RESEARCH MEMO NO. 386

SIGNAL-TO-NOISE RATIO IN IMAGE DISSECTORS

Prepared by

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The comparatively noise-free current amplification properties of the electron multiplier used in image dissectors makes it possible to observe directly in the output circuit the random shot noise present in the photocathode emission current. This shot noise obeys the familiar "white" noise law:

$$i^2_{nk} = 2e\, I_k\, \Delta f$$

where

i_{nk} = rms noise current (amps) at the photocathode

e = electronic charge $= 1.6 \times 10^{-19}$ coulombs

I_k = d-c current (amps) at the cathode corresponding to i_{nk}

Δf = bandwidth (cps)

Following current amplification by a factor, u, in the electron multiplier, such that:

i_{na} = rms anode noise current (amps) $= u i_{nk}$

and

I_a = d-c anode current (amps) $= u\, I_k$

the above relationship reduces to:

$$i^2_{na} = 2\, e\, u\, I_a\, \Delta f\, \sigma/(\sigma - 1)$$

where σ = effective gain/stage in the early stages of the electron multiplier. (The above considerations are discussed in more detail in Research Memo No. 337: "Noise in Image Dissector Tubes", and in Research Memo No. 309, "Noise in Multiplier Phototubes".)

The d-c anode current, I_a, as well as the corresponding rms anode noise current, i_{na}, can be a result not only of photoemission but also of thermionic emission and other dark current sources from the photocathode. However, for the purposes of this memo all dark current sources other than photoemissive will be disregarded since, as shown in Appendix B, the photocurrents necessary for reasonable signal-to-noise performance in tv-type operation completely surpass any possible dark current in presently available photocathodes. Under these conditions the d-c current, I_a, can be considered as a signal current and a signal current-to-rms noise current ratio, $(S/N)_{rms}$ can be defined as:

$$(S/N)_{rms} = I_a/i_{na} = \sqrt{\frac{I_a(\sigma - 1)}{2\, e\, u\, \sigma\, \Delta f}} = \sqrt{\frac{I_k(\sigma - 1)}{2\, e\, \sigma\, \Delta f}}$$

This equation can be reduced to a more useful approximate form by making the following simplifications.

First, the cathode current, I_k, is given by:

$$I_k = J_k\, a/m^2$$

where
- a = dissector defining aperture area (cm^2)
- J_k = d-c cathode current density ($amps/cm^2$)
- m = linear magnification, cathode-to-defining aperture.

Secondly, the bandwidth, Δf, will be assumed to be no greater than absolutely necessary to "see" a resolution element during scan. This is equivalent to "allowing" one-half cycle i.e. a half wavelength, per aperture dwell time, Δt, or:

$$\Delta t \cong \frac{1}{2\,\Delta t}$$

The "dwell time", Δf, is essentially the time permitted to remain on one element if a continuous scan were to be replaced by a step scan in which the scan is jumped instantaneously between contiguous resolution elements. Possible overlap considerations, as in the case of round elements, as well as aperture correction techniques are ignored herein since the above relationship in any case is only approximate.

Thirdly, the rms noise current used above is not a good measure of visible noise in an image presentation, a more useful noise magnitude being the "peak-to-peak" value.

While "peak-to peak" measurements are basically meaningless with random noise of the type under consideration,[1] nevertheless, in practice, an approximate peak-to-peak amplitude i_n (peak-to-peak) can be estimated visually from an oscilloscope presentation. Experimental correlations made in our lab, as well as theoretical noise considerations,[2] lead to the following approximation:

$$i_n \text{ (peak-to-peak)} \cong 7 \ i_n \text{ (rms)}$$

With the above three simplifications, the effective signal current-to noise current ratio, $(S/N)_{eff}$, becomes

$$(S/N)_{eff} \cong \frac{1}{7} \sqrt{\frac{J_k \, a \, \Delta t \, (\sigma - 1)}{m^2 \, a \, \sigma}}$$

For a given dissector design the effective signal-to-noise current ratio is proportional to

$$(S/N)_{eff} \sim \sqrt{J_k \, a \, \Delta t} = \sqrt{\text{cathode current density x aperture area x dwell time}}$$

Figure 1 shows a plot of $(S/N)_{eff}$ vs J_k for various values of the parameter "$a \Delta t$". The two dotted lines on this graph show the calculated characteristics of (1) an image dissector with a 3-inch diameter photocathode and a 5-mil aperture operated at standard tv scan rates (giving 360 tv-line resolution for a theoretical 100 percent modulation factor), and (2) an image dissector with a 1-inch diameter cathode, a 1-mil aperture, and an 8-cps horizontal readout rate (giving 600 tv-line resolution). The loss of S/N ratio expected in going from 5 mils to 1 mil is more than compensated for by the much slower scan rates in the second dissector, the end result being much better S/N ratio.

