more generally convenient for pentodes, in which the plate resistance is very high. Either circuit is equally valid for both triodes and pentodes.

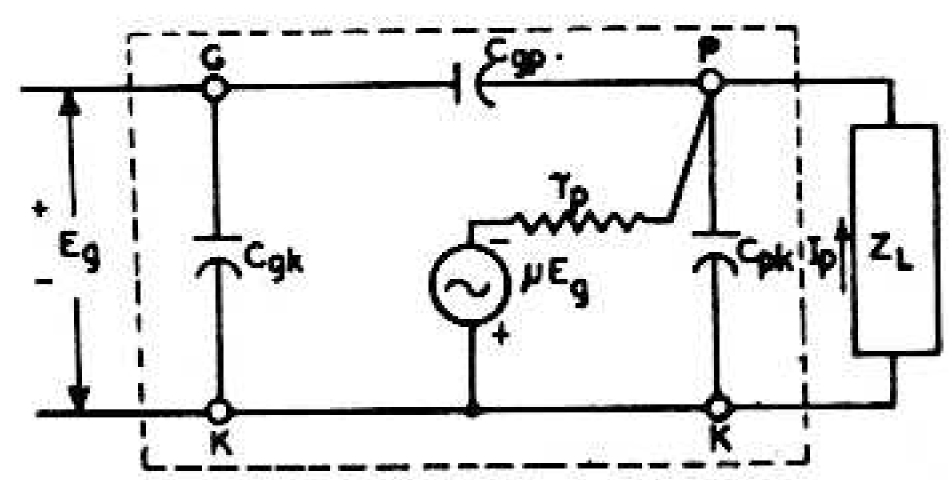

FIG. 2·38

Fig. 2.38. Equivalent circuit of valve on load, with interelectrode capacitances.

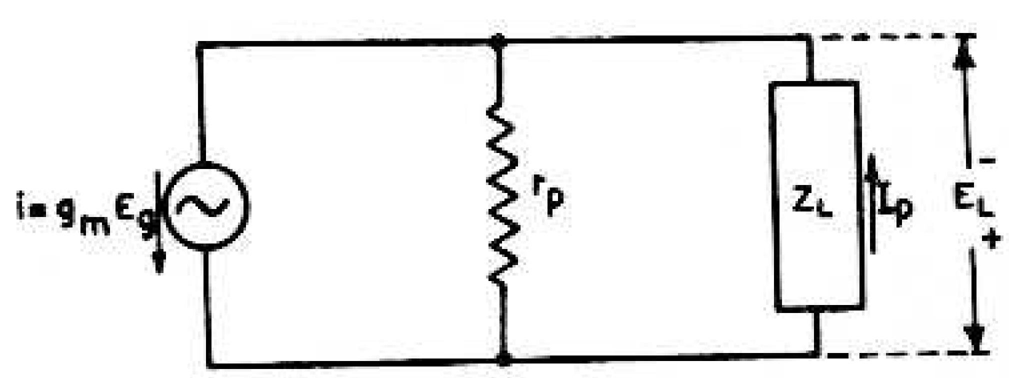

FIG. 2·39.

Fig. 2.39. Equivalent circuit of valve on load, using constant current generator.

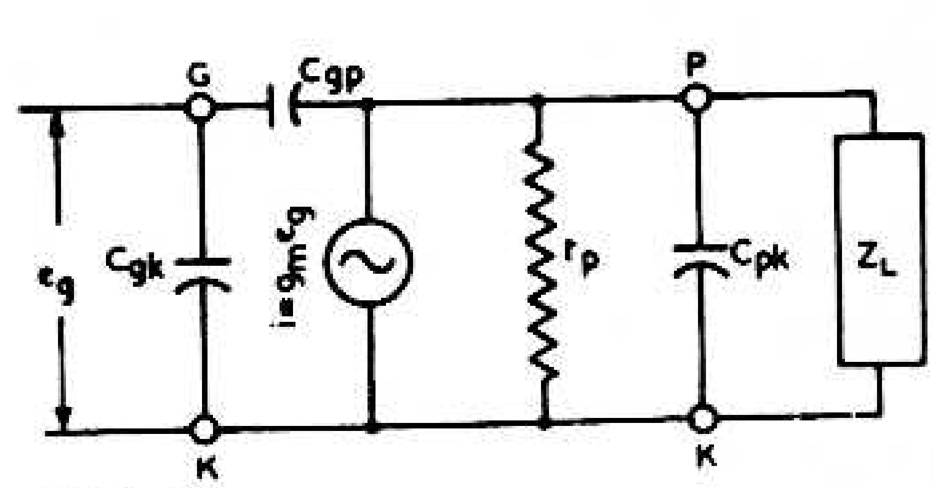

FIG. 2·40

Fig. 2.40. Equivalent circuit of valve on load, using constant current generator, with inter-electrode capacitances.

In the constant voltage generator equivalent circuit, the current varies with load impedance and plate resistance; in the constant current equivalent circuit, the voltage across the load and plate resistance varies with load impedance and plate resistance.

A constant current generator equivalent circuit, in which account is taken of capacitances, is shown in Fig. 2.40. This circuit may be applied at frequencies up to nearly 10 Mc/s, beyond which the inductances of the leads become appreciable. It will be seen that C_{pk} (which may be taken to include all capacitances from plate to cathode, and the output capacitance of a pentode) is shunted across both r_p and Z_L. In the case of a resistance-capacitance coupled stage, Z_L would be the resultant of R_L and R_g (following grid resistor) in parallel.

Maximum power output is obtained when the valve works into a load resistance equal to its plate resistance provided that the valve is linear and completely distortionless over the whole range of its working, and also that it is unlimited by maximum electrode dissipations or grid current. In practice, of course, these conditions do not hold and the load resistance is made greater than the plate resistance.

At frequencies of 10 Mc/s and above, the effects of the inductance of connecting leads (both internal and external to the valve) become appreciable. Although it is possible to draw an equivalent circuit for frequencies up to 100 Mc/s, in which each capacitance is split into an electrode part and a circuit part (Ref. B21 Fig. 38) the circuit is too complicated for analysis, and the new circuit elements that have been introduced cannot be measured directly from the external terminals alone.

At frequencies above about 50 Mc/s, transit time effects also become appreciable. The circuit which is commonly used for frequencies above 50 Mc/s is Fig. 2.47 in which the valve is treated as a four terminal network with two input and two output terminals. This is described in Sect. 8(iii)e.

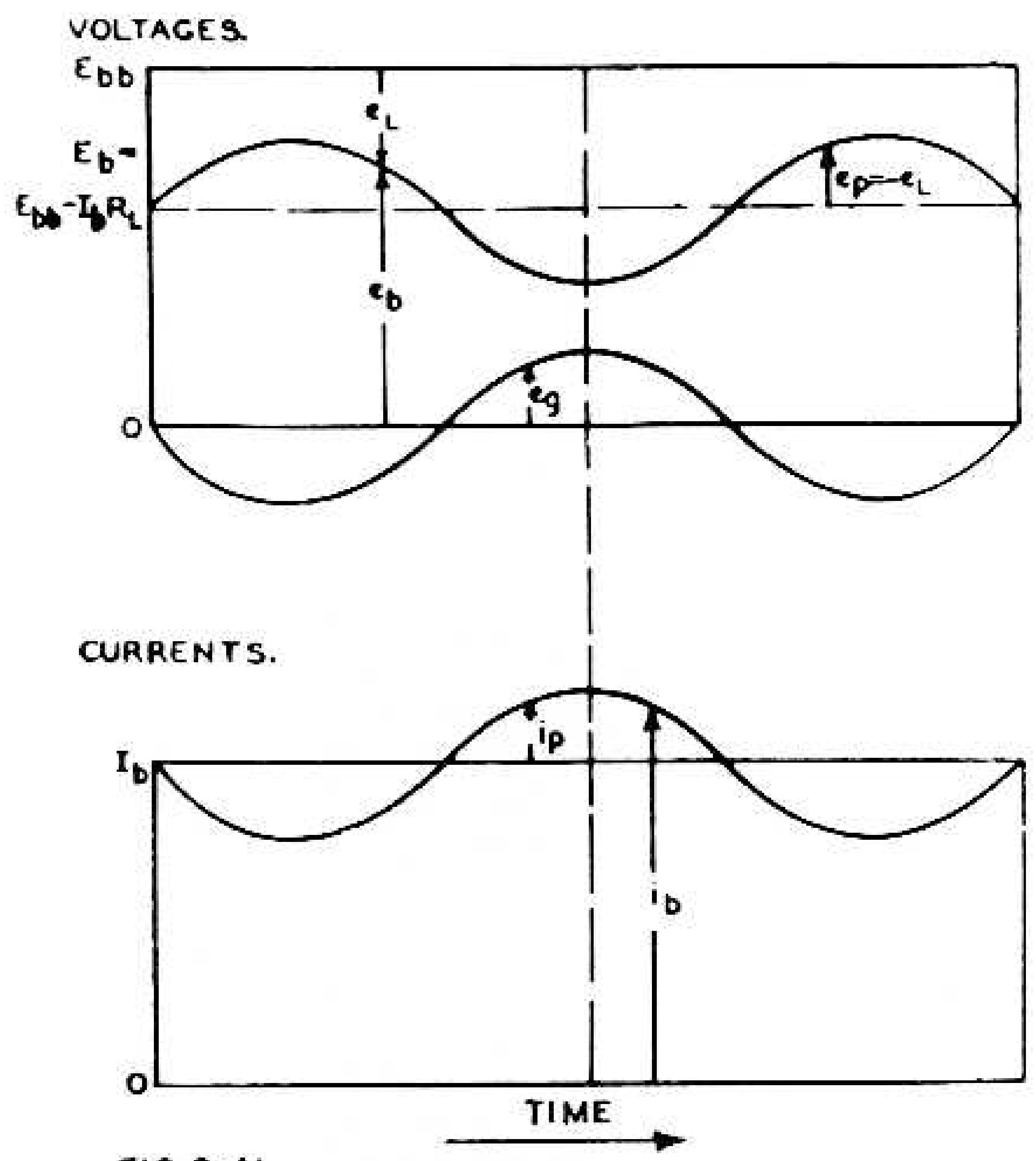

FIG. 2·41

Fig. 2.41. Voltage and current relationships in a resistance-loaded valve.