The first dissector is only marginal in performance at standard tv rates, the S/N ratio at the minimum safe cathode current density of 1 $\mu a/cm^2$ being only 1.3. This corresponds well with experimental evidence. The S/N performance can be and has been improved by increasing the cathode illumination and thus the current density, J_k, but this usually is possible only at the expense of shortened tube life due to irreversible photocathode fatigue.

Unfortunately, definitive data on permissible cathode emission current densities is not yet available experimentally. It varies from tube to tube, with the permissible fatigue permitted during tube life, and with the type of photocathode.

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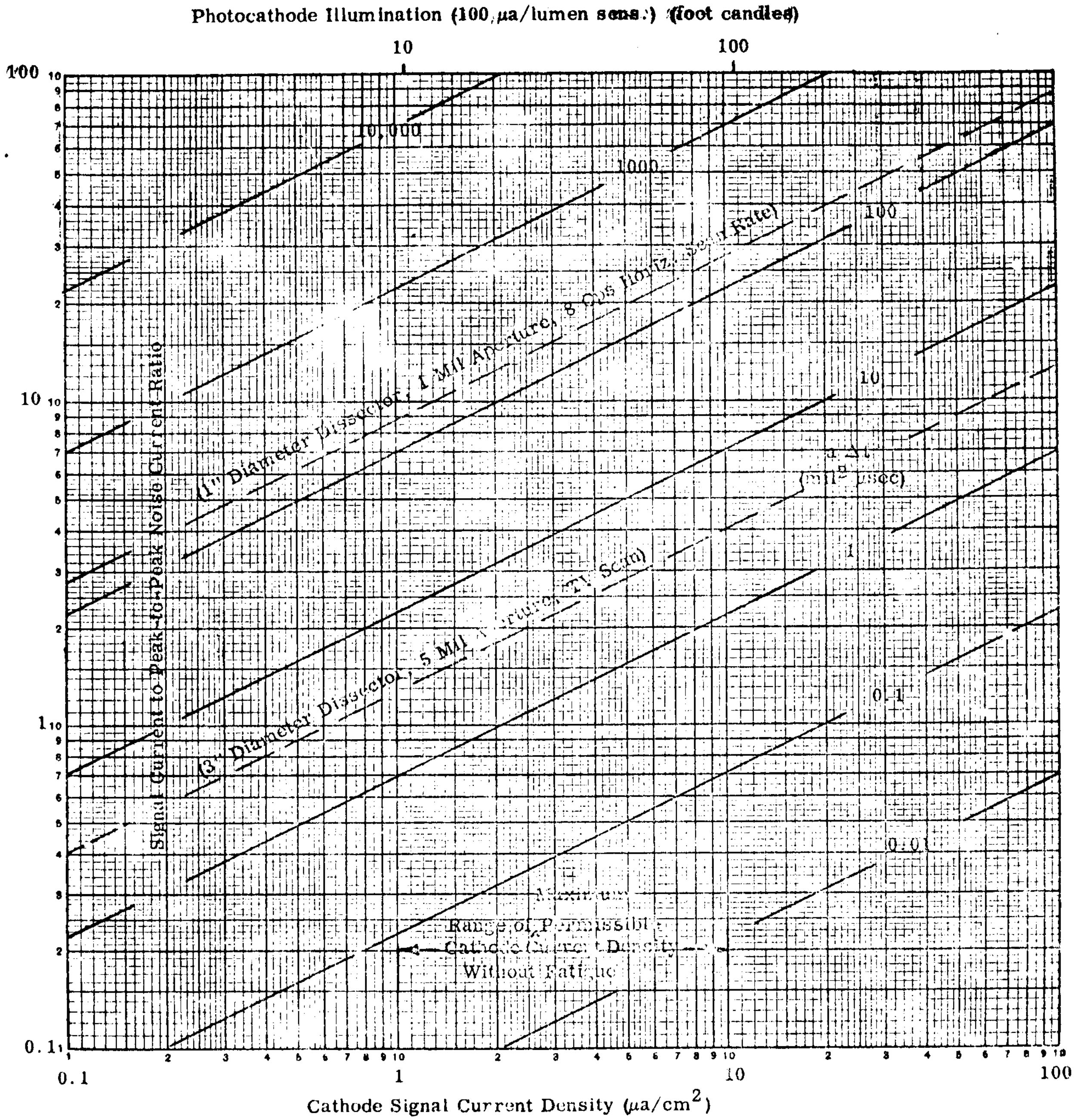
1 Noise pulses of any given magnitude always have a finite probability of occurring.
2 See, for example, Bell System Technical Journal, Volume 24, p 74.