(iii) Valve vectors

Vectors [see Chapter 6 Sect. 5(iv)] may be used to illustrate the voltage and current relationships in a valve, but great care must be taken on account of the special conditions. Vectors are normally restricted to the representation of the a.c. voltages and currents when the grid is excited with a sine-wave voltage limited to such a value that the operation is linear. The grid and output voltages (with respect to the cathode) are normally of opposite polarity when the load is resistive; under the conditions noted above this is almost the same as being 180° out of phase except there is no half-cycle time lag between them.

The voltage and current relationships for a resistance loaded valve are shown in Fig. 2.41; peak total plate current occurs with peak positive grid voltage and results in maximum voltage across the load (e_L) and minimum voltage from plate to cathode (e_b). It will be seen that $e_b + e_L = E_{bb}$ (the supply voltage) under all conditions, and that e_L is naturally measured in the downward direction from E_{bb}. If only alternating components are considered, a negative peak e_g corresponds to a positive peak e_p and a negative peak e_L. If the supply voltage E_{bb} is omitted from the equivalent circuit, we are left with $e_p = -e_L$.

Each case must be considered individually and the vectors drawn to accord with the conditions. The only general rule is that E_g and μE_g are always either in phase or of opposite phase.

Fig. 2.42 shows the vector diagram (drawn with respect to the cathode) of an amplifying valve with a resistance load and a.c. grid excitation. Commencing with the grid-to-cathode voltage E_g, the vector μE_g is drawn in the same direction but is μ times as large. The output voltage E_L is also in the same direction as μE_g, but smaller by the value $I_p r_p$. All of these voltages are with respect to the cathode and the centre-point of the vector diagram has accordingly been marked K. The a.c. component of the plate current (I_p) is in phase with E_L, since E_L is the voltage drop which it produces in R_L. The grid-to-plate voltage E_c is the sum of E_L and E_g owing to the phase reversal between grid and plate.

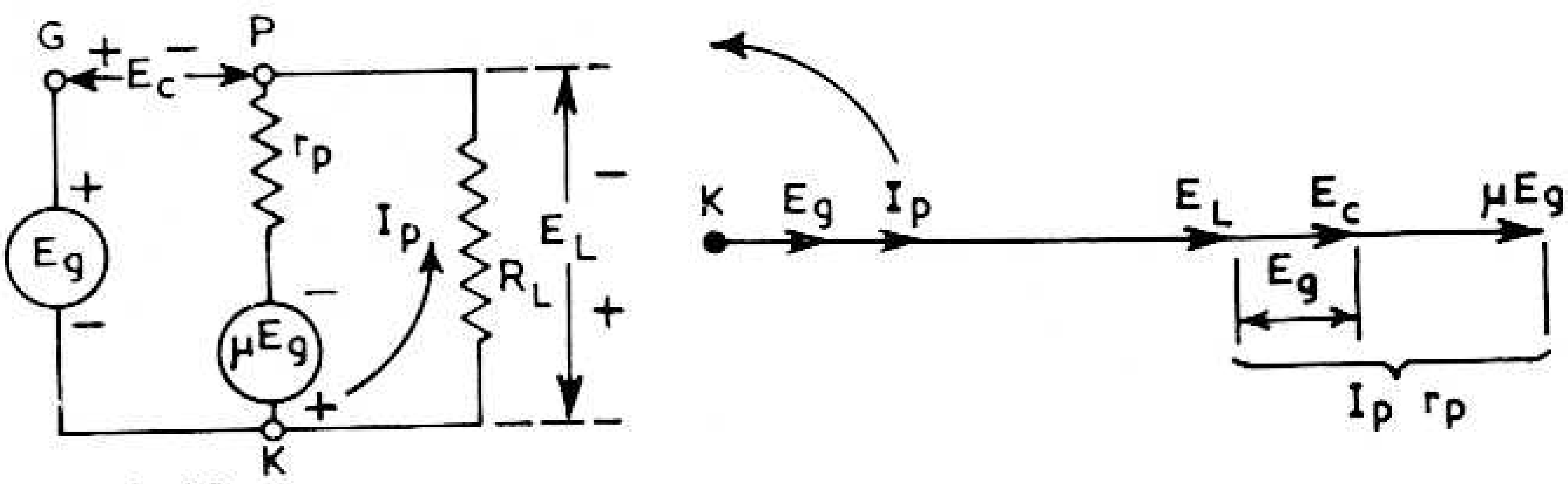

FIG. 2.42

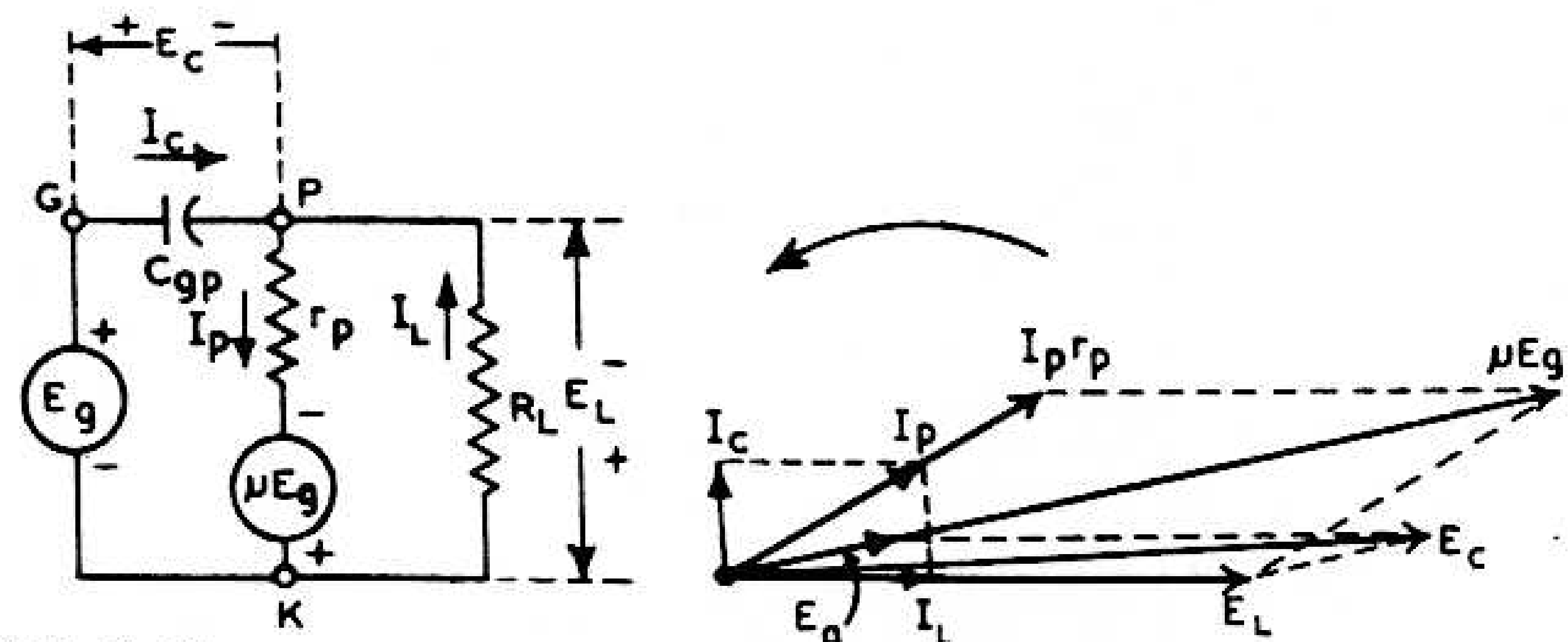

FIG. 2·43

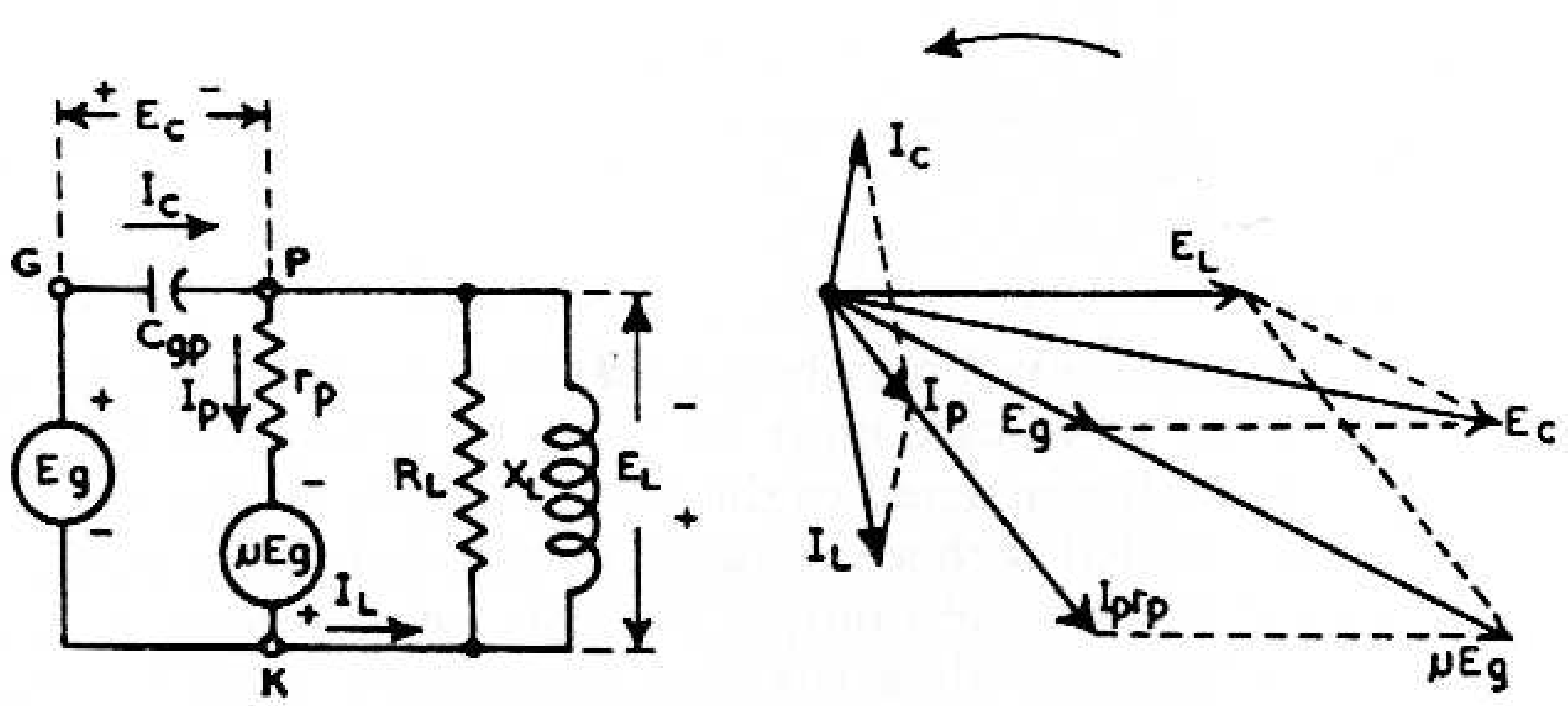

FIG. 2·44

Fig. 2.42. *Equivalent circuit and vector diagram of resistance-loaded valve.*

Fig. 2.43. *Vector diagram of valve with resistance load and capacitance from grid to plate.*