Assumptions:
i (noise, peak-to-peak) ≅ 7 i (noise, rms), SE ratio ≅ 2.6, I(dark) << ((signal), $\Delta f \cong \frac{1}{2\Delta t}$

Figure 1 Calculated Image Dissector Performance Characteristics

The above relationships show that the S/N ratio is not a function of cathode sensitivity or quantum efficiency provided sufficient input flux density can be provided to supply a fixed emission current density, J_k. This is quite contrary to the usual threshold type considerations where the input flux is limited in magnitude and the S/N ratio is therefore proportional to the square root of the cathode sensitivity.

Figure 2 shows a conversion type plot for computing the performance parameter "a Δt" from known system parameters. The basis for this figure is given in Appendix C. The two specific image dissector examples shown in Figure 1 are also shown in Figure 2.

In summary, the S/N performance of image dissectors limits them to applications in which high illumination levels[3] and/or slow scan rates occur. They do have the following advantageous characteristics:

a. high resolution, approaching 1000's of tv lines[4]

b. excellent linearity over a wide dynamic range, the output current being directly proportional to the input flux over many orders of magnitude.

c. variable scan rates, including stopped scan (d-c read-out), random access, etc.

d. operable with rapid camera panning

e. simplicity of design, with no electron gun

f. rugged construction principles, suitable to special environments

g. wide range of permissible operating temperatures, limited only by photocathode damage of high temperatures

h. fast response, of the order of nanoseconds

i. the ability to see random shot noise of the input radiation image, i.e. "background noise limited" operation.

j. readily calculable signal-to-noise characteristics, permitting accurate system pre-evaluation studies.

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3 A specialized technique for providing very high illumination levels without exceeding the average cathode current density limits has been described by Dr. G. Papp in ITTIL Research Communication No. 36 "On a Novel Application of the Image Dissector".

4 The limiting resolution of image dissectors has been treated by Dr. G. Papp in detail (IRE Trans. on Nuclear Science Vol NS-9 No. 2 p. 91-93, April 1962).