Fig. 2.44. *Vector diagram of valve with partially inductive load and capacitance from grid to plate.*

When the equivalent circuit includes more than one mesh, it is usual to proceed around each mesh in turn, using some impedance, common to both, as the link between each pair of meshes. For example Fig. 2.43 shows a valve with a capacitor C_{gp} from grid to plate, and a resistive load. Firstly, set down E_L in any convenient direction (here taken horizontally to the right) and I_L in the same direction; then draw I_c leading by approximately 90° (actually I_c leads E_c by 90°) and complete the parellelogram to find the resultant current I_p; then draw $I_p r_p$ in the same direction as I_p and complete the parallelogram to find the resultant μE_g—this completes the first mesh. Finally take E_g along μE_g and complete the parallelogram to find the resultant of E_g and E_L, which will be E_c.

If the load is partially inductive (Fig. 2.44) the plate current and $I_p r_p$ lag behind E_L and the resultant μE_g is determined by the parallelogram; E_g and E_L combine to give the resultant E_c; I_c leads E_c by 90°, and I_L is determined by completing the parallelogram of which I_c is one side and I_p the resultant.

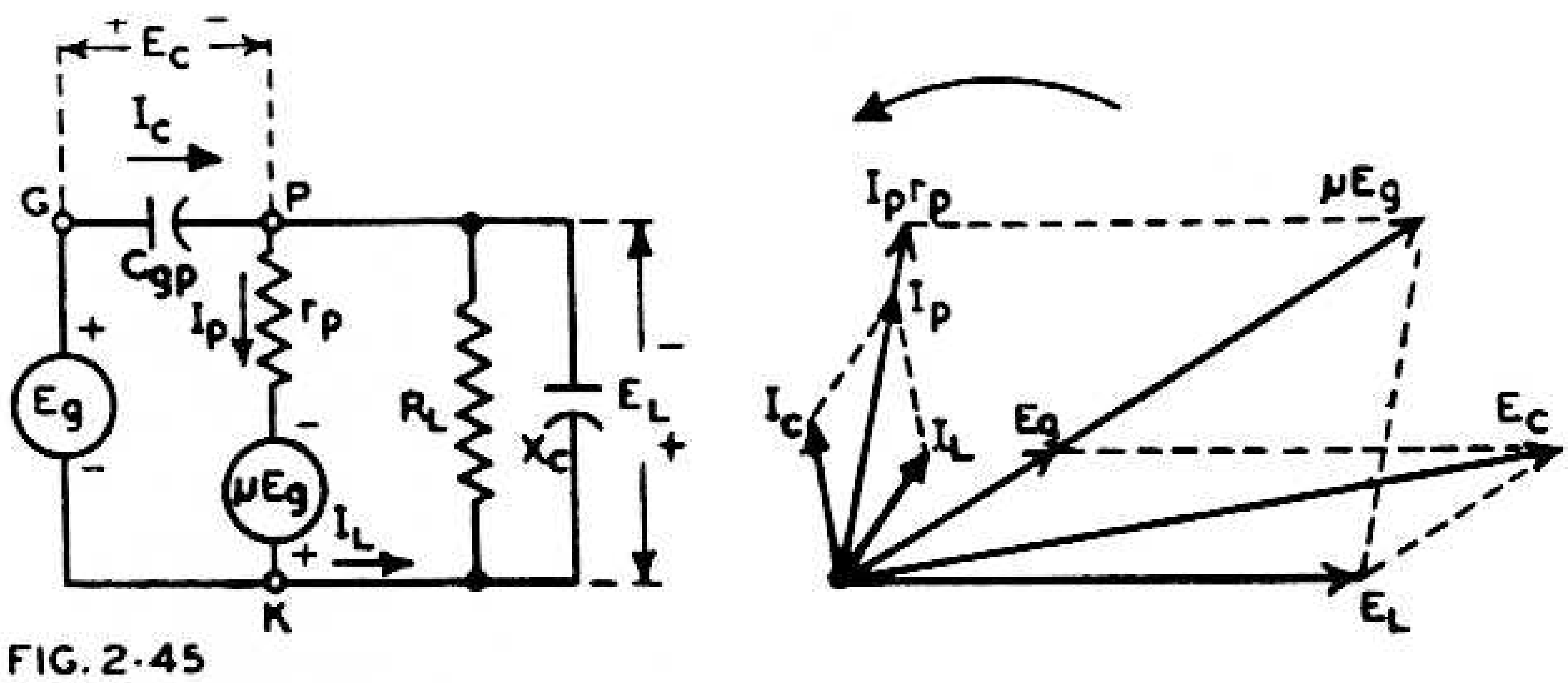

Fig. 2.45. *Vector diagram of valve with partially capacitive load and capacitance from grid to plate.*

With a partially capacitive load (Fig. 2.45) the plate current and $I_p r_p$ lead E_L, and the resultant μE_g is determined by the parallelogram; E_g and E_L combine to give the resultant E_c; I_c leads E_c by 90°, and I_L is determined by completing the parallelogram of which I_c is one side and I_p the resultant.

SECTION 8 : VALVE ADMITTANCES

(i) Grid input impedance and admittance (ii) Admittance coefficients (iii) The components of grid admittance—Input resistance—Input capacitance—Grid input admittance (a) with plate-grid capacitance coupling; (b) with both plate-grid and grid-cathode capacitance coupling; (c) with grid-screen capacitance coupling; (d) with electron transit time; (e) equivalent circuit based on admittances (iv) Typical values of short-circuit input conductance (v) Change of short-circuit-input capacitance with transconductance (vi) Grid-cathode capacitance (vii) Input capacitances of pentodes (published values) (viii) Grid-plate capacitance.

(i) Grid input impedance and admittance

When a valve is used at low audio frequencies, it is sometimes assumed that the grid input impedance is infinite. In most cases, however, this assumption leads to serious error, and careful attention is desirable to both its static and dynamic impedances.

As with any other impedance (see Chapter 4 Sect. 6) it may be divided into its various components :—

Component	Normal	Reciprocal
Resistive	Grid input resistance (r_g)	conductance (g_g)
Reactive	Grid input reactance (X_g)	susceptance (B_g)
Resultant	Grid input impedance (Z_g)	admittance (Y_g)

Normal values are measured in ohms, while reciprocal values are measured in reciprocal ohms (mhos). It is interesting to note that

a resistance of / a reactance of / an impedance of	is equivalent to	a conductance of / a susceptance of / an admittance of
1 megohm		1 micromho
0·1 megohm		10 micromhos
10 000 ohms		100 micromhos
1000 ohms		1000 micromhos = 1 mA/volt

The following relationships hold :

$$g_g = \frac{r_g}{r_g^2 + X_g^2}, \qquad r_g = \frac{g_g}{g_g^2 + B_g^2} \qquad (1)$$

$$B_g = \frac{X_g}{r_g^2 + X_g^2}, \qquad X_g = \frac{B_g}{g_g^2 + B_g^2} \qquad (2)$$

$$|Y_g| = \sqrt{g_g^2 + B_g^2} = 1/|Z_g|, \qquad Y_g = g_g + jB_g = 1/Z_g \qquad (3)$$

Similar relationships hold for other electrodes.

It is usual to carry out calculations with admittances, even though the resultant may then have to be changed to the form of an impedance. With a number of conductances (or susceptances) in parallel, the total conductance (or susceptance) is found by adding all together, with due regard to positive and negative quantities :—

$$\text{e.g. } g_g = g_1 + g_2 + g_3 + \ldots + g_k \qquad (4)$$

$$B_g = B_1 + B_2 + B_3 + \ldots + B_k \qquad (5)$$

Inductive reactance is regarded as positive.
Capacitive reactance is regarded as negative.
Inductive susceptance is regarded as positive.
Capacitive susceptance is regarded as negative.

With Complex Notation (see Chapter 6 Sect. 6) we have

$$Z_g = R_g + jX_g, \qquad Y_g = g_g - jB_g. \qquad (6)$$

(The " j " merely indicates a vector at 90° which must be added vectorially.)

(ii) Admittance coefficients

The operation of a valve may be expressed by the two equations

$$i_p = A\,e_g + B\,e_p \qquad (7)$$

$$i_g = C\,e_g + D\,e_p \qquad (8)$$

where A, B, C and D are complex values determined by the valve characteristics, being in the form of admittances and known as the Admittance Coefficients. The effect of these Admittance Coefficients may be understood more easily by considering two special cases, one with a short-circuited output (i.e. short-circuited from plate to cathode) and the other with a short-circuited input (i.e. short-circuited from grid to cathode).

Case 1 : Short-circuited output ($e_p = 0$).
From equation (7), $i_p = A\,e_g$
From equation (8), $i_g = C\,e_g$

where A is defined as the short-circuit forward admittance,
and C is defined as the short-circuit input admittance.

Case 2 : Short-circuited input ($e_g = 0$).
From equation (7), $i_p = B\,e_p$
From equation (8), $i_g = D\,e_p$

where B is defined as the short-circuit output admittance,
and D is defined as the short-circuit feedback admittance.

At frequencies up to about 10 Mc/s, the Admittance Coefficients are given approximately by:

Short-circuit forward admittance $(A) = g_m - j\omega C_{gp} \approx g_m$ (9)
Short-circuit output admittance $(B) = 1/r_p + j\omega(C_{pk} + C_{gp})$ (10)
Short-circuit input admittance $(C) = 1/r_g + j\omega(C_{gk} + C_{gp})$ (11)
Short-circuit feedback admittance $(D) = j\omega C_{gp}$ (12)

If the grid is negatively biased to prevent the flow of positive grid current, the grid resistance r_g becomes very high, and $1/r_g$ may be negligible in the expression for C.