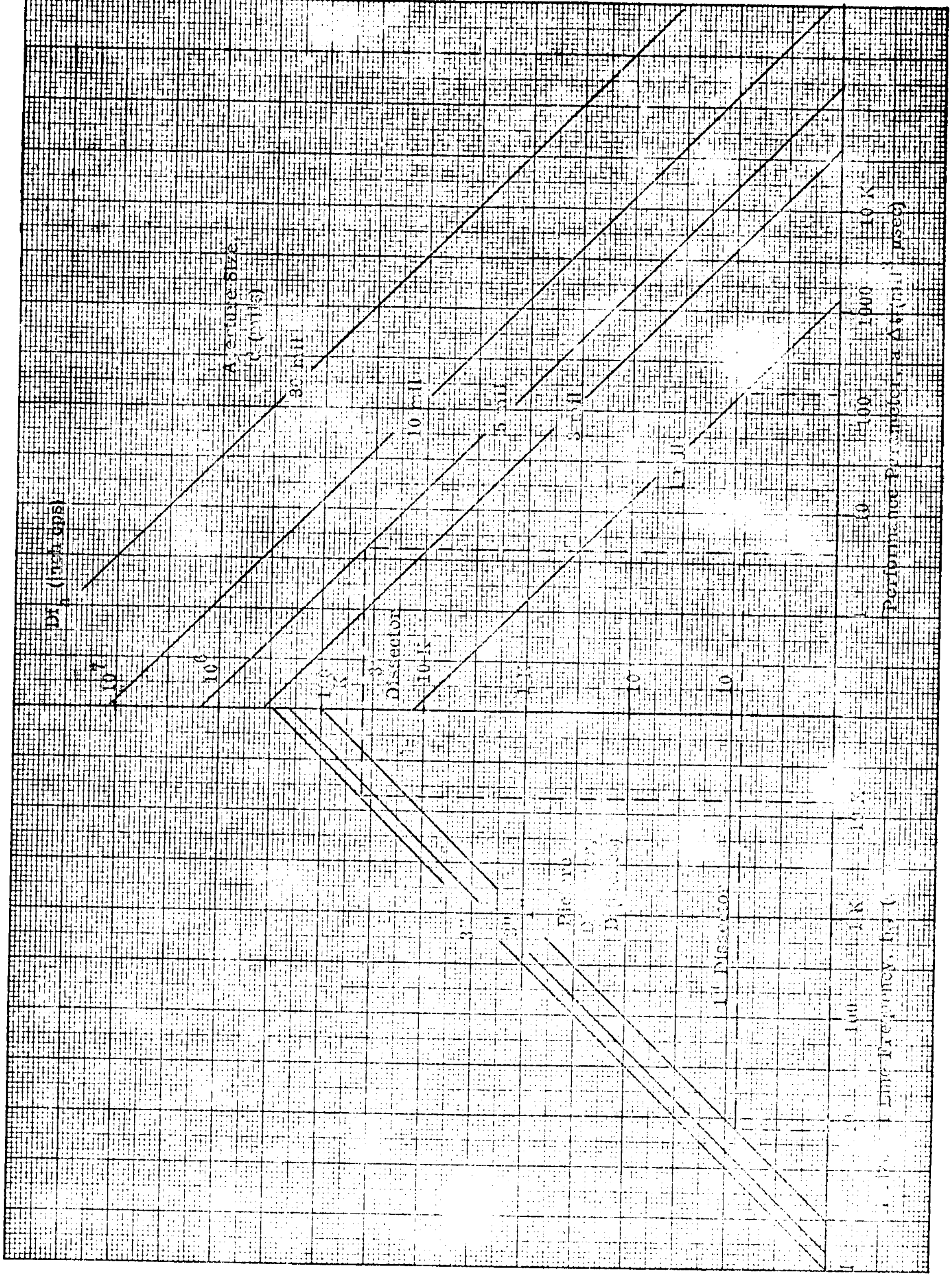

Figure 2 Dissector Dwell Time Characteristics

APPENDIX A

NUMERICAL CALCULATIONS

In plotting Figure 1 the following numerical data was used:

m = image magnification = 1

e = charge on the electron = 1.6×10^{-19} coulombs

σ = gain/stage = 2.6 (a typical value)

$J_k\ (\text{amps/cm}^2) = J_k\ (\mu\text{a/cm}^2) \times 10^{-6}$

$a\ (\text{cm}^2) = a\ (\text{mil}^2) \times 10^{-6} \times (2.54)^2$

$\Delta t\ (\text{sec}) = \Delta t\ (\mu\text{sec}) \times 10^{-6}$

giving

$$(S/N)_{eff} \cong \frac{1}{7}\sqrt{\frac{J_k\ (\mu a/cm^2) \times 10^{-6} \times a\ (mil^2) \times 10^{-6} \times (2.54)^2 \times \Delta t\ (\mu sec) \times 10^{-6} \times 1.6}{(1) \times 1.6 \times 10^{-19} \times 2.6}}$$

$$\cong 0.7\sqrt{J_k\ (\mu a/cm^2)\ a\ (mil^2)\ \Delta t\ (\mu sec)}$$

This is the equation plotted in Figure 1.