At frequencies above 10 Mc/s the Admittance Coefficients are somewhat modified, the capacitances and admittances containing a term which is proportional to the square of the frequency.

The short-circuit forward admittance (A) is affected by the transit time of electrons and the inductance of the cathode lead, thus causing a phase shift between anode current and grid voltage. This is treated in detail in Chapter 23 Sect. 5.

The short-circuit output admittance (B) is affected by the reduction in r_p which occurs with increasing frequency due to the capacitances and inductances of the electrodes. The capacitance term is practically constant.

The short-circuit input admittance (C) is affected by the transit time of electrons, the inductances of the electrodes (particularly the cathode) and the capacitance between grid and cathode. The capacitance term is practically constant.

The short-circuit feedback admittance (D) remains purely reactive even at very high frequencies, although it changes from capacitive at low frequencies, through zero, to inductive at high frequencies. This can cause instability in certain circumstances.

(iii) The components of grid admittance

Input resistance may be due to several causes :

1. Leakage between the grid and other electrodes.
2. Negative grid current (caused by gas or grid emission).
3. Positive grid current (may be avoided by negative grid bias).
4. Coupling between the grid and any other electrode presenting an impedance to the input frequency (e.g. C_{gp}).
5. Transit time of the electrons between cathode and grid (at very high frequencies only).

Input capacitance (C_{in}) is dependent on several factors :

1. The static (cold) capacitance (C_i) from the grid to all other electrodes, except the plate.

$$\text{For a pentode, } C_i = C_{gi \cdot k} + C_{g1 \cdot g2} \quad (13)$$
$$\text{For a triode, } C_i = C_{gk}. \quad (14)$$

2. The very slight increase in capacitance caused by thermal expansion of the cathode (0·1 to 0·6 $\mu\mu$F for the majority of r-f pentodes).
3. The increase in capacitance caused by the space charge and by conduction (0·5 to 2·4 $\mu\mu$F for r-f pentodes).
4. Coupling between the grid and any other electrode presenting an impedance to the input frequency ; this holds both with capacitive and inductive reactance (Miller Effect—see below).
5. Transit time of the electrons between cathode and grid (at very high frequencies only).

References to change of input capacitance : B13, B15, B16, B17, C1, C4, C5.

The measurement of interelectrode capacitances is covered in Chapter 3 Sect. 3(ii)g, together with some general comments and a list of references to their significance and measurement.

Grid Input Admittance

(a) With plate-grid capacitance coupling

In the circuit of Fig. 2.46A, in which $C_{g1 \cdot k}$ and $C_{g1 \cdot g2}$ are not considered, it may be shown* that

$$r_g = \frac{1}{g_g} = \frac{(g_p + G_L)^2 + (B_L + B_{gp})^2}{B_{gp}[g_m \cdot B_L + B_{gp}(g_p + G_L + g_m)]} \quad (15)$$

and

$$C_g = \frac{B_g}{\omega} = \frac{C_{gp}[(g_p + G_L + g_m)(g_p + G_L) + B_L(B_L + B_{gp})]}{(g_p + G_L)^2 + (B_L + B_{gp})^2} \quad (16)$$

where $g_p = 1/r_p$, $Y_L = G_L + jB_L = 1/Z_L$, $B_{gp} = 1/X_{Cgp} = 1/2\pi f C_{gp}$.
As an approximation, if $g_p \ll G_L$, and $B_{gp} \ll B_L$,

$$r_g = \frac{1}{g_g} = -\frac{1}{g_m \omega^2 L_o C_{gp}} \quad \text{when the load is inductive} \quad (17)$$

$$= \frac{C_L}{g_m C_{gp}} \quad \text{when the load is capacitive} \quad (18)$$

*Sturley, K. R. "Radio Receiver Design, Part 1" (Chapman & Hall, London, 1943) p. 37 *et seq*

$$C_g = C_{gp}\left[1 + \frac{g_m G_L}{G_L^2 + B_L^2}\right] \tag{19}$$

When B_L is infinite, i.e. when $Z_L = 0$, R_g is infinite and $C_g = C_{gp}$.

When the load is inductive, the input resistance is usually negative, thus tending to become regenerative, although for values of B_L between 0 and

$$-B_{gp}(g_p + G_L + g_m)/g_m,$$

the input resistance is positive.

When the load is capacitive, the input resistance is always positive, thus causing degeneration.

As an approximation, if $B_{gp} \ll g_m$ and $B_{gp} \ll (g_p + G_L)$, the positive and negative minimum values of r_g are given by

$$r_g\,(min) \approx \pm\frac{2(g_p + G_L)}{g_m B_{gp}} \tag{20}$$

and these occur at $B_L = \pm(g_p + G_L)$.

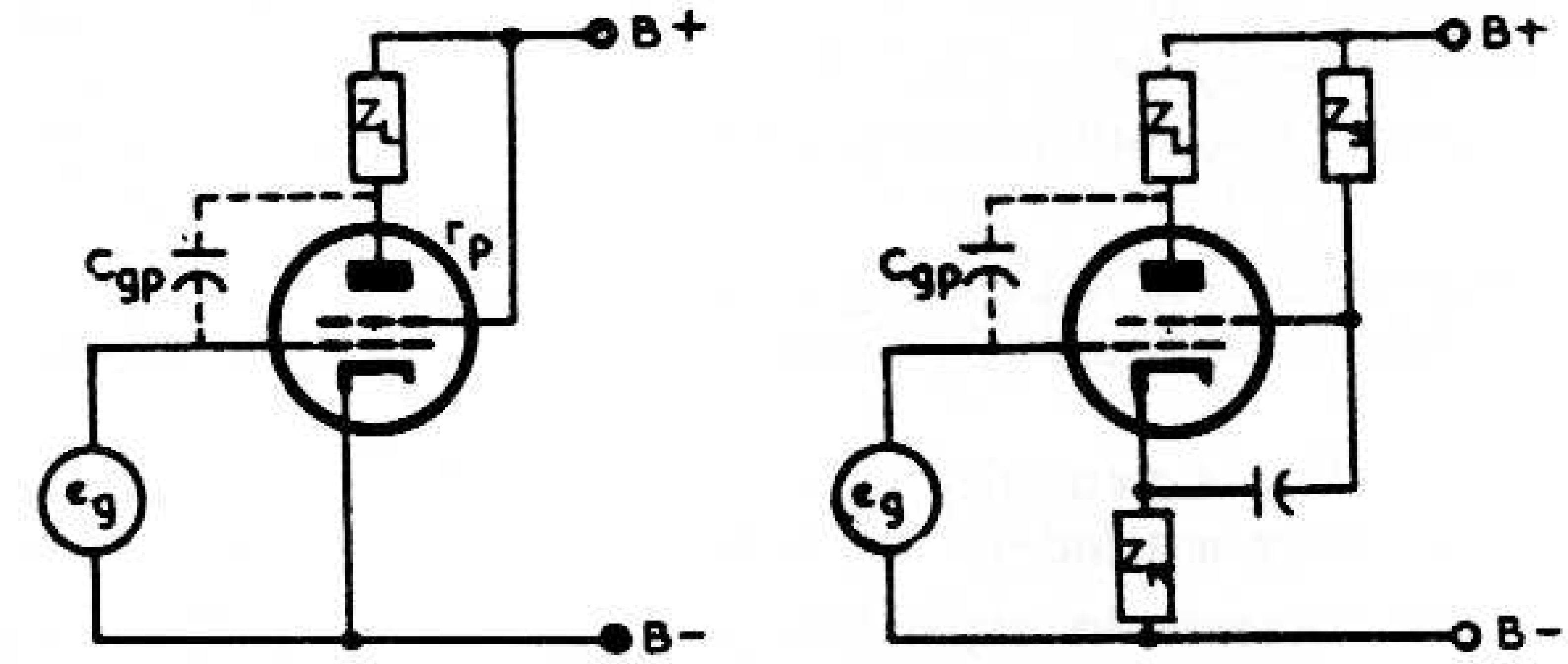

Fig. 2.46. *Conditions for deriving input admittance (A) with plate-grid capacitance coupling (B) general case including cathode circuit impedance.*

Similarly, the maximum value of the input capacitance is given by

$$C_g\,(max) = C_{gp}\left[1 + \frac{g_m r_p R_L}{r_p + R_L}\right] = C_{gp}\left[1 + \frac{\mu R_L}{r_p + R_L}\right] \tag{21}$$

which occurs at $B_L = -B_{gp}$. This is the well known " Miller Effect " [see Chapter 12 Sect. 2(xi) for a-f amplifiers]. The effect on the tuning of r-f amplifiers is treated in Chapter 23 Sect. 5, and on i-f amplifiers in Chapter 26 Sect. 7.

In the circuit of Fig. 2.46B, which includes an impedance Z_k in the cathode circuit, with the screen decoupled to the cathode, the input resistance is given approximately by

$$r_g = \frac{1}{g_g} \approx \frac{G_L^2 + B_L^2}{g_m B_{gp} B_L}\left[1 + \frac{g_m(g_m + 2G_k)}{G_k^2 + B_k^2}\right] \tag{22}$$

where B_{gp} and g_p are neglected in comparison with the other components, and $(B_L G_k - B_k G_L)$ is very small. Thus the reflected resistance is increased, and the damping decreased, as the result of the insertion of Z_k.

The input capacitance under these conditions is given by

$$C_g = C_{gp}\left[\frac{(G_L + g_m)G_L + B_L^2}{G_L^2 + B_L^2} - \frac{g_m[g_m^2 G_L + g_m(G_k G_L - B_k B_L)]}{(G_L^2 + B_L^2)[(G_k + g_m)^2 + B_k^2]}\right] \tag{23}$$

which is less than with $Z_k = 0$.

If the screen is by-passed to the cathode,

$$r_g = \frac{1}{g_g} = \frac{G_L^2 + B_L^2}{g_m B_{gp} B_L}\left[1 + \frac{g_t(g_t + 2G_k)}{G_k^2 + B_k^2}\right] \tag{24}$$

$$C_g = C_{gp}\left[\frac{(G_L + g_m)G_L + B_L^2}{G_L^2 + B_L^2} - \frac{g_m[g_t^2 G_L + g_t(G_k G_L - B_k B_L)]}{(G_L^2 + B_L^2)[(G_k + g_t)^2 + B_k^2]}\right] \tag{25}$$

where g_t = triode g_m (whole cathode current)
$= g_m (I_p + I_{g2})/I_p$.

(b) With both plate-grid and grid-cathode capacitance coupling

The circuit is as Fig. 2.46B with the addition of a capacitance C_{gk} between grid and cathode. The input resistance is given by

$$r_g = \frac{[(G_k + g_m)^2 + B_k^2][G_L^2 + B_L^2]}{g_m[B_{gp}B_L(G_k^2 + B_k^2) - B_{gk}B_k(G_L^2 + B_L^2)]} \qquad (26)$$

This becomes infinite when $B_{gp}B_L(G_k^2 + B_k^2) = B_{gk}B_k(G_L^2 + B_L^2)$, which is the condition for input resistance neutralization (see Chapter 26 Sect. 8 for i-f amplifiers). This condition may be put into the form

$$\frac{B_{gp}}{B_{gk}} = \frac{C_{gp}}{C_{gk}} = \frac{L_k}{L_p} \qquad (27)$$

Thus, by including an inductance $L_p = L_k C_{gk}/C_{gp}$ between the load and the plate, the input resistance may be increased to a very high value. The same effect may also be achieved by means of an inductance in the screen circuit.

The input capacitance under the conditions of Fig. 2.46B is given approximately by

$$C_g \approx C_{gp} + C_{gk} + g_m\left[\frac{C_{gp}\,G_L}{G_L^2+B_L^2} - \frac{C_{gk}(G_k + g_m)}{(G_k + g_m)^2 + B_k^2}\right] \qquad (28)$$

If $g_m \ll G_k$ it is possible to prevent change of input capacitance when g_m is varied (for example with a.v.c.), by making

$$\frac{C_{gp}}{C_{gk}} = \frac{R_k}{R_L}.$$

(c) With grid-screen capacitance coupling

Conditions as in Fig. 2.46B, but with screen by-pass capacitor.

$$r_g = \frac{(g_{g2} + G_s)^2 + (B_s + B_{g1\cdot g2})^2}{B_{g1\cdot g2}[g_{g1\cdot g2}B_sB_{g1\cdot g2}(g_{g2} + G_s + g_{g1\cdot g2})]} \qquad (29)$$

$$C_g = C_{g1\cdot g2}\left[\frac{(g_{g2} + G_s + g_{g1\cdot g2})(g_{g2} + G_s) + B_s(B_s + B_{g1\cdot g2})}{(g_{g2} + G_s)^2 + (B_s + B_{g1\cdot g2})^2}\right] \qquad (30)$$

where g_{g2} = screen conductance,
$g_{g1\cdot g2}$ = grid-screen transconductance,
and $B_{g1\cdot g2}$ = susceptance due to capacitance from $g1$ to $g2$.

In an r-f amplifier, $B_{g1\cdot g2}$ and $(g_{g2} + G_s)$ may usually be neglected in comparison with B_s, and thus

$$r_g \approx -\frac{1}{g_{g1\cdot g2}\,\omega^2 L_s C_{g1\cdot g2}} \quad \text{when } B_s \text{ is inductive} \qquad (31)$$

$$r_g \approx \frac{C_1}{g_{g1\cdot g2}\,C_{g1\cdot g2}} \quad \text{when } B_s \text{ is capacitive} \qquad (32)$$

The input resistance may be made infinite by making

$$C_{g1\cdot g2}/C_{gk} = L_k/L_s \qquad (33)$$

(d) With electron transit time

This subject is treated fully in Chapter 23 Sect. 5.

(e) Equivalent circuit based on admittances

In determining valve admittances at frequencies higher than approximately 10 Mc/s, it is not practicable to introduce voltages or measure them directly at the electrodes of a valve. The lead inductances and interelectrode capacitances form a network too complex for exact analysis. The most practical method of avoiding such difficulties is to consider the valve, the socket, and the associated by-pass or filter circuits as a unit, and to select a pair of accessible input terminals and a pair of accessible output terminals as points of reference for measurements. When such a unit is considered as a linear amplifier, it is possible to calculate performance in terms of four admittance coefficients. These are:

Y_{in} = short-circuit input admittance
= admittance measured between input terminals when the output terminals are short-circuited for the signal frequency.

Y_{for} = short-circuit forward admittance
= value of current at output terminals divided by the voltage between the input terminals, when the output terminals are short-circuited for the signal frequency.

Y_{out} = short-circuit output admittance
= admittance measured between output terminals when the input terminals are short-circuited for the signal frequency.

Y_{fb} = short-circuit feedback admittance
= value of current at the input terminals divided by the voltage between the output terminals, when the input terminals are short-circuited for the signal frequency.

Each of these admittances can be considered as the sum of a real conductance component and an imaginary susceptance component. In the cases of the input and output admittances, the susceptance components are nearly always positive (unless the valve is used above its resonant frequency) and it is, therefore, common practice to present the susceptance data in terms of equivalent capacitance values. The short-circuit input capacitance is the value of the short-circuit input susceptance divided by 2π times the frequency. The capacitance values are more convenient to work with than the susceptance values because they vary less rapidly with frequency and because they are directly additive to the capacitances used in the circuits ordinarily connected to the input and output terminals. However, when frequencies higher than 200 Mc/s and resonant lines used as tuning elements are involved, the use of susceptance values may be preferable.

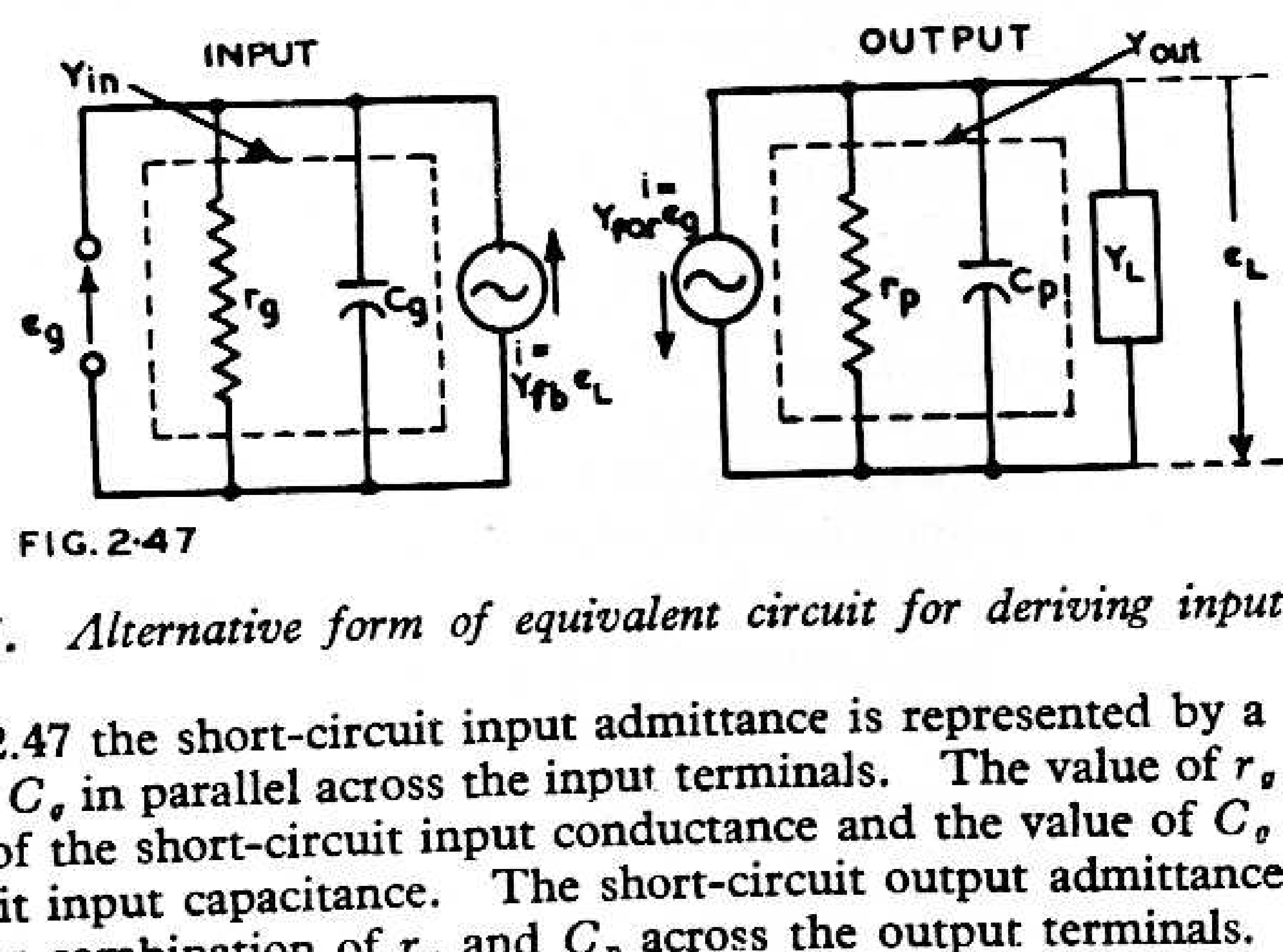

FIG. 2·47

Fig. 2.47. Alternative form of equivalent circuit for deriving input admittance

In Fig. 2.47 the short-circuit input admittance is represented by a resistor r_g and a capacitor C_g in parallel across the input terminals. The value of r_g is equal to the reciprocal of the short-circuit input conductance and the value of C_g is equal to the short-circuit input capacitance. The short-circuit output admittance is represented by a similar combination of r_p and C_p across the output terminals.

Since the input and output circuits are separated, allowance may be made for their interaction by an additional constant current generator in each. A constant current generator is shown at the output terminals producing a current equal to the product of the short-circuit forward admittance and the input voltage. A similar generator is shown at the input terminals producing a current equal to the product of the short-circuit feedback admittance and the output voltage.

The principal differences in the performance of receiving valves at high and low frequencies can be attributed to the variations of the short-circuit input conductance with frequency. The other short-circuit admittance coefficients, however, contribute to the input admittance actually observed in an operating circuit as follows:

$$\text{Voltage gain } (A) = \frac{Y_{for}}{Y_{out} + Y_L} \approx \frac{g_m}{Y_{out} + Y_L} \quad (34)$$

Added current at input terminals due to presence of load $= e_g A Y_{fb}$. (35)

Phase angle of added component = phase angle of voltage gain + phase angle of feedback admittance. (36)

Grid input admittance $(Y_g) = Y_{in} + AY_{fb}$

$$= Y_{in} + \frac{Y_{for}\,Y_{fb}}{Y_{out} + Y_L} \qquad (37)$$

See also Ref. B21.