Example 1:

For an image dissector with a 5-mil aperture, 3-inch cathode diameter, and operated at standard tv rates.

a = 25 mil^2 (assuming a square aperture)

Horizontal line length $= (4/5) \times 3'' = 2.4'' = \dfrac{2.4}{0.0005} = 480$ apertures/line

Horizontal line time $= \dfrac{1}{15{,}500}$ sec $= 64.5\ \mu$sec (neglecting retrace time)

$$\Delta t = \frac{64.5}{480} = 0.134 \ \mu sec$$

$$a\Delta t = (25)(0.134) = 3.36 \ mil^2 \ \mu sec$$

$$(S/N)_{eff} = 1.28 \sqrt{J_k(\mu a/cm^2)}$$

Example 2:

For an image dissector with 1 mil aperture, 1-inch photocathode diameter, and 8-cps horizontal scan rate, the corresponding numerical data is:

$$a = 1 \ mil^2 \quad \text{(assuming square aperture)}$$

Horizontal line length $= 4.5 \times 1'' = 0.8'' = \frac{0.8}{0.001} = 800$ apertures/line

Horizontal line time $= 1/8 = 0.125$ sec $= 125,000 \ \mu sec$

$$\Delta t = \frac{125,000}{800} = 156 \ \mu sec$$

$$a\Delta t = 156 \ mil^2 \ \mu sec$$

$$(S/N)_{eff} \cong 8.7 \sqrt{J_k(\mu a/cm^2)}$$

APPENDIX B

DARK CURRENT MAGNITUDES

An examination of Figure 1 shows that relatively large cathode current densities (above at least 0.1 $\mu a/cm^2$) are required if usable signal-to-noise ratios are to be achieved with image dissectors at "ordinary" raster scan rates. These required current densities are many orders of magnitude greater than dark current densities experienced even with S-1 photocathodes (whose maximum dark current density is seldom greater than 10^{-5} $\mu a/cm^2$). Thus photocathode dark current and dark current noise is never significant in ordinary dissector applications.[5] The limiting tube noise is therefore noise in signal under all ordinary conditions, i.e. random fluctuations of the signal itself.

If, however, large apertures and/or very slow scan rates occur, as in star tracking applications using such simplified tubes as the FW118, FW129, and FW130, then dark noise can be and is, in fact, usually encountered.

Photocathode thermionic emission dark current and dark noise can be included directly in the above computations if so desired, by merely replacing I_k in Equation 1 by a combined term, $I_{thermionic} + I_{photoemission}$, and carrying this sum, or its equivalent, throughout the remaining performance equations.

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5 Of course, other types of dark current and dark noise may occur in some cases, such as amplifier noise, leakage, pickup, etc.

APPENDIX C

DWELL TIME CONSIDERATIONS

The dwell time, Δt, and the product, $a\Delta t$, have interesting and rather complex physical significance. Dimensionally, $a\,\Delta t$ is in cm^2 sec or the equivalent. Physically it is equal to the amount of charge gathered per one element sample per unit cathode current density.

Under many conditions, when the aperture area, a, is decreased, the dwell time, Δt, must also be decreased (in order to allow time to scan the larger number of picture elements). For example, if the scan rates are fixed and the aperture diameter is reduced to 1/10 of its original size, the dwell time is also down by 1/10, "a" is down by 1/100, "$a\Delta t$" is reduced to 1/1000 of its original amplitude, and the S/N ratio is down by 1/31.4. This is a greater loss of performance than is directly apparent from the performance equations. Furthermore, with the smaller aperture, the number of horizontal lines should be increased (by 10 x in the above case) to make full use of the improved resolution. If this were done the end result for a fixed frame time would be a signal-to-noise current ratio down to 1/100 of its original value.

For the usual tv type application, with a 4 to 3 raster geometry inserted within a picture (photocathode) diameter, D, a horizontal line frequency, f_n, and a square aperture of side, d, the product, $a\Delta t$, is given by

$$a\,\Delta t = \frac{5d^3}{4Df_h} \quad (\text{cm}^2\ \text{sec})$$

$$a\,\Delta t\ (\text{mil}^2\ \mu\text{sec}) = \frac{5 \times 10^{-3}}{4} \quad \frac{d^3\ (\text{mil}^3)}{D(\text{in})\ f_h\ (\text{mc})}$$

This is the relationship used in plotting Figure 2.