The measurement of the four short-circuit admittances is covered in Chapter 3 Sect. 3(vi) A, B, C and D and also Refs. B17, B21.

(iv) Typical values of short-circuit input conductance

Pentodes tested under typical operating conditions (Ref. B17).

Type	Input conductance approx. (micromhos) f = 50	60	80	100	120	150 Mc/s	Mutual Conductance μmhos
6AB7	200	310	600	980			5000
6AC7	380	600	1200	1970			9000
6AG5	100	145	280	326	480		5000
6AK5	40	57	92	134	185		5100
6AU6	180	280	490	759	1100		5200
6BA6	150	230	410	603	950		4400
6CB6	125	170	300	460	(Ref. B20)		6200
6BJ6				275	(Ref. B19)		3800
6SG7	190	270	430	604	670		4700
6SH7	200	300	470	632	880		4900
6SJ7		260	380	528			1650
6SK7	138	190	320	503	660		2000
9001			44	60	96	141	1400
9003			48	66	100	145	1800
Z77	110 at 45 Mc/s (Data from M.O.V.)						7500

(v) Change of short-circuit input capacitance with transconductance

(f = 100 Mc/s). Ref. B17 unless otherwise indicated.

Type	Increase in capacitance ($\mu\mu F$) from cut-off to g_m = 1000	2000	4000	Typical operation $\mu\mu$F	μmhos
6AB7	0·55	1·0	1·7	1·8	5000
6AC7	0·65	1·2	1·8	2·4	9000
6AG5	0·5	0·8	1·25	1·4	5000
6AK5	0·3	0·6	1·0	1·1	5100
6AU6	0·6	1·1	2·0	2·5	5200
6BA6	0·75	1·4	2·2	2·2	4400
6BH6 (Ref. B18)				1·8	4600
6CB6 (Ref. B20)				1·54	6200
6BJ6 (Ref. B19)				1·6	3800
6SG7	0.8	1·5	2·2	2·3	4700
6SH7	0·75	1·3	2·05	2·3	4900
6SJ7	0·8	—	—	1·0	1650
6SK7	0·65	1·18	—	1·2	2000
9001	0·35	—	—	0·5	1400
9003	0·39	—	—	0·5	1800
Z77 (M.O.V.)				2·2	7500
Limits	0·3-0·8	0·6-1·5	1·0-2·2	0·43-2·38	

Value of unbypassed cathode resistor needed for complete compensation of input capacitance change with bias change (Ref. B18)

Valve type	Interelectrode capacitances C_{in}	C_{out}	C_{gp}	Unbypassed cathode resistor	Gain factor*
6BA6	5·5 $\mu\mu$F	5·0 $\mu\mu$F	0·0035 $\mu\mu$F	100 ohms	0·62
6AU6	5·5	5·0	0·0035	85	0·61
6AG5	6·5	1·8	0·025	50	0·75
6AK5	4·0	2·8	0·02	50	0·75
6BJ6	4·5	5·5	0·0035	135	0·59
6BH6	5·4	4·4	0·0035	110	0·59
Z77	7·4	3·1	0·009	60	0·64

*degeneration due to unbypassed cathode resistor (see below).

$$\text{Gain factor} = \frac{\text{gain with cathode unbypassed}}{\text{gain with cathode by-passed}} \quad (38)$$

$$= \frac{1}{1 + R_k g_m (I_b + I_{c2})/I_b} \quad (39)$$

where R_k = cathode resistor for complete compensation of input capacitance change with bias

$$\approx \frac{\triangle C}{C_{gk}\, g_m (I_b + I_{c2})/I_b}$$

$\triangle C$ = change in input capacitance in farads from normal operating condition to cut-off,

C_{gk} = grid-to-cathode capacitance in farads measured with valve cold,

g_m = mutual conductance in mhos at normal operating condition,

I_b = direct plate current in amperes

and I_{c2} = direct screen current in amperes.

(vi) Grid-cathode capacitance

The mathematical treatment of the effects of grid-cathode capacitance has been given above. Methods of neutralization are described in Chapter 26 Sect. 8.

The published grid-plate capacitances are usually in the form of a maximum value, without any indication of the minimum or average value. In some cases the average is fairly close to the maximum, while in others it may be considerably less. The average value is likely to vary from one batch to another, and from one manufacturer to another. Equipment should be designed to avoid instability with the maximum value, although fixed neutralization should be adjusted on an average value, determined by a test on a representative quantity of valves.

Effect of electrode voltages on grid-cathode capacitance—see Ref. B23.

(vii) Input capacitances of pentodes (published values)

	$\mu\mu$F
1. Indirectly heated	
High slope r-f (metal)	8 to 11
Ordinary metal r-f, All glass and miniature r-f	4·3 to 7
Small power amplifiers	5 to 6
Ordinary power amplifiers	6·5 to 10
Large power amplifiers	10 to 15
Pentode section of diode-pentodes :	
Metal	5·5 to 6·5
Glass	3 to 5·5
2. Directly heated	
2 volt r-f pentodes	5 to 6
1·4 volt r-f pentodes	2·2 to 3·6
2 volt power pentodes	8
1·4 volt power pentodes	4·5 to 5·5

(viii) Grid-plate capacitance

The grid-plate capacitance decreases with increasing plate current. Eventually the rate of change becomes very small and even tends to become positive. The total change in triodes does not usually exceed 0·06 $\mu\mu$F for high-mu types, or 0·13 $\mu\mu$F for low-mu voltage amplifiers, although it may exceed 2 $\mu\mu$F in the case of triode power amplifiers (Ref. B13).

SECTION 9: MATHEMATICAL RELATIONSHIPS

(i) General (ii) Resistance load (iii) Power and efficiency (iv) Series expansion; resistance load (v) Series expansion; general case (vi) The equivalent plate circuit theorem (vii) Dynamic load line—general case (viii) Valve networks—general case (ix) Valve coefficients as partial differentials (x) Valve characteristics at low plate currents.

(i) General

Valve characteristics may be represented mathematically as well as graphically (see Chapter 6 for mathematical theory).

The plate (or space) current is a function (F) of the plate and grid voltages and may be expressed exactly as

$$i_b = F(e_b + \mu e_c + e_1) \tag{1}$$

where e_1 is the equivalent voltage which would produce the same effect on the plate current as the combined effects of the initial electron velocity of emission together with the contact potentials. The amplification factor μ is not necessarily constant. There will be a small current flow due to e_1 when e_b and e_c are both zero.

As an approximation, when e_b and μe_c are large, e_1 may be neglected. The function in eqn. (1) may also be expressed approximately in the form

$$i_b \approx K(e_b + \mu e_c)^n \tag{2}$$

in which K is a constant. The value of n varies from about 1·5 to 2·5 over the usual operating range of electrode voltages, but is often assumed to be 1·5 (e.g. Conversion Factors) over the region of nearly-straight characteristics, and 2·0 in the region of the bottom bend (e.g. detection). We may take the total differential* of eqn. (2),

$$di_b = \frac{\partial i_b}{\partial e_b} de_b + \frac{\partial i_b}{\partial e_c} de_c \tag{3}$$

which expresses the change in i_b which occurs when e_b and e_c change simultaneously. Now—see (ix) below—provided that the valve is being operated entirely in the region in which μ, g_m and r_p are constant,

$$\frac{\partial i_b}{\partial e_b} = \frac{1}{r_p} \quad \text{and} \quad \frac{\partial i_b}{\partial e_c} = g_m,$$

so that

$$di_b = \frac{1}{r_p} de_b + g_m de_c. \tag{4}$$

If i_b is held constant,

$$\frac{1}{r_p} de_b + g_m de_c = 0,$$

and thus

$$g_m r_p = -\frac{de_b}{de_c} \quad (i_b \text{ constant}) \tag{5}$$

whence $g_m r_p = \mu$ [see (ix) below]. (6)

The treatment so far has been on the basis of the total instantaneous voltages and currents, e_b, e_c, i_b; it is now necessary to distinguish more precisely between the steady (d.c.) and varying (signal) voltages and currents.

*For total differentiation see Chapter 6 Sect. 7(ii).

Instantaneous total values	e_b	e_c	i_b
Steady (d.c.) values	E_{bo}	E_{cc}	I_{bo}
Instantaneous varying values	e_p	e_g	i_p
Varying (a.c.) values (r.m.s.)	E_p	E_g	I_p
Supply voltages	E_{bb}	E_{cc}	

For definitions of symbols refer to the list in Chapter 38 Sect. 6.

In normal operation each of the voltages and currents is made up of a steady and a varying component:

$$\left.\begin{aligned} i_b &= I_{bo} + i_p \\ e_b &= E_{bo} + e_p \\ e_c &= E_{cc} + e_g \end{aligned}\right\} \quad (7)$$

Eqn. (4) may therefore be extended in the form

$$d(I_{bo} + i_p) = \frac{1}{r_p} d(E_{bo} + e_p) + g_m d(E_{cc} + e_g).$$

But the differentials of constants are zero, and the relation between the varying components may be expressed in the form

$$i_p = \frac{e_p}{r_p} + g_m e_g \quad (8)$$

or

$$i_p = \frac{e_p + \mu e_g}{r_p}. \quad (9)$$

This only holds under the condition that μ, g_m and r_p are constant over the operating region.

(ii) Resistance load

If there is a resistance load (R_L) in the plate circuit,

$$e_b + e_L' = E_{bb} \quad (10)$$

where e_L' is the instantaneous total voltage across R_L.

Breaking down into steady and varying components,

$$(E_b + e_p) + (E_L + e_L) = E_{bb} \quad (11)$$

where e_L is the instantaneous varying voltage across R_L,
and E_L is the steady (d.c.) voltage across R_L.

Under **steady conditions** $e_p = e_L = 0$, and therefore

$$E_b + E_L = E_{bb}.$$

Now, by Ohm's Law, $E_L = I_b R_L$

Therefore

$$I_b = \frac{E_{bb} - E_b}{R_L}. \quad (12)$$

Eqn. (12) represents a straight line on the plate characteristics, passing through the points

$$E_b = E_{bb}, I_b = 0 \text{ and } E_b = 0, I_b = E_{bb}/R_L,$$

in other words, **the loadline.**

The quiescent operating point must satisfy both the equation for the valve characteristics (1) and that for the loadline (12), therefore

$$F(e_b + \mu e_c + e_1) = \frac{E_{bb} - E_b}{R_L}. \quad (13)$$

Under **varying conditions,** neglecting steady components, we may derive from equation (11) the relation

$$e_p + e_L = 0$$

i.e.

$$e_p = -e_L. \quad (14)$$

Also $e_L = i_p R_L$ by Ohm's Law,

Therefore

$$e_p = -i_p R_L. \quad (15)$$

Substituting this value of e_p in equation (9) we obtain

$$i_p = \frac{-i_p R_L + \mu e_g}{r_p}$$

i.e.
$$i_p = \frac{\mu e_g}{r_p + R_L} \tag{16}$$

which is a fundamentally important relation but which holds only in the region where μ, g_m and r_p are constant.

Substituting $-e_L$ (eqn. 14) in place of e_p in equation (9) we obtain, **for the valve alone,**

$$i_p = \frac{-e_L + \mu e_g}{r_p}$$

i.e.
$$e_L = \mu e_g - i_p r_p \tag{17}$$

which is **the basis of the constant voltage generator equivalent circuit** as in Sect. 7(i).

Eqn. (17) may be put into the form

$$e_L = r_p (g_m e_g - i_p). \tag{18}$$

This voltage e_L across the valve and the load can be developed by means of a current $g_m e_g$, passed through r_p in the opposite direction to i_p, so that the total current through r_p is $(g_m e_g - i_p)$. Eqn. (18) is **the basis of the constant current equivalent circuit** as in Sect. 7(ii).

The **voltage gain** (A) of an amplifying stage with a load resistance R_L is

$$A = \left|\frac{e_L}{e_g}\right| = \left|\frac{i_p R_L}{e_g}\right| = \left|\frac{\mu R_L}{r_p + R_L}\right|. \tag{19}$$

When the load is an impedance Z_L, the voltage gain may be shown to be

$$A = \left|\frac{\mu Z_L}{r_p + Z_L}\right| = \left|\frac{\mu}{1 + r_p/Z_L}\right| \tag{20}$$

where r_p and Z are complex values (see Chapter 6).

If $Z_L = R_L + jX_L$, the scalar value of A is given by

$$A = \mu \frac{\sqrt{R_L^2 + X_L^2}}{\sqrt{(r_p + R_L)^2 + X_L^2}}. \tag{21}$$

The voltage gain may also be put into the alternative form

$$A = \left| g_m \frac{r_p Z_L}{r_p + Z_L} \right| \tag{22}$$

$$\approx |g_m Z_L| \text{ if } r_p \gg Z_L. \tag{23}$$

(iii) Power and efficiency

When the operation of a valve as a Class A_1 amplifier is perfectly linear we may derive* the following :—

Zero-Signal :

Plate current		$= I_{bo}$.
Power input from plate supply	P_{bb}	$= E_{bb} I_{bo}$.
D.C. power absorbed in load	P_{dc}	$= I_{bo}^2 R_L = E_{Lo} I_{bo}$.
Quiescent plate dissipation	P_{po}	$= E_{bo} I_{bo}$.
But	E_{bb}	$= E_{bo} + E_{Lo}$.
Therefore	P_{bb}	$= E_{bo} I_{bo} + E_{Lo} I_{bo}$.
		$= P_{po} + P_{dc}$. (24)

Signal Condition :

Average value of total input $= P_{bb} = E_{bb} I_{bo}$ (25)
which is constant irrespective of the signal voltage.

*After book by M.I.T. Staff "Applied Electronics" (John Wiley & Sons Inc. New York, 1943) pp. 419-425.

Power absorbed by load $= P_L$

$$P_L = \frac{1}{2\pi}\int_o^{2\pi} i_b^2 R_L\, d(\omega t) \tag{26}$$

$$= \frac{1}{2\pi}\int_o^{2\pi} (I_{bo} + i_p)^2 R_L d(\omega t)$$

$$= \frac{1}{2\pi}\int_o^{2\pi} I_{bo}^2 R_L d(\omega t) + \frac{1}{2\pi}\int_o^{2\pi} 2I_{bo} i_p R_L d(\omega t) + \frac{1}{2\pi}\int_o^{2\pi} i_p^2 R_L d(\omega t) \tag{27}$$

$$= I_{bo}^2 R_L + 0 + P_{ac} \tag{28}$$

i.e. $P_L = P_{dc} + P_{ac}$

$$\text{where } P_{ac} = \frac{1}{2\pi}\int_o^{2\pi} i_p^2 R_L d(\omega t) \tag{29}$$

$$\text{Plate dissipation } P_p = \frac{1}{2\pi}\int_o^{2\pi} e_b i_b d(\omega t) \tag{30}$$

From eqns. (28) and (29) it will be seen that the power absorbed by the load increases when the signal voltage increases, but the power input remains steady; the plate dissipation therefore decreases as the power output increases,

i.e. $P_p = P_{bb} - P_L$ (31)

from (26), (28) $= E_{bb} I_{bo} - E_{Lo} I_{bo} - P_{ac}$ (32)

$= E_{bo} I_{bo} - P_{ac}$

$= P_{po} - P_{ac}$ (33)

where $P_{po} = E_{bo} I_{bo}$.

That is to say, the plate dissipation (P_p) is equal to the apparent d.c. power input to the valve (P_{po}) minus the a.c. power output.

$$\text{The plate efficiency } \eta_p = \frac{\text{power output}}{\text{d.c. power input}}$$

$$= \frac{P_{ac}}{E_{bo} I_{bo}}. \tag{34}$$

$$\left[\text{For non-linear operation} \quad \eta_p = \frac{P_{ac}}{E_b I_b}\right] \tag{35}$$

With sinusoidal grid excitation, linear Class A_1 valve operation and resistive load,

$$P_{ac} = E_p I_p = I_p^2 R_L.$$

Applying equation (16),

$$P_{ac} = \frac{\mu^2 E_g^2 R_L}{(r_p + R_L)^2}. \tag{36}$$

Differentiating with respect to R_L and equating to zero in order to find the condition for maximum power output,

$$\frac{dP_{ac}}{dR_L} = \mu^2 E_g^2 \left[\frac{(r_p + R_L)^2 - 2R_L(r_p + R_L)}{(r_p + R_L)^4}\right] = 0$$

i.e. when $(r_p + R_L)^2 - 2R_L(r_p + R_L) = 0$ (37)

or when $R_L = r_p$

and the maximum power output is

$$P_{acm} = \frac{\mu^2 E_g^{\ 2}}{4r_p} = \frac{E_g^{\ 2}}{4} \mu g_m. \tag{38}$$

The factor μg_m is a figure of merit for power triodes.

If the load is an impedance ($Z_L = R_L + jX_L$) the condition for maximum power output is when

$$R_L = \sqrt{r_p^{\ 2} + X_L^{\ 2}}. \tag{39}$$

In the general case, with a resistive load, the power output is given by eqn. (36) which may be put into the form

$$P_{ac} = \frac{\mu^2 E_g^{\ 2}}{r_p} \cdot \frac{1}{\dfrac{r_p}{R_L} + 2 + \dfrac{R_L}{r_p}} \tag{40}$$

If $R_L/r_p = 2$, the loss of power below the maximum is only 11%, while if $R_L/r_p = 4$ the loss of power is 36%, so that "matching" of the load is not at all critical.

The treatment above is correct for both triodes and pentodes provided that both are operated completely within the linear region, that is with limited grid swing. A pentode is normally operated with a load resistance much less than the plate resistance on account of the flattening of the output voltage characteristic which would otherwise occur at low plate voltages.

This subject is considered further in Chapter 13, under practical instead of under ideal conditions.

(iv) Series expansion ; resistance load

Except in eqn. (1), which is perfectly general, certain assumptions have been made regarding linearity and the constancy of μ, g_m and r_p which restrict the use of the equations. If it is desired to consider the effects of non-linearity in causing distortion in amplifiers and in producing detection or demodulation, it is necessary to adopt a different approach.

The varying component of the plate current of a valve may be expressed in the form of a series expansion:

$$i_p = a_1 e + a_2 e^2 + a_3 e^3 + a_4 e^4 + \ldots \tag{41}$$

This form may be derived* from eqn. (1), and it may be shown that

$$a_1 = \frac{\mu}{r_p + R_L} \tag{42}$$

$$a_2 = -\frac{\mu^2 r_p}{2(r_p + R_L)^3} \cdot \frac{\partial r_p}{\partial e_b} \tag{43}$$

$$a_3 = \frac{\mu^3 r_p}{6(r_p + R_L)^5} \left[(2r_p - R_L)\left(\frac{\partial r_p}{\partial e_b}\right)^2 - r_p (r_p + R_L) \frac{\partial^2 r_p}{\partial e_b^{\ 2}} \right] \tag{44}$$

If μ is assumed to be constant (this is only approximately true for triodes and not for pentodes)

$$e = e_g + \frac{v_p}{\mu}$$

where v_p = instantaneous value of plate excitation voltage. (Normally for an amplifier $v_p = 0$ and $e = e_g$).

The value of $\partial r_p/\partial e_b$ may be determined by plotting a curve of r_p versus e_b for the given operating bias, and drawing a tangent at the point of operating plate voltage. The value of $\partial^2 r_p/\partial e_b^{\ 2}$ may be determined by plotting a curve of ∂r_p versus ∂e_b and treating in a similar manner.

The higher terms in the series expansion (41) diminish in value fairly rapidly, so that a reasonably high accuracy is obtained with three terms if the valve is being used as an amplifier under normal conditions with low distortion.

*Reich, H. J. "Theory and Applications of Electron Tubes" (2nd edit.). McGraw-Hill, New York and London, 1944), pp. 74-77.

The first term $a_1e = \mu e/(r_p + R_L)$ is similar to eqn. (16) above, which was regarded as approximately correct for small voltage inputs ; that is to say for negligible distortion.

The first and second terms

$$i_p = a_1e + a_2e^2$$

express the plate current of a " square law detector " which is closely approached by a triode operating as a grid or plate (" anode-bend ") detector with limited excitation voltage.

The second and higher terms are associated with the production of components of alternating plate current having frequencies differing from that of the applied signal—i.e. harmonics and (if more than one signal frequency is applied) intermodulation frequencies.

For example, with a single frequency input,

$$e = E_m \sin \omega t \tag{45}$$

Therefore

$$e^2 = E_m{}^2 \sin^2 \omega t = \tfrac{1}{2}E_m{}^2 - \tfrac{1}{2}E_m{}^2 \cos 2\omega t \tag{46}$$

and

$$e^3 = \tfrac{3}{4}E_m{}^3 \sin \omega t - \tfrac{1}{4}E_m{}^3 \sin 3\omega t. \tag{47}$$

The second term (e^2) includes a d.c. component ($\tfrac{1}{2}E_m{}^2$) and a second harmonic component. The third term includes a fundamental frequency component ($\tfrac{3}{4}E_m{}^3 \sin \omega t$) and a third harmonic component.

If the input voltage contains two frequencies (f_1 and f_2) it may be shown that the second term of eqn. (41) produces

a d.c. component
a fundamental f_1 component
a second harmonic of f_1
a fundamental f_2 component
a second harmonic of f_2
a difference frequency component $(f_1 - f_2)$
a sum frequency component $(f_1 + f_2)$

The third term of equation (41) produces

a fundamental f_1 component
a third harmonic of f_1
a fundamental f_2 component
a third harmonic of f_2
a difference frequency component $(2f_1 - f_2)$
a difference frequency component $(2f_2 - f_1)$
a sum frequency component $(2f_1 + f_2)$
a sum frequency component $(2f_2 + f_1)$

In the case of an A-M mixer valve, f_2 may be the signal frequency and f_1 the oscillator frequency. The normal i-f output frequency is $(f_1 - f_2)$ while there are spurious output frequencies of $(f_1 + f_2)$, $(2f_1 + f_2)$, $(2f_2 + f_1)$, $(2f_1 - f_2)$ and $(2f_2 - f_1)$. Even though no oscillator harmonics are injected into the mixer, components with frequencies $(2f_1 + f_2)$ and $(2f_1 - f_2)$ are present in the output, thus demonstrating mixing at a harmonic of the oscillator frequency.

If the input voltage contains more than two frequencies, or if the terms higher than the third are appreciable, there will be greater numbers of frequencies in the output. The effect of this on distortion is treated in Chapter 14.

It may be shown that the effect of the load resistance, particularly when it is greater than the plate resistance, is to decrease the ratio of the harmonics and of the intermodulation components to the fundamental. This confirms the graphical treatment in Chapter 12.

(v) Series expansion : general case

The more general case of a series expansion for an impedance load and variable μ has been developed by Llewellyn* and the most important results are given in most text books.†

(vi) The equivalent plate circuit theorem

It was shown above (eqn. 41) that the plate current may be expressed in the form of a series expansion. If the distortion is very low, as may be achieved with low input voltage and high load impedance, sufficient accuracy may be obtained by making use of only the first term in the equation, i.e.

$$i_p = \frac{\mu e}{r_p + Z_L} \tag{48}$$

where $e = e_g$ (for amplifier use)
and Z_L = impedance of the plate load at the frequency of the applied voltage.
For amplifier use this may be put into the form

$$I_p = \frac{\mu E_g}{r_p + Z_L} \tag{49}$$

This is the same as eqn. (16), except that R_L has been replaced by Z_L.

This is the basis of the Equivalent Plate Circuit Theorem which states‡ that **the a.c. components of the currents and voltages in the plate (load) circuit of a valve** may be determined from an equivalent plate circuit in one of two forms—

(1) a fictitious constant-voltage generator (μE_g) in series with the plate resistance of the valve, or
(2) a fictitious constant-current generator ($I = g_m E_g$) in parallel with the plate resistance of the valve.

These are applied in Sect. 7 of this chapter.

If a distortionless Class A amplifier or its equivalent circuit is excited with an alternating grid voltage, the a.c. power in the load resistor R_L (i.e. the output power) is $I_p^2 R_L$.

The d.c. input from the plate supply to the valve and load (in the actual case) is $P_{bb} = I_{bo} E_{bb}$. Under ideal Class A_1 conditions the d.c. current I_{bo} remains constant, since the a.c. current is symmetrical and has no d.c. component.

Now the a.c. power input from the generator is

$$P_g = \mu E_g I_p \tag{50}$$

But
$$I_p = \frac{\mu E_g}{(r_p + R_L)}$$

Therefore
$$\mu E_g = r_p I_p + R_L I_p$$

and
$$P_g = \mu E_g I_p = r_p I_p^2 + R_L I_p^2. \tag{51}$$

In this equation

P_g = a.c. power input from generator
$r_p I_p^2$ = a.c. power heating plate
$R_L I_p^2$ = a.c. power output = P_{ac}.

The a.c. power P_g can only come from the d.c. power P_{po} dissipated in the valve, which decreases to the lower value P_p when the grid is excited.

The total plate dissipation (P_p) is therefore

$$P_p = P_{po} - P_g + r_p I_p^2 \tag{52}$$

where P_{po} = d.c. plate dissipation.

This may be put into the form

$$P_p = P_{po} - (P_g - r_p I_p^2)$$
$$= P_{po} - P_{ac} \tag{53}$$

where $P_{ac} = R_L I_p^2 = (P_g - r_p I_p^2)$ = a.c. power output.

*Llewellyn, F. B. Bell System Technical Journal, 5 (1926) 433.
†such as Reich, H. J. "Theory and Application of Electron Tubes," p. 75.
‡for a completely general definition see Reich, H. J. (letter) "The equivalent plate circuit theorem," Proc. I.R.E. 33.2 (Feb., 1945) 136.

The statement may therefore be made, that the plate dissipation is equal to the d.c. plate dissipation minus the a.c. power output.

A more general statement covering all types of valve amplifiers and oscillators is that the plate input power is equal to the plate dissipation plus the power output.

This analysis, based on the equivalent plate circuit, reaches a conclusion in eqn. (46) which is identical with eqn. (33) derived from a direct mathematical approach. It is, however, helpful in clarifying the conditions of operation of a distortionless Class A_1 amplifier.

The preceding treatment only applies to amplifiers ($e = e_g$ in eqn. 41), but it may be extended to cover cases where the load impedance contains other e.m.f's, by using the principle of superposition—see Chapter **4** Sect. 7(viii).

It is possible to adopt a somewhat similar procedure to develop the **Equivalent Grid Circuit,** or that for any other electrode in a multi-electrode valve.

(vii) Dynamic load line—general case

If the a.c. plate current is sinusoidal,

$$i_p = I_{pm} \sin \omega t$$

and $$e_p = -I_{pm}|Z_L| \sin(\omega t + \theta)$$

where $$\theta = \tan^{-1} X_L/R_L.$$

From this it is possible to derive*

$$e_p^2 + 2e_p i_p R_L + i_p^2|Z_L^2| = I_{pm}^2 X_L^2 \qquad (54)$$

which is the equation of an ellipse with its centre at the operating point, this being the dynamic path of operation.

(viii) Valve networks ; general case

The ordinary treatment of a valve and its circuit—the Equivalent Plate Circuit Theorem in particular—is a fairly satisfactory approximation for triodes or even pentodes up to frequencies at which transit-time effects become appreciable. If it is desired to calculate, to a higher degree of precision, the operation of a valve in a circuit, particularly at high frequencies, a very satisfactory approach is the preparation of an equivalent network which takes into account all the known characteristics. This method has been described† in considerable detail, and those who are interested are referred to the original article.

(ix) Valve coefficients as partial differentials

Valve coefficients, as well as other allied characteristics, may be expressed as partial differential coefficients—see Chapter 6 Sect. 7(ii).

Partial differential coefficients, designated in the form $\frac{\partial y}{\partial x}$ are used in considering the relationship between two of the variables in systems of three variables, when the third is held constant.

" $\frac{\partial y}{\partial x}$ " is equivalent to " $\frac{dy}{dx}$ (z constant) " when there are three variables, x, y and z. Partial differentials are therefore particularly valuable in representing valve coefficients.

Let e_p = a.c. component of plate voltage,
e_g = a.c. component of grid voltage,
and i_p = a.c. component of plate current.

(These may also be used with screen-grid or pentode valves provided that the screen voltage is maintained constant, and is completely by-passed for a.c.).

*Reich, H. J. " Theory and application of electron tubes " (2nd edit.), p. 99.
†Llewellyn, F. B. and L. C. Peterson " Vacuum tube networks," Proc. I.R.E. 32.3 (March, 1944), 144.

Then $\mu = -\frac{\partial e_p}{\partial e_g}$ (i_p = constant), (55)

or more completely $+\frac{\partial i_p/\partial e_g}{\partial i_p/\partial e_p}$,

$g_m = +\frac{\partial i_p}{\partial e_g}$ (e_p = constant). (56)

$r_p = +\frac{\partial e_p}{\partial i_p}$ (e_g = constant), (57)

or more correctly* $+\frac{1}{\partial i_p/\partial e_p}$

In a corresponding manner the gain (A) and load resistance (R_L) of a resistance-loaded amplifier may be given in the form of total differentials—

$$\left|A\right| = \left|\frac{de_p}{de_g}\right|\dagger \quad (58)$$

and

$$R_L = -\frac{de_p}{di_p}. \quad (59)$$

Particular care should be taken with the signs in all cases, since otherwise serious errors may be introduced in certain calculations.

(x) Valve characteristics at low plate currents

In the case of diodes and diode-connected triodes at very low plate currents (from 1 to about 100 microamperes) an increment of plate voltage of about 0·21 volt produces a 10-fold increase of plate current. If the $\log_{10}$ of current is plotted against plate voltage, the result should approximate to a straight line with a slope of 1/0·21.

In the case of triodes operating as triodes the relationship of plate current to grid-cathode voltage is still approximately logarithmic, up to a value of plate current which varies from type to type, but the slope of the curve is decreased by the plate-grid voltage. The decrease in slope is approximately proportional to the grid bias and therefore to $1/\mu$ times the plate voltage. The curve at a given plate voltage is, in general, steeper for a high-mu than for a low-mu triode. Over the region in which the logarithmic relationship holds, the mutual conductance is proportional to the plate current. For a given plate voltage and plate current, the g_m in the low-current region is greater for high-mu than for low-mu triodes, regardless of ratings. Also, for a given triode at a given plate current, g_m is greater than at lower plate voltages. Maximum voltage gain in a d-c amplifier is obtained if the valve is operated at as low a plate voltage as possible, and at a plate current corresponding to the top of the straight portion of the characteristic when $\log_{10}$ of current is plotted against the grid voltage.

With pentodes at low plate currents, the maximum gain is obtained when the screen voltage is as low as is permissible without resulting in the flow of positive grid current.

Reference A12 pp. 414-418.

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*The simple inversion of partial differentials cannot always be justified.

†A is a complex quantity which represents not only the numerical value of the stage gain but also the phase angle between the input and output voltages. The vertical bars situated one on each side of A and its equivalent indicate that the numerical value only is being considered.

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